Exterior Solution for Round Wire (Section 2.5) PhL 11.26.13

Comments 12/5/13. I think at this time 11.26 Section 2.5 might have been about an *exterior solution* for the round wire! Perhaps here is where I developed that solution (since removed from Ch 2). I have not studied what appears below. I see I was debating which Helmholtz equation to use, which β. Since I am planning a comprehensive solution to this problem for Appendix D (not done yet) I won't worry about details below. I now know that the round wire is really the central conductor of a coaxial cable, that there is surface charge, the E_r is going to dominate in the exterior, and other things I did not think of when writing this. Here I even got that $H_1^{(1)}(\beta r)$ for exterior B and maybe $H_0^{(1)}(\beta r)$ for exterior E, but I think all that stuff is completely wrong. I will archive this early attempt nevertheless.

In this section the E and B fields are functions only of r. Therefore $\nabla^2 = \nabla^2_{2D}$ but to reduce clutter we shall write the 2D Laplacian as ∇^2.

In Section 2.1 we solved Maxwell's equations for the E and B fields inside a round wire with results summarized in box (2.1.26) in terms of Bessel functions J_0 and J_1. The arguments of these functions have phase $e^{j3\pi/4}$ which causes them to be complex, and this led to a restatement of the solutions for E and B in terms of the real Kelvin functions as shown in (2.2.11) and (2.2.12). Here we shall complete the solution to the problem by considering the region exterior to the wire.

Question: (it did not take long!) Why do I ignore displacement current in (2.1.5) ? Ah, the reason is that in the conductor we approximate $E \approx 0$? Then $j\omega\varepsilon E \approx 0$. I comment on this elsewhere but not at the place I assume it. This won't work in the dielectric. Maybe I should just use the ODE which I quote in (2.1.12) as saying $(\nabla^2 + \beta^2)\,\mathbf{E}(\mathbf{x},\omega) = 0$ inside the wire. This would be true also outside the wire since $\rho = 0$ at both places, not sure about the boundary. How would this go?

$$(\nabla^2 + \beta^2)\,\mathbf{E}(\mathbf{x},\omega) = 0$$

Suppose you maintain the assumptions of (2.1.4) in terms of field directions, so $\mathbf{E} = E\hat{\mathbf{z}}$. Then this thing says

$$(\nabla^2 + \beta_1^2)\,E(r) = 0$$

as a scalar Laplace equation where now β is for the exterior medium. Solutions are

$$E(r) = C' J_0(\beta_1 r) + D' Y_0(\beta_2 r)$$

Backup a moment. I know this

Damped *field* wave equations (1.3.36) : $\beta_i^2 = \omega^2\mu_i\xi_i$

$(\nabla^2+\beta_1^2)\mathbf{E} = (1/\varepsilon_1)\,\Sigma_{i=2}^{N+1}\text{grad}\,\rho_i$ $(\nabla^2+\beta_1^2)\mathbf{B} = 0$ // region 1

$(\nabla^2+\beta_2^2)\mathbf{E} = 0$ $(\nabla^2+\beta_2^2)\mathbf{B} = 0$ // region 2

$(\nabla^2+\beta_3^2)\mathbf{E} = 0$ $(\nabla^2+\beta_3^2)\mathbf{B} = 0$ // region 3 (1.5.27)

Here 1 = dielectric and 2 = conductor, so inside either material we are going to have a homo equation. Thus we have

$$(\boldsymbol{\nabla}^2 + \beta^2)\, \mathbf{B}(\mathbf{x},\omega) = 0$$

But B is in the θ direction, unlike E which was in the z direction, so this time we really do have to use cylindrical coordinates for the vector ∇^2 operator. Maybe avoid using this equation? But how am I going to find B in the exterior region?? I need an equation for it. OK, suppose $\mathbf{B} = B(r)\,\hat{\boldsymbol{\theta}}$. I just went and added this to my diffop doc, so we then have

$$\nabla^2\mathbf{B} = \hat{\mathbf{r}}\,[\nabla^2 B_r - 2r^{-2}\partial_\theta B_\theta - r^{-2}B_r] + \hat{\boldsymbol{\theta}}\,[\nabla^2 B_\theta + 2r^{-2}\partial_\theta B_r - r^{-2}B_\theta] + \hat{\mathbf{z}}\nabla^2 B_z$$

$$= \hat{\mathbf{r}}\,[-2r^{-2}\partial_\theta B_\theta] + \hat{\boldsymbol{\theta}}\,[\nabla^2 B_\theta - r^{-2}B_\theta]$$

$$= \hat{\boldsymbol{\theta}}\,[\nabla^2 B_\theta - r^{-2}B_\theta]$$

Thus, the equation $(\boldsymbol{\nabla}^2 + \beta^2)\, \mathbf{B}(\mathbf{x},\omega) = 0$ becomes

$$\nabla^2 B_\theta - r^{-2}B_\theta + \beta^2 B_\theta = 0$$

I have done this recently elsewhere, but I will just do it here again rather than search all over. Call it B, and then we have

$$r^{-1}\partial_r(r\partial_r B) - r^{-2}B + \beta^2 B = 0$$

$$\partial_r(r\partial_r B) - r^{-1}B + r\beta^2 B = 0$$

$$r B'' + B' - r^{-1}B + r\beta^2 B = 0$$

$$r^2 B'' + rB' - B + r^2\beta^2 B = 0$$

$$r^2 B''(r) + rB'(r) + (r^2\beta^2 - 1)B(r) = 0$$

This is almost Bessel's with n=1.

$x = \beta r$ $\quad f(x) = B(x/\beta) = B(r)$

$f(x) = B(r)$	$B(r) = f(x)$
$f'(x) = (1/\beta)B'(x/\beta) = (1/\beta)B'(r)$	$B'(r) = \beta\, f'(x)$
$f''(x) = (1/\beta)^2 B''(x/\beta) = (1/\beta)^2 B''(r)$	$B''(r) = \beta^2 f''(x)$

So equation becomes

$$r^2 \beta^2 \; f''(x) + r \beta \, f'(x) \; + (r^2\beta^2 - 1) f(x) = 0$$

$$x^2 \; f''(x) + x \, f'(x) \; + (x^2 - 1) f(x) = 0$$

Solutions are like $J_1(x) = J_1(\beta r)$. DONE.

So, what we find is that

$$B(r) = B(a) \, H_1^{(1)}(\beta r) / H_1^{(1)}(\beta a) \qquad r > a$$

Aside: Now I need some argument to say that $H_0^{(1)}(\beta_1 r)$ will be the solution. The usual argument is the large r behavior. So get NIST going again. But wait, I already comment on this somewhere. In I.3 I am doing the Green's for Helmholtz in 2D. But I never quote the result from NIST. OK, I have added this in Appendix I, and I will refer to it. However, in the dielectric the phase of β is not $3\pi/4$. It is this

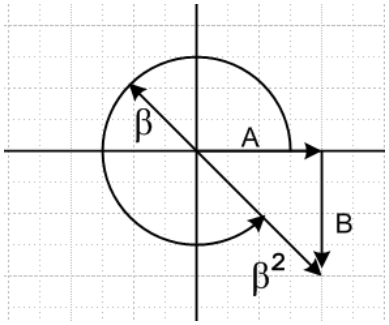
$$\beta_1^2 = \mu\varepsilon_1\omega^2 - j\omega\mu_1\sigma_1 \; = \omega^2\mu_1 \, (\, \varepsilon_1 - j\sigma/\omega) \; = \omega^2\mu_1 \, \xi_1 \qquad \xi_1 \equiv \varepsilon_1 - j\sigma_1/\omega \quad . \qquad (1.5.1)$$

Ouch. In the dielectric, σ_1 is very small and we have roughly $\beta_1^2 = \mu\varepsilon_1\omega^2 - j\varepsilon$ But write this as

$$\xi_1 \equiv \varepsilon_1 - j\sigma_1/\omega = \varepsilon_1 + (-j) \, \sigma_1/\omega \; = \varepsilon_1 + e^{j3\pi/2} \, \sigma_1/\omega$$

$\mathrm{Re}\xi_1 = \varepsilon_1$ $\qquad$ $\mathrm{Im}\xi_1 = -\sigma_1/\omega$ $\qquad$ ouch! Won't this cause $H^{(1)}$ problems???

Although β_1^2 has a negative imaginary part, according to our selected branch of square root function, the quantity β_1 has a positive imaginary part as shown in this picture where $\beta^2 = A - j\,B$.



For more details see Comment and Figure in Appendix I ****.

Thus, the $H_0^{(1)}(\beta_1 r)$ does indeed decay for large r.

So here is the proposed exterior solution for E. It has n = 0 because $\mathbf{E} = E\hat{\mathbf{z}}$,

$$E(r) = K \, H_0^{(1)}(\beta_1 r)$$

Here I rewrite the interior solution using 2 subscripts for the conductor.

Interior Solution of a Round Wire (2.1.26)

$\frac{\partial [\, rB(r)]}{\partial r} = (\mu_2\sigma_2)\,[\, r\,E(r)\,]$ *(2.1.7)* $\qquad \frac{\partial E(r)}{\partial r} = j\omega B(r)$ *(2.1.9)*

$E(r) = E(a)\,[J_0(\beta_2 r) / J_0(\beta_2 a)]$ *(2.1.22)*

$J(r) = J(a)\,[J_0(\beta_2 r) / J_0(\beta_2 a)]$ // above times σ_2, $J = J_z$ = current density

$B(r) = -\,(\beta_2/j\omega)\,E(a)\,[J_1(\beta_2 r) / J_0(\beta_2 a)] = +\,B(a)\,[J_1(\beta_2 r) / J_1(\beta_2 a)]$ *(2.1.25)*

$\beta_2 = e^{j3\pi/4}\,(\sqrt{2}\,/\delta) = e^{j3\pi/4}\,\sqrt{\omega\mu_2\sigma_2}$ and $\beta_2^2 = -2j/\delta^2$ *(2.1.21), (2.1.14)*

$\delta \equiv \sqrt{2/\omega\mu_2\sigma_2}$ = skin depth *(2.1.20)*

The B field is in the $\hat{\theta}$ direction and so is a tangential field, so we call upon

$H_{t2} - H_{t1} = K_z^{free}$ or $(1/\mu_2)B_{t2} - (1/\mu_1)B_{t1} = K_z^{free}$ *(1.1.44)*

We assume there are no free longitudinal surface currents on the surface of the round wire (only the longitudinal bulk current), so therefore

$(1/\mu_2)B_{t2} = (1/\mu_1)B_{t1}$

Thus, our matching condition is

$(1/\mu_2)\,B(a)\,[J_1(\beta_2 a) / J_1(\beta_2 a)] = (1/\mu_1)\,K\,H_0^{(1)}(\beta_1 a)$

or

$(1/\mu_2)B(a) = (1/\mu_1)\,K\,H_0^{(1)}(\beta_1 a)$

Comment: We still have this unknown in one form or the other, B(a) and K. This matching condition does not determine either one.

What about the idea that far away, it looks like a thin wire, and then we can use the Green's function solution from Appendix I. Our problem for a thin wire is this

Back up. First, write the two wave equations of interest

$(\nabla^2 + \beta^2)\,\mathbf{E} = 0$

$(\nabla^2 + \beta^2)\,\mathbf{B} = 0$

Now assume that in both regions that

$\mathbf{E} = E(r)\,\hat{\mathbf{z}}$ => only E_z

$\mathbf{B} = B(r)\,\hat{\boldsymbol{\theta}}$ => only B_θ

Use cylindrical coordinates in which

$$\nabla^2\mathbf{E} = \hat{\mathbf{r}}\,[\nabla^2 E_r - 2r^{-2}\partial_\theta E_\theta - r^{-2}E_r] + \hat{\boldsymbol{\theta}}\,[\nabla^2 E_\theta + 2r^{-2}\partial_\theta E_r - r^{-2}E_\theta] + \hat{\mathbf{z}}\nabla^2 E_z$$
$$= + \hat{\mathbf{z}}\nabla^2 E_z = + \hat{\mathbf{z}}\nabla^2 E(r)$$

$$\nabla^2\mathbf{B} = \hat{\mathbf{r}}\,[\nabla^2 B_r - 2r^{-2}\partial_\theta B_\theta - r^{-2}B_r] + \hat{\boldsymbol{\theta}}\,[\nabla^2 B_\theta + 2r^{-2}\partial_\theta B_r - r^{-2}B_\theta] + \hat{\mathbf{z}}\nabla^2 B_z$$
$$= \hat{\mathbf{r}}\,[-2r^{-2}\partial_\theta B_\theta] + \hat{\boldsymbol{\theta}}\,[\nabla^2 B_\theta - r^{-2}B_\theta]$$
$$= \hat{\boldsymbol{\theta}}\,[\nabla^2 B_\theta - r^{-2}B_\theta] \qquad \text{since } B_\theta = B_\theta(r)$$

Then the two wave equations become

$$(\nabla^2 + \beta^2)E(r) = 0$$
$$(\nabla^2 + \beta^2 - r^{-2})B(r) = 0$$

Using $\psi = \psi(r)$,

$$\nabla^2\psi = r^{-1}\partial_r(r\partial_r\psi) + r^{-2}\partial_\theta^2\psi + \partial_z^2\psi = r^{-1}[r\partial_r^2\psi + \partial_r\psi] = \psi'' + r^{-1}\psi'$$

we write our two equations then as

$$E''(r) + r^{-1}E'(r) + \beta^2 E(r) = 0$$
$$B''(r) + r^{-1}B'(r) + (\beta^2 - r^{-2})B(r) = 0$$

or

$$r^2 E''(r) + rE'(r) + \beta^2 r^2 E(r) = 0$$
$$r^2 B''(r) + rB'(r) + (r^2\beta^2 - 1)B(r) = 0$$

Letting $B(r) = b(x)$ and $E(r) = e(x)$ where $x = \beta r$, we find for example for B (and similarly for E)

$b(x) = B(r)$ => $B(r) = b(x)$

$b'(x) = \partial_x b(x) = \partial_x B(x/\beta) = (1/\beta)B'(x/\beta) = (1/\beta)\,B'(r)$ => $B'(r) = \beta\, b'(x)$

$b''(x) = (1/\beta)^2\, B''(r)$ => $B''(r) = \beta^2\, b''(x)$

The two equations then become

$$r^2\beta^2\, e''(x) + r\,\beta\, e'(x)E'(r) + \beta^2 r^2\, e(x) = 0$$
$$r^2\beta^2\, b''(x) + r\,\beta\, b'(x)E'(r) + (\beta^2 r^2 - 1)\, b(x) = 0$$

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$x^2\, e''(x) + xe'(x) + x^2 e(r) = 0$ solutions like $e(x) = J_0(x) = J_0(\beta r) = E(r)$

$x^2\, b''(x) + xb'(x) + (x^2 - 1)b(x) = 0$ solutions like $b(x) = J_1(x) = J_1(\beta r) = B(r)$

where we recognize the equations in x as standard Bessel equations as in Spiegel 24.1.

Now we have to select which *kind* of Bessel function to use in each region. For r < a we use

$E(r) = J_0(\beta_2 r)$			
$B(r) = J_1(\beta_2 r)$	r < a	region 2	conductor
$E(r) = H_0^{(1)}(\beta_1 r)$			
$B(r) = H_1^{(1)}(\beta_1 r)$	r > a	region 1	dielectric

Normalizing at the surface we can write

$E(r) = E(a-)\, J_0(\beta_2 r)/ J_0(\beta_2 a)$			
$B(r) = B(a-)\, J_1(\beta_2 r)/ J_1(\beta_2 a)$	r < a	region 2	conductor
$E(r) = E(a+)\, H_0^{(1)}(\beta_1 r)/ H_0^{(1)}(\beta_1 a)$			
$B(r) = B(a+)\, H_1^{(1)}(\beta_1 r)/ H_1^{(1)}(\beta_1 a)$	r > a	region 1	dielectric

where a+ means just outside the round wire and a- just inside the wire. We now call up our boundary conditions. The $\mathbf{E} = E(r)\,\hat{\mathbf{z}}$ field is tangential so box (1.1.50) tells us that

$$E_{t1} = E_{t2} \quad \Rightarrow \quad E(a+) = E(a-) \equiv E(a)$$

The $\mathbf{B} = B(r)\,\hat{\boldsymbol{\theta}}$ is also tangential, so from box (1.1.50),

$$(1/\mu_2)B_{t2} = (1/\mu_1)B_{t1} \quad \Rightarrow \quad (1/\mu_2)B(a-) = (1/\mu_1)B(a+) \equiv (1/\mu_2)B(a)$$

since we assume there is no free surface current on the surface of our wire (only the bulk volume current). The symbols E(a) and B(a) are those just inside the wire and were used in Section ***. We then have

$E(r) = E(a)\, J_0(\beta_2 r)/ J_0(\beta_2 a)$			
$B(r) = B(a)\, J_1(\beta_2 r)/ J_1(\beta_2 a)$	r < a	region 2	conductor
$E(r) = E(a)\, H_0^{(1)}(\beta_1 r)/ H_0^{(1)}(\beta_1 a)$			
$B(r) = (\mu_1/\mu_2)B(a)\, H_1^{(1)}(\beta_1 r)/ H_1^{(1)}(\beta_1 a)$	r > a	region 1	dielectric

Comment: so here we are with our still undetermined E(a) and B(a) constants.

If we apply Ampere's Law just inside the wire surface we get

$$2\pi a H(a) = I \quad \Rightarrow \quad 2\pi a B(a)/\mu_2 = I \quad \Rightarrow \quad B(a) = \mu_2 I/(2\pi a)$$

so we now have one of these two constants.

From

$$\text{curl } \mathbf{E} = -\partial_t \mathbf{B} \qquad \Leftrightarrow \qquad \oint_C \mathbf{E} \bullet d\mathbf{s} = -\partial_t \left[\int_S \mathbf{B} \bullet d\mathbf{S} \right] \qquad (1.1.36)$$

we showed in (2.1.8) applied to a certain math loop inside the wire (region 2) that

$$\frac{\partial E(r)}{\partial r} = j\omega B(r)$$

Using the region 2 equations above this says

$$\partial_r [E(a)\, J_0(\beta_2 r)/ J_0(\beta_2 a)] = j\omega\, [B(a)\, J_1(\beta_2 r)/ J_1(\beta_2 a)]$$

Now $\partial_r\, J_0(\beta_2 r) = \beta_2\, J_0'(\beta_2 r) = -\beta_2\, J_1(\beta_2 r)$ using Spiegel 24.7. Thus the above becomes

$$-[E(a)\, \beta_2\, J_1(\beta_2 r)/ J_0(\beta_2 a)] = j\omega\, [B(a)\, J_1(\beta_2 r)/ J_1(\beta_2 a)]$$

Now evaluate this at r = a to get

$$-[E(a)\, \beta_2\, J_1(\beta_2 a)/ J_0(\beta_2 a)] = j\omega\, [B(a)\, J_1(\beta_2 a)/ J_1(\beta_2 a)]$$

$$-[E(a)\, \beta_2/ J_0(\beta_2 a)] = j\omega\, [B(a) / J_1(\beta_2 a)]$$

$$E(a) = \{ \ (-j\omega/\beta_2)\, [J_0(\beta_2 a)/ J_1(\beta_2 a)] \ \}\, B(a)$$

which is consistent with (2.1.25). Thus we have our final frequency domain solutions:

$$\frac{J_0(\beta r)}{J_0(\beta a)} \qquad \frac{J_1(\beta r)}{J_1(\beta a)}$$

$$\frac{\mu_2 I}{2\pi a} \qquad \frac{J_0(\beta_2 r)}{J_0(\beta_2 a)} \qquad \frac{J_1(\beta_2 r)}{J_1(\beta_2 a)}$$

$$\frac{H_0^{(1)}(\beta_1 r)}{H_0^{(1)}(\beta_1 a)} \qquad \frac{H_1^{(1)}(\beta_1 r)}{H_1^{(1)}(\beta_1 a)}$$

$E(r) = E(a)\, J_0(\beta_2 r)/ J_0(\beta_2 a)$

$B(r) = B(a)\, J_1(\beta_2 r)/ J_1(\beta_2 a)$ $\qquad r < a \qquad$ region 2 $\qquad$ conductor

$E(r) = E(a)\, H_0^{(1)}(\beta_1 r)/ H_0^{(1)}(\beta_1 a)$

$B(r) = (\mu_1/\mu_2)B(a)\, H_1^{(1)}(\beta_1 r)/ H_1^{(1)}(\beta_1 a)$ $\quad r > a \quad$ region 1 $\quad$ dielectric

where

$B(a) = [\mu_2 I/(2\pi a)]$

$$E(a) = [\mu_2 I/(2\pi a)]\{\ (-j\omega/\beta_2)\ [J_0(\beta_2 a)/ J_1(\beta_2 a)]\ \} = \frac{\mu_2 I}{2\pi a}\ \frac{(-j\omega)}{\beta_2}\ \frac{J_0(\beta_2 a)}{J_1(\beta_2 a)}$$

Now rewrite the above results

$$E(r) = \frac{\mu_2 I}{2\pi a}\ \frac{(-j\omega)}{\beta_2}\ \frac{J_0(\beta_2 a)}{J_1(\beta_2 a)}\ \frac{J_0(\beta_2 r)}{J_0(\beta_2 a)} = \frac{\mu_2 I}{2\pi a}\ \frac{(-j\omega)}{\beta_2}\ \frac{J_0(\beta_2 r)}{J_1(\beta_2 a)}$$

$$B(r) = \frac{\mu_2 I}{2\pi a}\ \frac{J_1(\beta_2 r)}{J_1(\beta_2 a)} \qquad \text{region 2}$$

$$E(r) = \frac{\mu_2 I}{2\pi a}\ \frac{(-j\omega)}{\beta_2}\ \frac{J_0(\beta_2 a)}{J_1(\beta_2 a)}\ \frac{H_0^{(1)}(\beta_1 r)}{H_0^{(1)}(\beta_1 a)} \qquad \text{region 1}$$

$$B(r) = (\mu_1/\mu_2)\ [\mu_2 I/(2\pi a)]\ \frac{H_1^{(1)}(\beta_1 r)}{H_1^{(1)}(\beta_1 a)} \quad = \quad [\mu_1 I/(2\pi a)]\ \frac{H_1^{(1)}(\beta_1 r)}{H_1^{(1)}(\beta_1 a)} \qquad \text{region 1}$$

Restate:

$$B(r) = \frac{\mu_2 I}{2\pi a}\ \frac{J_1(\beta_2 r)}{J_1(\beta_2 a)} \qquad E(r) = \frac{\mu_2 I}{2\pi a}\ \frac{(-j\omega)}{\beta_2}\ \frac{J_0(\beta_2 r)}{J_1(\beta_2 a)} \qquad \text{region 2} \qquad r < a$$

$$B(r) = \frac{\mu_1 I}{2\pi a}\ \frac{H_1^{(1)}(\beta_1 r)}{H_1^{(1)}(\beta_1 a)} \qquad E(r) = \frac{\mu_2 I}{2\pi a}\ \frac{(-j\omega)}{\beta_2}\ \frac{J_0(\beta_2 a)}{J_1(\beta_2 a)}\ \frac{H_0^{(1)}(\beta_1 r)}{H_0^{(1)}(\beta_1 a)} \qquad \text{region 1} \qquad r > a$$

Question: Why can't I do Ampere's law outside at radius r and get $2\pi r B/\mu_1 = I$ so that

$B(r) = \mu_1 I/2\pi r$ $\quad$ region 1 $\quad$ r>a

Answer: if you draw your loop, you have to include with the current $j\omega\varepsilon_1 E(r)$ in the dielectric!

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The interior solution agrees with the solution shown in box (2.1.26) but now we know the constants. According to this same box we have

$$\beta_2 = e^{j3\pi/4}\sqrt{\omega\mu_2\sigma_2}$$

which has phase factor $e^{j3\pi/4}$ which motivated us to replace the J_n Bessel functions with their Kelvin function equivalents

$$J_\nu(e^{j3\pi/4}z) = \mathrm{ber}_\nu(z) + j\,\mathrm{bei}_\nu(z) \quad . \qquad (2.2.1)$$

In region 1 which we imagine not to be a conductor, σ_1 is relatively small and so

$$\beta_1^2 = \mu_1\varepsilon_1\omega^2 - j\omega\mu_1\sigma_1 = \omega^2\mu_1(\varepsilon_1 - j\sigma_1/\omega) = \omega^2\mu_1\,\xi_1 \qquad \xi_1 \equiv \varepsilon_1 - j\sigma_1/\omega \qquad (1.5.1)$$

so that

$$\beta_1^2 \approx \mu_1\varepsilon_1\omega^2 = \text{real} \quad \text{(small imaginary part as per ***)}$$

Thus we have no need to write out the exterior $H_1^{(1)}(\beta_1 r)$ functions in terms of Kelvin functions.

Question: What happens regarding the complex nature of these functions. B(a) is real, E(a) has phase shown relative to B.

Question: How can you "understand" this solution:

$$B(r) = \frac{\mu_2 I}{2\pi a}\,\frac{J_1(\beta_2 r)}{J_1(\beta_2 a)} \qquad E(r) = \frac{\mu_2 I}{2\pi a}\,\frac{(-j\omega)}{\beta_2}\,\frac{J_0(\beta_2 r)}{J_1(\beta_2 a)} \qquad \text{region 2} \qquad r < a$$

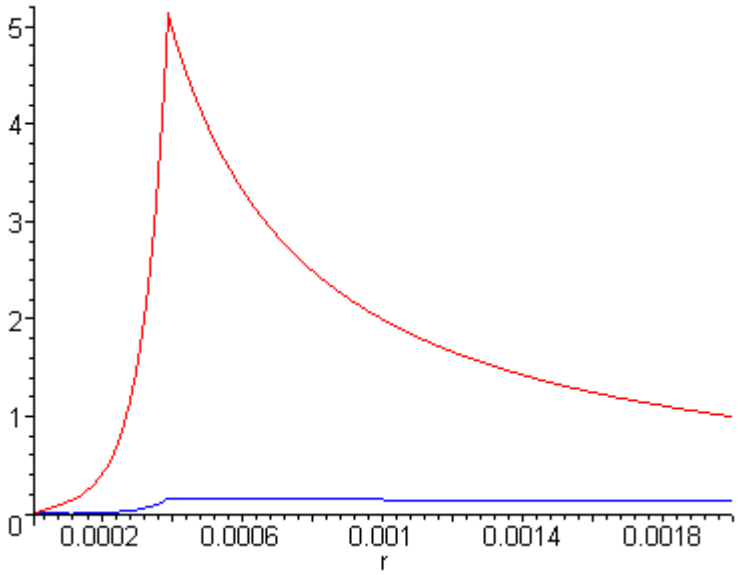
$$B(r) = \frac{\mu_1 I}{2\pi a}\,\frac{H_1^{(1)}(\beta_1 r)}{H_1^{(1)}(\beta_1 a)} \qquad E(r) = \frac{\mu_2 I}{2\pi a}\,\frac{(-j\omega)}{\beta_2}\,\frac{J_0(\beta_2 a)}{J_1(\beta_2 a)}\,\frac{H_0^{(1)}(\beta_1 r)}{H_0^{(1)}(\beta_1 a)} \qquad \text{region 1} \qquad r > a$$

where

$$B(a) = \frac{\mu_2 I}{2\pi a} \qquad E(a) = \frac{\mu_2 I}{2\pi a}\,\frac{(-j\omega)}{\beta_2}\,\frac{J_0(\beta_2 a)}{J_1(\beta_2 a)}$$

Generally everything is complex. The Hankel functions for real argument are complex since the real part is J and the imaginary part is Y.

I have now made some abs value plots and get things like this



where the B field is red and in gauss, and the E field is in V/m. Are these plots believable? E decays very slowly as $r^{-1/2}$ from the Hankel function.