Contradiction Bug PhL 9.25.13

Resolved. I thought there was a rigid 90 degree relation between complex E and B, but not so.

I have shown in lines that for the round wire, we have the following fields

$$E(r) = E(a)\,[J_0(\beta r) / J_0(\beta a)]$$

$$B(r) = -(\beta/j\omega)\,E(a)\,[J_1(\beta r) / J_0(\beta a)]$$

The ratio of these fields is given by

$$B(r)/E(r) = -(\beta/j\omega)\,[J_1(\beta r) / J_0(\beta r)] \qquad \beta = e^{j3\pi/4}\sqrt{\omega\mu\sigma}$$

I now evaluate this in two different ways for small ω.

Method 1. I use these expansions from Spiegel p 136

$$J_0(x) \approx 1 + \text{order}(x^2) \qquad x = \beta r = r\, e^{j3\pi/4}\sqrt{\omega\mu\sigma} = \text{small for small } \omega$$

$$J_1(x) \approx x/2 + \text{order}(x^3).$$

Thus, we have

$$[J_1(\beta r) / J_0(\beta r)] \approx (\beta r/2)$$

and so

$$B(r)/E(r) \approx -(\beta/j\omega)\,[(\beta r/2)] = -\beta^2 r/[2j\omega] = -(-j\omega\mu\sigma)r/[2j\omega] = (\mu\sigma)r/[2] = \mu\sigma r/2.$$

This ratio then has zero phase for small ω. If μ = 1 and σ = 1 and r = 1 then B(r)/E(r) = 1/2

Maple agrees with this calculation of 1/2 so no contradiction yet. Maple can plot the phase of the ratio:

```
restart; with(plots):
beta := (w) -> exp(I*3*Pi/4)*sqrt(w*mu*sigma);
```

$$\beta := w \to e^{\left(\frac{3}{4} I \pi\right)} \sqrt{w \mu \sigma}$$

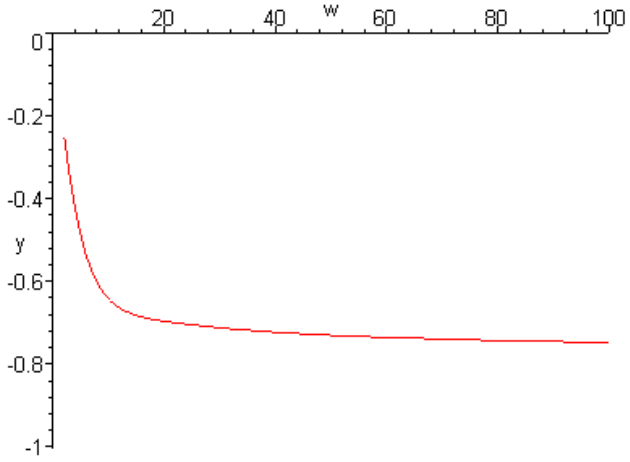
```
BoverE := (w) -> -(beta(w)/(I*w))*BesselJ(1,beta(w)*r)/BesselJ(0,beta(w)*r);
```

$$BoverE := w \to \frac{I \beta(w)\, \mathrm{BesselJ}(1, \beta(w)\, r)}{w\, \mathrm{BesselJ}(0, \beta(w)\, r)}$$

```
r := 1: mu := 1: sigma := 1:
evalf(BoverE(0.0000001));
```

$$.4999999998 - .6462616452\, 10^{-8}\, I$$

```
plot(argument(evalf(BoverE(w))),w = 0..100,y=-1..0);
```



This shows that the phase of the ratio B(r)/E(r) is not constant in ω.

Method 2. I show in Fact (I.3.7) of "complex E and B..." that this ratio must be a constant.
[but this Fact is wrong!]

Therefore we have a contradiction.