Chap 2 wave intro PhL 3.26.05

This was my draft of what to add at the start of Chapter 2 to try to give a context and meaning for the Chapter 2 calculations. I thought things were fine, but then they were not fine. I installed this piece into lines doc, but will have to do something about it when I get my "meaning of Ch 2" understood.

12/5/13. Basically this is what is now installed in lines doc.

The Implicit Wave Context and the Skin Effect

In the sections below we don't explicitly consider the notion that a wave is traveling down our round wire, but that is in fact what is happening and this fact deserves a few comments before we delve into the interior solution of the wire.

Our stated assumption below is only that we assume the z dependence of E and B fields is weak enough that we can just ignore it. For example, if the z dependence were some e^{-jkz} factor, then any ∂_z derivative is proportional to k, and if we just assume k is very small, we throw out such derivatives. In effect then a field E(x,y,z) is replaced by E(x,y) and ∇^2 by ∇^2_{2D}.

To put this in more context, we can imagine that **E** really does have a wave form

$$\mathbf{E}(x,y,z,t) = e^{j(\omega t - kz)}\, \mathbf{E}(x,y)$$

so that the frequency domain field is

$$\mathbf{E}(x,y,z,\omega) = e^{-jkz}\, \mathbf{E}(x,y,\omega) .$$

The E field has to satisfy two damped wave equations, one inside and one outside the wire ($\nabla^2 = \nabla^2_{3D}$), as was shown in (1.5.27) ,

$$(\nabla^2 + \beta^2)\, \mathbf{E}(x,y,z,\omega) = 0 \qquad \text{inside wire} \qquad \beta = (j-1)\sqrt{\omega\mu\sigma/2}$$
$$(\nabla^2 + \beta_d^2)\, \mathbf{E}(x,y,z,\omega) = 0 \qquad \text{outside wire} \qquad \beta_d = \omega\sqrt{\mu_d\varepsilon_d}$$

where we use subscript d to indicate dielectric properties. The two β expressions come from the general form (1.5.1) and the first will be explained more below so we accept it for now. If we insert our wave form for **E**(x,y,z,ω) these two equations become

$$(\nabla^2_{2D} + \beta^2 - k^2)\, \mathbf{E}(x,y,\omega) = 0 \qquad \text{inside wire} \qquad \beta = (j-1)\sqrt{\omega\mu\sigma/2} \text{ = complex}$$
$$(\nabla^2_{2D} + \beta_d^2 - k^2)\, \mathbf{E}(x,y,\omega) = 0 \qquad \text{outside wire} \qquad \beta_d = \omega\sqrt{\mu_d\varepsilon_d} \text{ = real}$$

In the second equation, we then make the ansatz assumption that $k = \beta_d$ which basically says that our wave form $e^{j(\omega t - kz)}\, \mathbf{E}(x,y)$ really does describe a wave traveling down the wire with $k = \beta_d$. This k is then related to the speed of light in the dielectric and is the expected value of k for, say, a radio or light wave travelling through the dielectric with no wire present. In the first equation, since the conductor has

such a large σ, |β| is a huge number and |β| >> k (unless ω is very small), so k really plays no role in the first equation for such ω. We then have

$$(\nabla^2_{2D} + \beta^2)\,\mathbf{E}(x,y,\omega) \approx 0 \qquad \text{inside wire} \qquad \beta = (j-1)\sqrt{\omega\mu\sigma/2} \;= \text{complex}$$
$$\nabla^2_{2D}\,\mathbf{E}(x,y,\omega) = 0 \qquad \text{outside wire}$$

The second equation says that the E field outside the wire must solve the 2D (vector) Laplace equation. We shall hear more about this in later chapters.

It is the first equation for the wire interior that we study below in this section, and hopefully we have now put that equation into the context of a wave travelling down the wire.

Although the first equation seems to allow k to be a free small parameter, when we examine where the two solutions meet at the boundary r = a, we conclude that k in the first equation must be β_d.

If β_d has a small imaginary part due to conductivity of the dielectric (see (1.5.1)), the factor $e^{-j\beta_d z}$ says that the wave slowly damps out as it travels down the wire due to dielectric ohmic loss, as it well should.

On the other hand, β is huge and has equal real and imaginary parts. Due to our axial symmetry, the first equation really says $(\nabla^2_{2D} + \beta^2)\,\mathbf{E}(r,\omega) = 0$ which can be through of as a "wave equation" in the radial direction. Of course it is a *damped* wave equation of a very extreme sort. As we move in from the surface of the wire toward the center, we claim that over a distance in which the "wave" phase changes by about π/2, the amplitude is already down by a factor 1/e, so one can roughly say that the wave basically damps out before the wave even goes 1/2 wavelength. This is the skin effect described below.

To understand this effect, it is useful to consider a 1D version of the situation. Imagine zooming the camera in very close to the left surface of the round wire, so that we see a half space of conductor on the right and a half space of dielectric on the left. Let the radial direction be called x which increases into the conductor with x = 0 at the interface. Then the inside-wire wave equation above says

$$(\partial_x^2 + \beta^2)\,E(x,\omega) = 0 \;\; .$$

The solution to this equation is (we select a particular sign for the phase)

$$E(x,\omega) = E(0,\omega)\,e^{+j\beta x} = E(0,\omega)\exp\{\, j\,[(j-1)\sqrt{\omega\mu\sigma/2}\,]x\}$$

$$= E(0,\omega)\,\exp\{-\sqrt{\omega\mu\sigma/2}\;x\}\;\exp\{\,-j\sqrt{\omega\mu\sigma/2}\;x\}$$

$$= E(0,\omega)\,\exp\{-x/\delta\}\;\exp\{\,-j\,x/\delta\} \qquad \delta \equiv \sqrt{2/(\omega\mu\sigma)}$$

$$= E(0,\omega)\,e^{-x/\delta}\,e^{-jx/\delta}\;.$$

Thus as we move from x=0 to the right, in distance δ the E field amplitude drops to 1/e and the phase has changed by π/2. Quantity δ is called the skin depth, and this is probably the most basic way to understand the notion of the skin effect. It is a result forced by the Helmholtz equation having a complex parameter β of the type shown.