The E dot B equals 0 problem PhL 1.27.14

See red comment in the v1 doc.

I have to say something about my little proof around (3.7.1). It is not valid below a certain ω and I would like to make an estimate.

I get to this point

$$C\,\mathbf{E}\bullet\mathbf{B} = C\,E\,B\cos\theta = (\partial_x B_y - \partial_y B_x)B_z + (\partial_y B_z - \partial_z B_y)B_x + (\partial_z B_x - \partial_x B_z)B_y$$

$$C\,\mathbf{E}\bullet\mathbf{B} = \text{curl}\,\mathbf{B}\bullet\mathbf{B} = \hat{\mathbf{r}}\,[\,r^{-1}\partial_\theta B_z - \partial_z B_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z B_r - \partial_r B_z] + \hat{\mathbf{z}}\,[\,r^{-1}\partial_r(rB_\theta) - r^{-1}\partial_\theta B_r\,]$$

$$\bullet\ [\,\hat{\mathbf{r}}\ B_r + \hat{\boldsymbol{\theta}}\,B_\theta + \hat{\mathbf{z}}\,B_z\,]$$

$$= B_r[\,r^{-1}\partial_\theta B_z - \partial_z B_\theta] + B_\theta[\partial_z B_r - \partial_r B_z] + B_z\,[\,r^{-1}\partial_r(rB_\theta) - r^{-1}\partial_\theta B_r\,]$$

Let's just assume $B_r = 0$ for simplicity. Then

$$C\,\mathbf{E}\bullet\mathbf{B} = \text{curl}\,\mathbf{B}\bullet\mathbf{B} = +B_\theta[\partial_r B_z] + B_z[\,r^{-1}\partial_r(rB_\theta)\,]$$

$$= B_\theta[\partial_r B_z] + B_z[\,r^{-1}B_\theta + \partial_r B_\theta\,]$$

where

$$C = j(\beta^2/\omega) = j\,[\omega\mu(\,j\omega\varepsilon + \sigma)/j]/\omega = \mu(\,j\omega\varepsilon + \sigma)$$

where μ and σ are for the dielectric. So then

$$C\,\mathbf{E}\bullet\mathbf{B} = C\,E\,B\cos\theta = \mu(\,j\omega\varepsilon + \sigma)\,E\,B\cos\theta = (\,j\omega\varepsilon + \sigma)\,E\,H\cos\theta$$

So then,

$$C\,E\,B\cos\theta = B_\theta[\partial_r B_z] + B_z[\,r^{-1}B_\theta + \partial_r B_\theta\,]$$

But $B \approx B_\theta$ so then roughly

$$C\,E\cos\theta = [\partial_r B_z] + B_z[\,r^{-1} + (\partial_r B_\theta)/B_\theta]$$

$$\mu(\,j\omega\varepsilon + \sigma)\,E\cos\theta = [\partial_r B_z] + B_z[\,r^{-1} + (\partial_r B_\theta)/B_\theta]$$

$$(\,j\omega\varepsilon + \sigma)\,E\cos\theta = [\partial_r H_z] + H_z[\,r^{-1} + (\partial_r H_\theta)/H_\theta]$$

I need estimates for E and B.

E D = V(z) $2\pi a\, H = i(z)$ at surface

D = transverse dimension H = field outside round wire

$H = i(z)/2\pi a$ $1/E = D/V(z)$

$H/E = i(z)/V(z) * D/2\pi a = (D/2\pi a)\, 1/Z_0$

so then roughly we have

$H/E = 1/Z_0$ amps/m // volts/m = mhos

so that

$1/E = 1/(Z_0 H_\theta)$

Then above becomes

$(j\omega\varepsilon + \sigma)\, E \cos\theta = [\partial_r H_z] + H_z [\, r^{-1} + (\partial_r H_\theta)/H_\theta]$

$(j\omega\varepsilon + \sigma) \cos\theta = \{ [\partial_r H_z] + H_z [\, r^{-1} + (\partial_r H_\theta)/H_\theta] \}/ (Z_0 H_\theta)$

Now yes, if you set $H_z \equiv 0$, you get $\cos\theta = 0$ so $\theta = 90°$. But let's keep only a few terms so that

$(j\omega\varepsilon + \sigma) \cos\theta = \{ H_z [\, D^{-1}] \}/ (Z_0 H_\theta) = (1/DZ_0)\, (H_z/H_\theta)$

Dimensions? σ = mho/m DZ_0 = m ohm $1/DZ_0$ = mho/m checks

Let's now assume that $\sigma = 0$ for dielectric, then we have

$j\omega\varepsilon \cos\theta = (1/DZ_0)\, (H_z/H_\theta)$

$\cos\theta \approx (1/DZ_0)\, (H_z/H_\theta)(1/\omega\varepsilon_0)$

A condition for $E \bullet B = 0$ is then

$(1/DZ_0)\, (H_z/H_\theta)(1/\omega\varepsilon_0) << 1$

$\omega >> (1/DZ_0\varepsilon_0)\, (H_z/H_\theta)$

Now assume $\omega > 60$ Hz so at least we have

$Z_0 = K\ 30\Omega$

$\omega >> (1/DK30\varepsilon_0)\ (H_z/H_\theta)$

$\omega^2 >> (1/DK30\varepsilon_0)^2\ (H_z/H_\theta)^2$

Then from below

$$\omega^2 >> (1/DK30\varepsilon_0)^2\ \frac{\omega\varepsilon_0}{\sigma} = (1/DK30)^2\ (1/\varepsilon_0)\ \frac{\omega}{\sigma}$$

$\omega >> (1/DK30)^2\ (1/\sigma\varepsilon_0)$

Now $(1/\sigma\varepsilon_0) \approx 1/[\ 10^8\ 10^{-11}] = 10^3$

So $\omega >> (1/DK)^2$

This is a very low frequency! Suppose D = 1 cm = 10^{-2} m and suppose K = 2. then

$\omega >> [\ 2\ 10^{-2}\ ]^{-1} = 100/2 = 50$ Hz.

Then

$D\ Z_0\ \varepsilon_0 = D\ K\ 30\Omega\ 10^{-11}$ dim = m ohm farad/m = ohm-farad = sec OK

Then

$\omega >> 10^{11} / (30DK) *\ (H_z/H_\theta)$

Now for my round wire I think I had

$$B_z(r,0) = (\beta'/\omega)\ \frac{K_0}{2}\ J_0(x)$$

$$B_\varphi(r,0) = (\beta'/\omega)\ \left(\frac{\beta_d}{\beta'} + \frac{\beta'}{\beta_d}\right)\frac{K_0}{2}\ J_1(x)$$

Then

$H_z/H_\theta = (\beta_d/\beta)\, J_0(x)/J_1(x) \approx (\beta_d/\beta) = \left(\frac{\mu}{\mu_d}\ \frac{\sigma}{\omega\varepsilon_d}\right)^{-1/2}$ from below (D.2.2)

call this

$$H_z/H_\theta \approx \left(\frac{\sigma}{\omega\varepsilon_0}\right)^{-1/2}$$

Then I get

$\omega >> 10^{11} / (30DK) * (H_z/H_\theta)$

$\omega^2 >> 10^{22} / (30DK)^2 \frac{\omega\varepsilon_0}{\sigma}$

$\omega >> 10^{22} / (30DK)^2 \frac{\varepsilon_0}{\sigma} = 10^{22} / (30DK)^2 * 10^{-11} / 10^8$

$\omega >> 10^{11} / (30DK)^2 / 10^8$

$\omega >> 10^3 / (30DK)^2$

$2\pi f >> 10^3 / (30DK)^2$

$f >> (1/6)(1000\ Hz) / (30 * .01m * 5)^2$

$(1/6)(10^5\ Hz) / (30 * 1 * 5)^2 = 10^5 / 135000 = 100000/ 135000 = 1\ Hz$???

curl $\mathbf{B} = \hat{\mathbf{r}} [r^{-1}\partial_\theta B_z - \partial_z B_\theta] + \hat{\boldsymbol{\theta}} [\partial_z B_r - \partial_r B_z] + \hat{\mathbf{z}} [r^{-1}\partial_r(rB_\theta) - r^{-1}\partial_\theta B_r]$

Now keep only B_θ