Sisyphus Paradox of the Day 11_27_13 PhL 11.27.13

See History section in "Finding a Meaning" for what is going on here.

1. In my new "section 2.5" I have assumed the following logic thread:

(a) First, we know from Maxwell that (dielectric does not conduct we shall say)

$$(\nabla^2 + \beta_d^2)\mathbf{E}(x,y,z,\omega) = 0 \qquad \text{in any region of dielectric where } \beta_d = \omega\sqrt{\mu_d\varepsilon_d}$$

This is "the wave equation" for the **E** field in a region of space absent any current and any charge. It appears in my Chapter 1 as (1.2.1). It applies therefore to the region "outside" my round wire.

(b) Then without any justification, I made the ***ansatz*** that the E field has "weak z dependence" so that we can basically ignore $\partial_z \mathbf{E}(x,t,z,\omega)$. In that case, the above becomes

$$(\nabla^2_{2D} + \beta_d^2)\mathbf{E}(x,y,z_0,\omega) = 0 \qquad \text{region 1}$$

where I just pick z_0 to represent the z value, since **E** is basically assumed constant in z.

(c) With this assumption, and assuming $\mathbf{E} = E_z\hat{\mathbf{z}}$, I write Section 2.5 and I obtain various results solving the ODE which is $(\nabla^2_{2D} + \beta_d^2)E_z(r) = 0$. For example, I find that (1 = dielectric, $\beta_1 = \beta_d$)

$$E(r) = \frac{\mu_2 I}{2\pi a}\,\frac{(-j\omega)}{\beta_2}\,\frac{J_0(\beta_2 a)}{J_1(\beta_2 a)}\,\frac{H_0^{(1)}(\beta_1 r)}{H_0^{(1)}(\beta_1 a)} \sim \text{constant} * H_0^{(1)}(\beta_1 r) \qquad r > a$$

I was very pleased with myself for finding this sensible result for the E field outside the wire, although I did note at the time that it seemed to decay with r "rather slowly" which seemed odd.

Now here is the paradoxical issue:

In what physical situation would the above ansatz apply?

I find that I am unable to construct any such situation for the round wire.

Failed Situation #1.

At first I thought I could construct some kind of "long wavelength" situation for the wire where

$$\mathbf{E}(x,y,z_0,\omega) = e^{-jkz}\,\mathbf{E}_0(x,y,\omega)$$

where somehow k is very small so we can ignore it. When this form is put into the wave equation above, the result is

$$(\nabla^2_{2D} + \beta_d^2 - k^2)\mathbf{E}_0(x,y,\omega) = 0 \qquad \text{region 1}$$

Then I just have to assume that $k << \beta_d$ and then I have my ansatz scenario exactly! So I then said the physical situation was just "some long wavelength wave" going down the wire.

However, waves do not go faster than c, and generally even in a transmission line only go at roughly velocity c. They go at the wave velocity in the dielectric medium I am pretty sure. There is then a relationship between k and ω which is $v = \omega/k$. One cannot arbitrarily set ω and k at the same time. In fact, what you have is

$$k = \omega/v = \omega\sqrt{\mu\varepsilon} = \beta_d$$

This says that k are β_d are the same thing! Thus, the correct wave equation outside the dielectric must be this:

$$(\nabla^2_{2D} + 0)\mathbf{E}_0(x,y,\omega) = 0 \qquad \text{region 1}$$

This equation is in complete conflict with the form I assumed above in region 1

$$(\nabla^2_{2D} + \beta_d^2)\mathbf{E}(x,y,z_0,\omega) = 0 \qquad \text{region 1}$$

and therefore my section 2.5 is completely wrong, or I have done something else wrong somewhere.

This then is my Paradox of the Day. I am not yet sure what implication this has for my Chapter 2 and the interior wire work I did there.

Idea: Maybe try to rewrite Section 2.5 in light of the new equation.