Transmission Line document Overviews PhL 9.11.13

These are very early overviews. The doc was so complicated, I was having trouble remembering what earlier sections were about, so I wrote some little overviews here. All this stuff is superseded by the newer overview and summary section.

Chapter 1. Basic Equations.

Section 1.1 states Maxell's equations and related equations in SI units followed by some comments.

Section 1.2 writes a pair of coupled vector wave equations for fields **H** and **E**.

Section 1.3 rewrites these equations in terms of the potentials **A** and φ. A certain gauge is then selected which simplifies these equations so they are completely decoupled. The decoupled equations are then written in relativistic notation to motivate their similar appearance.

Section 1.4 discusses the approach of using retarded potentials, but this approach is not taken.

Section 1.5 finds the Green's function solution to either of the decouple potential wave equations and then transforms these equations from the time domain to the ω frequency domain.

Chapter 2

Section 2.1 applies the two Maxwell curl equations to the azimuthally symmetric round wire in isolation. Sinusoidal time dependence at ω is assumed and the curl equations show that $E_z(r)$ and thus $J_z(r)$ must decrease as one moves away from the surface at r = a into the wire interior with a drop-off distance on the order of $\delta \equiv \sqrt{2/\omega\mu\mu_0\sigma}$ known as the skin depth. The fields are expression in terms of Bessel functions.

Section 2.2 replaces the complex-argument Bessel function solution for $E_z(r)$ with real-argument Kelvin functions and then with the associated Modulus and Phase functions. It is then shown that, $|E_z(r)| = |E_z(a)| \sqrt{a/r}\ e^{-(a-r)/\delta}$ for $2\delta \le r \le a$ where we see that the drop off is exponential from the surface.

Section 2.3 computes the surface impedance Z_s of the round wire in terms of Bessel functions and then finds a simple form for the low frequency limit which is $Z_s(\omega) = (1+j)/[2\pi a\sigma\delta]$ which increases as δ decreases as ω increases. Intuitively, impedance increases as current flow is constrained to the wire's surface region.

Section 2.4 then considers the surface impedance for a conductor which is part of a transmission line and shows the notion of "active surface area" where the skin effect occurs and where Z_s is raised.

Appendix A 1.1: Gauge Invariance

There is a many-to-one relationship between potentials **A** and φ, and fields **E** and **B**. That is to say, given a pair (**A**,φ), there are many other pairs (**A'**,φ') which yield the exact same fields **E** and **B**. This freedom in selecting the potentials is known as gauge invariance. This appendix shows that it is possible to select a set of potentials (**A'**,φ') such that div**A'** can be any reasonable scalar function f. Selecting that scalar function is known as "selecting a gauge". One gauge of interest is called the Lorentz Gauge. Given any starting (**A**,φ), it is possible to use an alternate (**A'**,φ') such that $\partial_\mu A'^\mu = 0$ in relativistic notation, and in normal notation this gauge has div **A'** = -∂φ/∂t, so in this gauge choice f = -∂φ/∂t. The appendix builds up to this final conclusion through a graduated set of steps Fact 0 through Fact 5. These steps involve the use of Green's Functions, parts integrations, and various vector identities involving div and curl.

Appendix B 1.2: The Complex Dielectric Constant.

In SI units, the normal dielectric "constant" of a medium is $\varepsilon\varepsilon_0$ where ε is dimensionless and ε_0 is a certain constant having appropriate dimensions. If the medium is a conducting medium with conductivity σ, it is convenient for certain purposes to define $\xi \equiv \varepsilon\varepsilon_0 + \sigma/(j\omega)$ to be the "complex" dielectric constant. If σ = 0, one has $\xi \equiv \varepsilon\varepsilon_0$. The term "complex dielectric constant" for ξ has the honor of being a double misnomer. It is obviously not a constant in ω, and secondly, the quantity ε is itself already complex. Conventionally one can define ε' ≡ Re(ε) and ε"≡ -Im(ε) so that ε = ε' - jε", and then when conductivity is included one has $\xi \equiv \varepsilon'\varepsilon_0 - j\varepsilon''\varepsilon_0 + \sigma/(j\omega) = \varepsilon'\varepsilon_0 - (j/\omega)[\sigma + \omega\varepsilon''\varepsilon_0]$ where now the real and imaginary parts of ξ are segregated. Defining $\sigma_{\mathbf{eff}} \equiv \sigma + \omega\varepsilon''\varepsilon_0$ gives $\xi = \varepsilon'\varepsilon_0 - j\sigma_{\mathbf{eff}}/\omega$. The ratio - Im(ξ)/Re(ξ) = $\sigma_{\mathbf{eff}}/[\omega\varepsilon'\varepsilon_0]$ is known as the "loss tangent" of the medium and is written $\tan_{\mathbf{L}}$. This is the subject of **Part 3** of the appendix.

Since the symbol σ is already used for conductivity, the symbol n is used for surface charge. As an example, **Part 1** considers a parallel plate capacitor (area A) with a conducting dielectric and an applied voltage which has the usual $e^{j\omega t}$ time dependence. It is shown than the current through the capacitor is given by $I = j\omega A n_{\mathbf{eff}}$ where $n_{\mathbf{eff}} \equiv n(\xi/\varepsilon\varepsilon_0)$ is an effective surface charge density on the plates of the capacitor. Expressions are then found for the real and imaginary parts of the complex admittance Y of the capacitor. One finds that Y = jωC' where C' = (ξ/εε0)C .

Part 2 then shows that, except during an initial time period, there can be no free charge density in a conducting medium.

Appendix 2.1 = C

Parts 1 and 2 show that, for a wire of uniform cross section, the DC surface impedance per unit length is the same as the DC wire resistance per unit length.

Part 3 computes first the internal and then the external inductance of a round wire.

Part 4 shows how to compute the internal and external inductance of a wire of arbitrary cross section. A certain logarithmic divergence in the external problem is encountered and dealt with.

Appendix 2.2 = D

The problem considered is finding E and B fields in a round wire when these fields do not have azimuthal symmetry. This would be the case when a round wire is one conductor of a general transmission line.

Part 1 outlines the method which involves doing a complex Fourier transform of the Helmholtz equation in cylindrical coordinates such that the azimuthal angle φ is replaced by an integer m which is the expansion index. This process is called a "partial wave expansion" in the text. Thus, **E**(r,φ) is replaced by **E**(r,m). The surface charge density n(φ) is similarly expanded and replaced by N_m coefficients. A solution is found for E_z(r,m) in terms of Bessel functions. The m = 0 term in the expansion corresponds to the azimuthally symmetric situation of a round wire in isolation.

Part 2 then finds corresponding solutions for E_r(r,m) and E_φ(r,m).

Part 3 states the partial wave solutions after normalizing them to the total current I in the wire. A closed form result is then given for the surface impedance Z_s.

Part 4 takes the low-frequency limit of the Part 3 results. This limit is roughly that δ >> a.

Part 5 obtains results for the potentials φ and **A** in the partial wave expansion.
