Old section 3.4 retired 10.11.14

3.4 Size of E fields in conductor and dielectric; conservation of total current at a boundary

We know from (1.1.48) that the following E field condition applies at a boundary between two media, where n refers to the normal component,

$\xi_1 E_{n1} = \xi_2 E_{n2}$ // frequency domain *(1.1.48)*

or

$(\varepsilon_1 + \sigma_1/j\omega)\, E_{n1} = (\varepsilon_2 + \sigma_2/j\omega)\, E_{n2}$. (3.4.1)

Let 1 = dielectric and 2 = conductor. From (3.3.8) we set $\xi_1 \approx \varepsilon_1$ (at least for f < 10 GHz), and from (2.2.3) we set $\xi_2 \approx \sigma_2/j\omega$ for f << 10^9 GHz (for polyethylene and copper), so (3.4.1) then reads

$$\varepsilon_1 E_{n1} \approx (\sigma_2/j\omega) E_{n2} \quad \Rightarrow \quad \text{ratio} = \left|\frac{E_{n1}}{E_{n2}}\right| \approx \frac{\sigma_2}{\omega\varepsilon_1} \quad . \tag{3.4.2}$$

We can look at some typical numbers,

$\varepsilon_1 = 2.3\ \varepsilon_0$ (polyethylene) (3.4.3)
$\sigma_2 = 5.81 \times 10^7$ mho/m (copper)
$\varepsilon_0 = 8.85 \times 10^{-12}$ farad/m

so

$$\frac{\sigma_2}{\omega\varepsilon_1} = \frac{5.81 \times 10^7}{2\pi f\, 2.3\ 8.85 \times 10^{-12}} \approx (1/2)10^{18}/f \approx 10^9/[2f(\text{GHz})] \tag{3.4.4}$$

At a high frequency of f ≈ 500GHz the ratio in (3.4.2) is ~ 10^6, and at lower frequencies the ratio only increases. Thus, we arrive at these useful facts:

Fact 1: The total current in a dielectric is dominated by displacement current, while that in a conductor is dominated by conduction current. (3.4.5)

Fact 2: At a boundary between a good dielectric and a good conductor, the normal E field is at least 1 million times larger in the dielectric than it is in the conductor for frequencies under 500 GHz. (3.4.6)

Fact 3: This large jump in E_n at the boundary must be supported by a significant surface charge density n on the boundary since, according to (1.1.47), $n = \varepsilon_1 E_{n1} - \varepsilon_2 E_{n2} \approx \varepsilon_1 E_{n1}$. (3.4.7)

Imagine now a tiny patch of area (bordered in red) on the surface between a conductor and a dielectric,

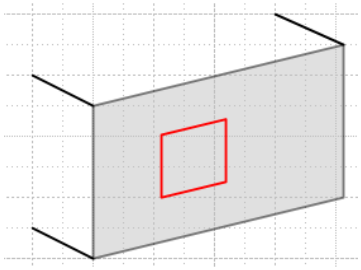


Fig 3.2

Defining a total current $J_{tot,n} \equiv j\omega\varepsilon E_n + \sigma E_n$, as in (2.2.1), we have shown that this total current flows right through the area patch but changes its nature from mostly conduction current on one side to mostly displacement current on the other side. In the next section, we identify the normal direction with the local radial direction. Then the total current passing through a tiny square patch like that in Fig 3.2 can be regarded as being "fed" by the radial current J_r just inside the conductor where $J_r = \sigma E_r$. This current feeds the surface charge on the boundary which in turn creates a large E field and thus a large displacement current in the dielectric. We sometimes refer to this mechanism as "charge pumping".