Old sections 1.6 (e) and (f) PhL 10.3.14

(e) A Pitfall to Avoid

Notice that

$$\mathbf{E}(\mathbf{x},t) = \mathbf{e}(\mathbf{x},t) + j\,\mathbf{e}'(\mathbf{x},t) \quad \Rightarrow$$
$$\mathbf{E}\hat{}(\mathbf{x},\omega) = \int_{-\infty}^{\infty} dt\, \mathbf{E}(\mathbf{x},t)\, e^{-j\omega t} = \int_{-\infty}^{\infty} dt\, [\mathbf{e}(\mathbf{x},t) + j\,\mathbf{e}'(\mathbf{x},t)]\, e^{-j\omega t}$$

$$= \mathbf{e}\hat{}(\mathbf{x},\omega) + j\,\mathbf{e}'\hat{}(\mathbf{x},\omega)\,. \qquad (1.6.17)$$

Whereas $\mathbf{e}(\mathbf{x},t)$ and $\mathbf{e}'(\mathbf{x},t)$ are the real and imaginary parts of $\mathbf{E}(\mathbf{x},t)$, the functions $\mathbf{e}\hat{}(\mathbf{x},\omega)$ and $\mathbf{e}'\hat{}(\mathbf{x},\omega)$ are *not* the real and imaginary parts of $\mathbf{E}\hat{}(\mathbf{x},\omega)$ since in general $\mathbf{e}\hat{}(\mathbf{x},\omega)$ and $\mathbf{e}'\hat{}(\mathbf{x},\omega)$ are both complex functions. In this document we shall never deal with transforms of the type $\mathbf{e}\hat{}(\mathbf{x},\omega)$ or $\mathbf{e}'\hat{}\,(\mathbf{x},\omega)$.

(f) Overloaded Notation and Maxwell's Equations in ω space

In this section we have carefully denoted the Fourier Transform of f(t) as f^(ω) which is a notation used by Stakgold and others (though Stakgold has our (1.6.8) phases negated as in his equation (5.32)). In the rest of this document, however, we represent the Fourier Transform of f(t) as f(ω) to avoid a proliferation of hat ^ symbols. Since the functions f(t) and f(ω) are completely different functions, the symbol f is "overloaded" (in the sense of overloaded variable names in computer languages) and we trust the reader to understand that f(ω) always means f^(ω). It is the presence of the argument ω that cues the reader to this fact. This overloaded notation has already been used in Section 1.5 and we continue it right here:

In the Maxwell and related equations which include the ∂_t operator, if the fields are expanded onto their Fourier transformed components using (1.6.8b), then using the rule (1.6.9) one may instantly write the frequency-domain version of these equations, just as in the example of Section 1.5. For example,

$$\text{curl}\ \mathbf{H}(\mathbf{x},\omega) = j\omega\mathbf{D}(\mathbf{x},\omega) + \mathbf{J}(\mathbf{x},\omega) \qquad (1.6.18)$$

$$\text{curl}\ \mathbf{E}(\mathbf{x},\omega) = -j\omega\mathbf{B}(\mathbf{x},\omega) \qquad (1.6.19)$$

$$\text{div}\ \mathbf{J}(\mathbf{x},\omega) = -j\omega\rho(\mathbf{x},\omega)\quad . \qquad (1.6.20)$$

Other equations in the Section 1.1 list have the same form but in terms of the frequency-domain functions. For example,

$$\mathbf{J}(\mathbf{x},\omega) = \sigma(\mathbf{x})\,\mathbf{E}(\mathbf{x},\omega) \qquad (1.6.21)$$

where we momentarily allow $\sigma(\mathbf{x})$ to have spatial dependence but not time dependence.