The applied Ja issue PhL 10.13.13

Comments 12/5/13. I was battling here with two issues. The first is the historical J_a "applied current" bugaboo which I think I have permanently banished from the Kingdom. The second issue had to do with whether β is real or complex. This was all before I had the multiple region concept where we apply a King gauge for region 1 to all regions so then in the end when the dielectric region current is absorbed you get a complex β working in all regions! That thing really was a breakthrough for me. This issue is directly related to the J_a applied current issue. I kept getting a homo wave equation for A with no driving current, so the Helm integral was 0 and I had no King like solution, so I was just *making up* J_a in order to have some driving current. I then had an elaborate iteration scheme based on that. It was horrible.

This matter just seems hazy to me and I want it to be clean and not hazy.

Scenario #1: Suppose I were solving the twin-lead problem where I know J_z will be a bit off center in each conductor. I could start off by assuming that J_z was uniform, and this would be my "applied" value. Using that J_z, I could compute H both inside and outside my one conductor then I could add these H's for both conductors to get a total H and then a total B field. From that I could somehow get the E field, and from that I could somehow get the E_z field inside the conductor, and I would find that it was not uniform, so J_z is then not be uniform. I would compute this new J_z and I could compare it to my starting J_z and I would see a difference. Perhaps then I could start with a modified J_z that was off-center and then redo the whole calculation and in that way **iterate** until I get a final solutions. If the first correction is very small, then my initial ansatz for J_z was reasonable and gives a reasonable result for the fields.

Now in my opening section the original uniform J_z would be called J_a, z. The correction I guess would be called **J**, and then $J_T = \mathbf{J} + \mathbf{J_a}$.

So do I have any evidence of **J** as being a "small correction" in anything I have done?

In the above scenario, one way to compute B would be to compute vector potential A using the special gauge choice shown in 1.3.4, which then gives

$$(\nabla^2 - \mu\varepsilon\,\partial_t^2 - \mu\sigma\,\partial_t)\mathbf{A} = -\mu\mathbf{J_a} \qquad (1.3.5)$$

$$(\nabla^2 - \mu\varepsilon\,\partial_t^2 - \mu\sigma\,\partial_t)\varphi = -(1/\varepsilon)\rho_T \quad . \qquad (1.3.6)$$

where now we see J_a as a "driving" inhomo term. I end up soon with

$$\mathbf{A}(\mathbf{x},\omega) = \frac{\mu}{4\pi}\int \mathbf{J_a}(\mathbf{x}',\omega)\,\frac{e^{-j\beta(\omega)R}}{R}dV' \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad (1.5.9)$$

$$\varphi(\mathbf{x},\omega) = \frac{1}{4\pi\varepsilon}\int \rho_T(\mathbf{x}',\omega)\,\frac{e^{-j\beta(\omega)R}}{R}dV' \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad \text{write as surface integral?} \qquad (1.5.10)$$

So there you are. Use the $\mathbf{J_a}$ and do the integral and you get **A**. But this is only a particular solution, and in theory you can add to this any solution of

$$(\nabla^2 - \mu\varepsilon\,\partial_t^2 - \mu\sigma\,\partial_t)\mathbf{A} = 0$$

What do I know about possible solutions of this homo equation? Let's first go to the ω domain where we then have

$$(\nabla^2 + \beta^2)\,\mathbf{A}(\mathbf{x},\omega) = -\mu\mathbf{J_a}(\mathbf{x},\omega) \tag{1.5.1}$$

$$(\nabla^2 + \beta^2)\,\varphi(\mathbf{x},\omega) = -(1/\varepsilon)\rho_T(\mathbf{x},\omega) \tag{1.5.2}$$

$$\beta^2 \equiv \omega^2\mu\,\xi \tag{1.5.3}$$

$$\xi \equiv \varepsilon - j\sigma/\omega \quad . \tag{1.5.4}$$

Then the question becomes: what do we know about solutions to

$$(\nabla^2 + \beta^2)\,\mathbf{A}(\mathbf{x},\omega) = 0$$

$$(\nabla^2 + \beta^2)\,\varphi(\mathbf{x},\omega) = 0$$

The first is a messy vector Helmholtz equation!

But now go back to

$$\varphi(\mathbf{x},\omega) = \frac{1}{4\pi\varepsilon}\int \rho_T(\mathbf{x}',\omega)\,\frac{e^{-j\beta(\omega)R}}{R}\,dV'$$

I already argue that if this includes ALL charge density, then it is the complete answer and in the Laplace case the only homo adder solution is 0.

Do I know anything about the solution to this problem:

$$(\nabla^2 + \beta^2)\,\varphi(\mathbf{x},\omega) = 0 \qquad \text{and } \varphi(\infty) = 0$$

Well consider instead this equation which agrees with the above almost everywhere

$$(\nabla^2 + \beta^2)\,\varphi(\mathbf{x},\omega) = \delta(\mathbf{r})$$

This gives $\varphi = e^{i\beta r}/4\pi\, r$ and it drops off as in (1.5.8).

But what about generic homo solutions?

$$\varphi = -(\beta^2/2)x^2 \qquad \partial_x\varphi = -\beta^2 x \qquad \partial_x^2\varphi = -\beta^2$$

So in theory you could have this solution floating around, but it blows up at ∞. Perhaps the same kind of thing happens with the vector Helmholtz equation.

Consider this example from 3D electrostatics.

Suppose I take two spheres with charges +1 and -1 which have radius a and are distance b apart. I could start by assuming a uniform σ distribution on each sphere. I could then use that to compute the E field of each sphere and then add these up (field of two point charges!). I could then use ρ = div E to compute the actual σ on those two spheres. It would be non-uniform. But then for that particular E field, I know exactly what surface charge creates it, so I at least have the solution of a certain problem. If the error is small (and it would be if the spheres were far apart), then I have an approximate solution to the problem of the field of two charged spheres.

What is the exact solution to this problem? Have I ever done this? No. I found a paper that gives the exact solution in terms of an infinite series of image charges, fine. I stored it. The problem also appears as a Problem in blue Jackson page 86. The first image charge is sphere center and corresponds to my simple "applied" solution noted above.

I am wondering now if there might be a better notation for things.

1.1 Maxwell's Equations in a Medium

Our working set of equations is the following:

curl $\mathbf{H} = \partial\mathbf{D}/\partial t + \mathbf{J} = \partial\mathbf{D}/\partial t + [\ \Delta\mathbf{J} + \mathbf{J_a}\ ]$	Maxwell curl H equation	(1.1.1)
curl $\mathbf{E} = -\partial\mathbf{B}/\partial t$	Maxwell curl E equation	(1.1.2)
div $\mathbf{D} = \rho = [\ \Delta\rho + \rho_a\ ]$	Maxwell div D equation	(1.1.3)
div $\mathbf{B} = 0$	Maxwell div B equation	(1.1.4)
$\mathbf{B} = \mu\mathbf{H}$	magnetic permeability μ	(1.1.5)
$\mathbf{D} = \varepsilon\mathbf{E}$	electric permeability ε (dielectric constant)	(1.1.6)
$\mathbf{J} = \sigma\mathbf{E}$	Ohm's Law (σ = conductivity)	(1.1.7)
$\mathrm{div}(\mathbf{J}) = -\partial\rho/\partial t$	Equation of Continuity	(1.1.8)

Suppose I do the above where now J and ρ are the "total" quantities and the Δ items are "corrections" in the sense of the above spheres discussion. What implications would this have? Let's cut and paste and edit a few things.

Section 1.1

(e) The symbols $\mathbf{J_a}$ and ρ_a stand for certain "applied" current and charge densities, and in terms of these symbols, the total current and charge density are given by $\mathbf{J} = \Delta\mathbf{J} + \mathbf{J_a}$ and $\rho = \Delta\rho + \rho_a$. These applied current and charge densities only exist in a certain approximation philosophy as outlined below. In the exact world in which Maxwell's equations apply everywhere, one has $\mathbf{J_a} = 0$ and $\rho_a = 0$, and in this case **$\Delta\mathbf{J} = 0$ and** $\Delta\rho = 0$. In the approximation model one regards the applied $\mathbf{J_a}$ (and corresponding ρ_a) as a current density which is "applied to" or "driven into" the conductors of a radiating antenna or a transmission line, as if this could be done outside the realm of Maxwell's equations. From the assumed applied $\mathbf{J_a}$ and ρ_a one computes the fields **E** and **B** using Maxwell's equations and then hopes that these fields in turn produce **J** and ρ on the conductors which are consistent with the assumed applied $\mathbf{J_a}$ and ρ_a. [this seems pretty reasonable in this new notation.]

$$\text{div}\,\mathbf{D} = \rho \quad \Leftrightarrow \quad \int_V \rho\, dV = \int_S \mathbf{D} \bullet d\mathbf{A} \qquad (1.1.13)$$

$$\text{div}\,\mathbf{E} = \rho/\varepsilon \quad \Leftrightarrow \quad (1/\varepsilon)\int_V \rho\, dV = \int_S \mathbf{E} \bullet d\mathbf{A} \quad \text{if } \varepsilon \text{ is constant in space} \qquad (1.1.14)$$

$$\text{div}(\mathbf{J}) = -\partial_t\rho \quad \Leftrightarrow \quad -\partial_t[\int_V \rho\, dV] = \int_S \mathbf{J} \bullet d\mathbf{A} \qquad (1.1.16)$$

$$\text{curl}\,\mathbf{H} = \partial\mathbf{D}/\partial t + \mathbf{J} \quad \Leftrightarrow \quad \oint \mathbf{H} \bullet d\mathbf{s} = \int_S [\partial_t \mathbf{D} + \mathbf{J}] \bullet d\mathbf{A} \qquad (1.1.18)$$

$$(1/\mu)\,\text{curl}\,\mathbf{B} = \varepsilon\,\partial\mathbf{E}/\partial t + \mathbf{J} \quad \Leftrightarrow \quad \oint \mathbf{B} \bullet d\mathbf{s} = \mu \int_S [\varepsilon\,\partial_t \mathbf{E} + \mathbf{J}] \bullet d\mathbf{A} \qquad (1.1.19)$$

These certainly look nicer.

Section 1.2

In my verification doc, I have now rederived (1.2.1) and (1.2.2) with these results (either pair is OK)

$(\nabla^2 - \mu\varepsilon\,\partial_t^2)\mathbf{H} = -\,\text{curl}\,\mathbf{J}$
$(\nabla^2 - \mu\varepsilon\,\partial_t^2)\mathbf{E} = \mu\partial_t\mathbf{J} + (1/\varepsilon)\,\text{grad}\,\rho$ // driven wave equations

$(\nabla^2 - \mu\varepsilon\,\partial_t^2 - \mu\sigma\partial_t)\mathbf{H} = 0$
$(\nabla^2 - \mu\varepsilon\,\partial_t^2 - \mu\sigma\partial_t)\mathbf{E} = (1/\varepsilon)\,\text{grad}\,\rho$ // damped wave equations if $J = \sigma E$

I think I will only put the first form, and I can put in a Jackson reference if I want. But actually I need the second form later, In the ω domain we have

$(\nabla^2 + \mu\varepsilon\,\omega^2)\mathbf{H} = -\,\text{curl}\,\mathbf{J}$
$(\nabla^2 + \mu\varepsilon\,\omega^2)\mathbf{E} = \mu\partial_t\mathbf{J} + (1/\varepsilon)\,\text{grad}\,\rho$ // driven wave equations

$(\nabla^2 + \mu\varepsilon\,\omega^2 - j\mu\sigma\omega)\mathbf{H} = 0$
$(\nabla^2 + \mu\varepsilon\,\omega^2 - j\mu\sigma\omega)\mathbf{E} = (1/\varepsilon)\,\text{grad}\,\rho$ // damped wave equations if $J = \sigma E$

So I might want to define

$\beta_0^2 = \mu\varepsilon\omega^2$

$\beta^2 = \mu\varepsilon\omega^2 \ -j\mu\sigma\omega \ = \mu\omega^2 [\varepsilon \ -j\sigma/\omega] = \mu\omega^2 [\varepsilon +\sigma/j\omega]$ = what I have been using all along

Thus, we can write

$(\nabla^2 + \beta_0^2)\mathbf{H} = -$ curl $\mathbf{J}$
$(\nabla^2 + \beta_0^2)\mathbf{E} = \mu\partial_t\mathbf{J} + (1/\varepsilon)$ grad ρ // driven wave equations

$(\nabla^2 + \beta^2)\mathbf{H} = 0$
$(\nabla^2 + \beta^2)\mathbf{E} = (1/\varepsilon)$ grad ρ // damped wave equations if $J = \sigma E$

inside a conductor this last item will have $\rho = 0$, then it will find use ahead.

Section 1.3

These too I have rederived in the verifications doc, with these results

$(\nabla^2 - \mu\varepsilon\,\partial_t^2)\mathbf{A} = \text{grad}\,[\mu\varepsilon\,\partial_t\varphi + \text{div}A\,] - \mu\mathbf{J}$ (1.3.2)
$\nabla^2\varphi + \partial_t\,(\text{div}\,\mathbf{A}) = -(1/\varepsilon)\rho$ (1.3.3)

Then use of this gauge condition

$\text{div}A = -\mu\varepsilon\,\partial_t\varphi$ (1.3.4)

gives these results again with Jackson refs:

$(\nabla^2 - \mu\varepsilon\,\partial_t^2)\mathbf{A} = -\mu\mathbf{J}$ (1.3.5)
$(\nabla^2 - \mu\varepsilon\,\partial_t^2)\varphi = -(1/\varepsilon)\rho$ (1.3.6)

Relativity note

Cleans up just fine, but have to do all the words!

Section 1.4 retarded potentials

Just remove the T subscripts

Section 1.5

Change example to match the equation!

I see big trouble brewing here now! The equation after the above repair is this:

$$(\nabla^2 - \mu\varepsilon\,\partial_t^2)\varphi = -(1/\varepsilon)\rho$$

There is no damping term, so my whole β^2 discussion will be wrong !!!!! Yikes!! I will just edit right here and see what happens:

$$(\nabla^2 + \mu\varepsilon\omega^2)\,\varphi(\mathbf{x},\omega) = -(1/\varepsilon)\rho(\mathbf{x},\omega)$$

or

$$(\nabla^2 + \beta^2)\,\varphi(\mathbf{x},\omega) = -(1/\varepsilon)\rho(\mathbf{x},\omega)$$

where β^2 is the following complex "Helmholtz parameter" [of operator $(\nabla^2 + \beta^2)$]

$$\beta^2 = \mu\varepsilon\omega^2 = \omega^2/v^2$$

Holy cow, this is going to be a huge change. How are we going to get that $3/4\pi$ special phase in the round wire situation?

Let's hold right here in the middle of Section 1.5

Jump ahead to 2.1 on the round wire. When I get to (2.1.12) I could quote my new

OK, enough goofing around here. I really need to do a MASSIVE edit on the entire lines doc! Be sure to do a very good backup save!! I have to read every single paragraph.