The r=0 BC idea for Ephi PhL 11.29.13

This was my attempt to say that the second boundary condition of App D (c) shown below is that E_φ vanishes at r = 0 instead of r = a. This idea later got completely rejected as non-sense, and the r = 0 BC was then reinforced in text at the start of App D.

Question: If you have fields $E_r(r,m)$ and $E_\varphi(r,m)$ as functions of r, what can you say about these functions when they are evaluated at r = 0 ? Think of this as a polar coordinates question. This is a singular point of the coordinate system of course.

If you had $E_r \neq A > 0$ at r=0, say, then somehow E_r points in all directions at once. I want to argue that therefore $E_r = 0$ at such an origin. But if you take a uniform E field and plop down an origin somewhere, that does not mean $E_r = 0$.

(c) Application of the Boundary Conditions

We have two boundary conditions to impose:

$$E_r(r=a,m) = (j\omega/\sigma)\, N_m \qquad \text{(D.2.18)}$$

$$E_\varphi(r=0,m) = 0 \qquad \text{(D.2.19)}$$

The first is the radial charge pumping condition shown in (D.1.23) above, while the second a statement of the fact that, since there is no "vortex line" of infinite B field at r = 0, $E_\varphi(r=0)$ cannot point in any direction and must therefore be 0.

These two boundary conditions serve to determine the two constants a_m and K_m, though a bit of algebra is required. The first step is to use (D.2.11) for E_r and (D.2.15) for E_φ to write out the two boundary conditions as

$$a_m\, x_a^{-1}\, J_m(x_a) + \frac{K_m}{2}\, J_{m+1}(x_a) = (j\omega/\sigma)\, N_m \qquad (1)$$

$$\lim_{x\to 0} \left[- a_m\, (1/x)\, J_m(x) + \left(\frac{K_m}{2} + \frac{a_m}{m} \right) J_{m+1}(x) \right] = 0 \ . \qquad (2)$$

Now we know that

$$J_0(x) \approx 1$$
$$J_m(x) \approx (x/2)^m/m! \qquad J_{m+1}(x) = (x/2)^{m+1}/(m+1)!$$
$$(1/x)J_m(x) \approx 2^{-m}/m! * x^{m-1}$$

For any $m \geq 0$, the second term in (2) vanishes so we then have

$$\lim_{x\to 0} [- a_m \ (1/x) \ J_m(x)] \ = 0 \ . \qquad (2)'$$

For m = 0, a_0 must vanish, otherwise the limit is infinite.

For m > 0 we have

$$\lim_{x\to 0} [- a_m \ (1/x) \ (x/2)^m/m!] \ = 0 \ . \qquad (2)'$$

or

$$\lim_{x\to 0} [- a_m \ (x/2)^{m-1}/m!] \ = 0 \ . \qquad (2)'$$

For m = 1 we find

$$\lim_{x\to 0} [- a_1 \ (x/2)^0/1!] \ = 0 \ . \qquad (2)'$$

which says $a_1 = 0$. For m > 1, this limit yields no conclusion about a_m.

Status: This means we don't have enough information to determine the constants for m > 1.