Confusions in Chap 3 PhL 10.8.13

Reviewed this doc today 10.13.13, added last section below, think it is all under control.

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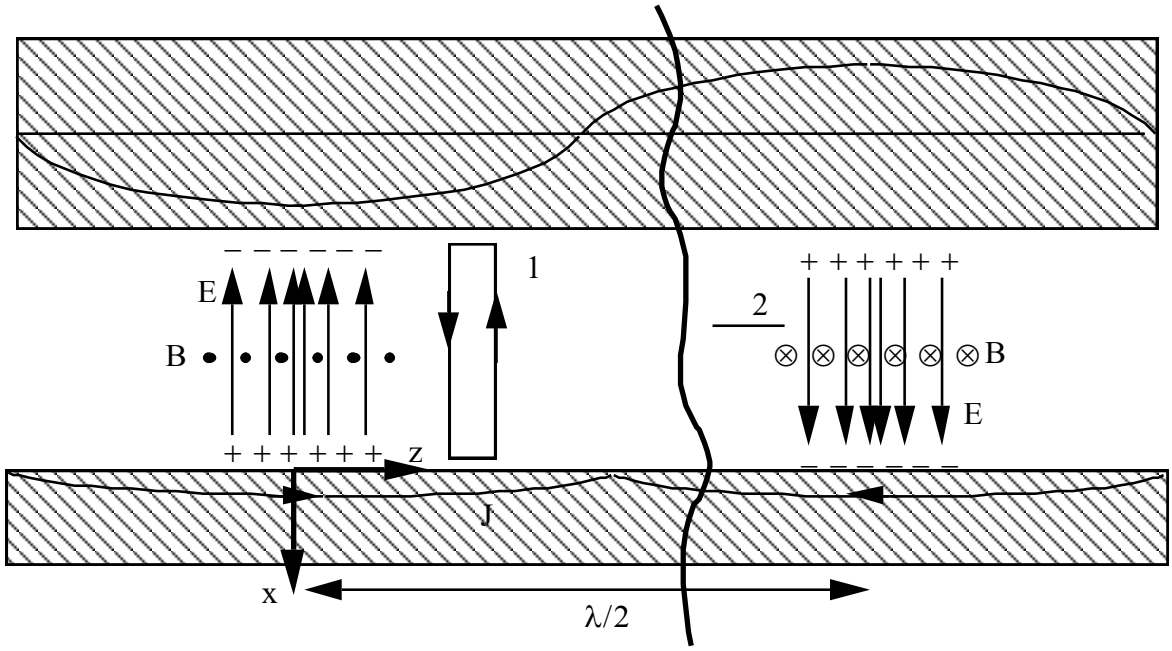
3. (10/13/13) What about those charge moments η_m in the round wire 7

1. Invalidating the connection between I and $I_{transverse}$ (hence J_z and J_r)

This chapter claims to be a qualitative description of a transmission line, but it seems very weak in many places.

3.1 no charge inside a conductor, this is fine.
3.2 charge on surface is very thin, fine.
3.3 loss tangent and dielectric : I guess OK, not clear where this fits in.
3.4 normal conservation of current at a surface -- I think I buy this section.
3.5 idea line: what do the 9 quantities look like? I guess OK
3.6 real line: what do they look like. first signs of trouble.

In Section 3.6 the flow slows down a lot. I think I am missing some basic rules of the line. So let's get right to my "famous" picture,



I claim that ALL quantities are sinusoidal, but remember they have phases that this type of picture might not be showing. The above shows one λ worth of line action and it is a photo at one instant in time. The upper sine wave I guess applies to the E and B fields,

So my first question: what about the relative phase of things? I can look at the round wire solution in box D.4.6 which is for E fields only. The E_z and E_φ field is at 0 phase, while the E_r field is at ±j phase. Thus, J_z and J_r really are 90 degrees out of phase. Why exactly is this? Let's scan thru App D and see where this comes from. We do have those "conversion rules" D.1.15 which include j's. Then in D.1.12 we see how $\partial_\varphi E_\varphi$ and $\partial_\varphi E_r$ appear in the vector Laplacian. The ∇^2 part shown in D.1.13 does have a ∂_r but this is sort of the "baseline" since final equations all have ∂_r in them, so ∂_r does not create a j factor. So in a very direct way, $\partial_\varphi E_\varphi$ and $\partial_\varphi E_r$ in those equations cause the j factors to appear in D.1.17 and 18. And the same thing puts the two j's into D.1.19. You get a j wherever there was a ∂_φ or a ∂_z in some equation.

Now look at solutions. I first find the E_z solution from D.1.16 and it has no j's, it is the baseline. The solution is D.1.27 and the constant there will be real I think. It is called C_{zm}. Later I have $K_m \equiv 2j\ (\beta_d/\beta')\ C_{zm}$ where the β's might be complex. Now in D.2.22 I eventually show K_m = real if the N_m are real, so I guess this says that K_m has j times the phase as the surface charge. But $x_a = \beta' a$ and $\beta' \approx \beta = \omega\sqrt{\mu\xi}$ from box D.4.6 and $\xi \approx \sigma/(j\omega)$. In general $\beta \equiv e^{j3\pi/4}\sqrt{\omega\mu\sigma}$ which has that funny phase so x_a then also has that funny phase. So all the J functions in the solution are complex! You cannot tell anything about phase from the leading factors, ouch!

So looking at my round wire solutions might not be all that helpful. Maybe need to look directly at some basic DE's.

div $\mathbf{E} = \rho/\varepsilon_0$: If I put a Gaussian box around some +++ charge in the above figure, it would say

$$(1/\varepsilon)\int_V \rho_T\, dV = \int_S \mathbf{E} \bullet d\mathbf{A}$$

$$(1/\varepsilon)\ ndA = EdA \qquad \Rightarrow \qquad E(z) = n(z)/\varepsilon$$

so of course these two are in phase. I guess I could integrate this over λ/2 to get

$$\langle E(z)\rangle = \langle n(z)\rangle/\varepsilon$$

But I think continuity is more important,

$$-\partial_t[\int_V \rho_T\, dV] = \int_S \mathbf{J}_T \bullet d\mathbf{A}$$

Again for the same Gaussian box,

$$-\partial_t ndA = J_r dA \qquad \Rightarrow \qquad J_r(z) = -\partial_t n(z)$$

Again we are at some point on a surface so radial has a clear meaning. This is the radial pumping idea.

Now what is the total displacement current flowing across the dielectric in a half wave? I think we can say that $J_r(r,\varphi,z)$ converts to displacement current at each point on a conductor. So we really want

$$I_{transverse} = \int dA\, J_r(r,\varphi,z) = -\partial_t \int dA\, n(r,\varphi,z) = -\partial_t Q_{halfwave}$$

where the dA integral is over the entire surface of one conductor in a half wave chunk of z.

Meanwhile, the total current in the conductor in the z direction is

$$I = \int dA_z J_z(r,\varphi,z)$$

Is there any connection between these two currents??? Well, J_z flows inside and on the surface I guess, so not directly connected to the surface charge. A single DC wire I guess has no surface charge but does have a current.

Tentative conclusion: In general, there is no simple relationship between I and $I_{transverse}$. I once suggested that one was twice the other or equal to the other or some such thing, but that seems wrong now. In any line, I is the current fed in at one end, whereas I_t will be related to the capacitance between the conductors. In a DC line, there is some I, and $I_t = 0$.

[But I have now reversed this conclusion! There is a connection, and the result is now installed as sections 3.7 (c) and (d). So in some sense $I_{transverse}$ = I after all. This is all cleared up now once I got the picture right, but I was definitely messed up on this! This involves the J_r/J_z ratio and I wasted lots of time editing in and out various explanations (see below)!]

I have to carefully remove statements to this effect from Section 3!

For example, in section 3.5 I am trying to explain why I think $J_r << J_z$. Maybe this is not true in a transmission line with a very large capacitance! [it is true] Soon after this I have this paragraph which seems completely wrong to me now:

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Estimate of J_r under the surface. The current J_r is the radial conduction current in the conductor which "feeds" the displacement current in the dielectric, as discussed in Section 3.4 above. This displacement current emits from a relatively large area of the conductor. We can estimate this area as the product of λ/2 in the longitudinal direction, and some active perimeter p in the transverse plane. As shown in Figure 2 below, over a distance of λ/2, we expect the total displacement current to be 2I [huh? Is I the total current in a conductor?] , where I is the peak current. Since we are doing ballpark estimates, we omit factors of 2 and make this estimate for the average radial current,

$$\langle J_r \rangle \sim I/(\lambda\, p) \sim (I\, f) / (pv) \qquad (3.5.1)$$

where we have replaced λ = v/f where v is the speed of light in the dielectric and f the frequency. We expect this dependence on f since the displacement current "fed" by this current is proportional to f. As a rough but conservative estimate of the size of J_r , let v ≈ c, p ≈ 2.5 mm, I ≈ 100 mA, f = 1 GHz. This

perimeter applies to a small #20 wire as found in the center of Belden 8281 cable. Then $J_r \approx 1.33 \times 10^2$ amps/m^2. As we shall see in later examples, this is a relatively "small" current density, so we have indicated this in the above table (region 2). For larger conductor size and lower frequency, J_r is even less.

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This is a very major malfunction in my document. With the round wire in App D, you could have a very large J_r if it happened that the charge moments η_m were very large. My claim for region 2 in the IDEAL analysis of Section 3.5 that E_r = tiny was based on J_r = small, and this now has no support at all! I am not sure how to approach this problem.

// I just added a lines 1.1 subsection showing how fields continue through a boundary, later I will adjust references to these conclusions.

Estimate of J_r under the surface. The claim we want to make is that $|E_r| << |E_z|$ for a transmission line operating in the "transmission line limit". This means the transverse dimensions of the line are small compared to the wavelength of the wave going down the line which is $\lambda = 2\pi/\beta_d$ where β_d is the wavenumber in the dielectric. We appeal to the case study in Appendix D of a round wire which carries a certain longitudinal wave equation solution described in that Appendix. The solution is given in box (D.4.6) which shows that E_r and E_φ have a factor $(a\beta_d)$ which E_z does not have, and the transmission line limit means $(a\beta_d) << 1$. Thus, $|E_r| << |E_z|$ both inside and just below the surface of a conductor. If the conductor has conductivity σ, then it follows that $|J_r| << |J_z|$ as well inside the conductor.

The fact that $|E_r| << |E_z|$ is difficult to prove at this point for a general transmission line geometry, so we lean on this round wire result until we have more tools available. Right now we are just outlining qualitative aspects of the transmission line so this is not a critical assumption.

Estimate of J_r under the surface. The current J_r is the radial conduction current in the conductor which "feeds" the displacement current in the dielectric, as discussed in Section 3.4 above. This displacement current emits from a relatively large area of the conductor. We can estimate this area as the product of $\lambda/2$ in the longitudinal direction, and some active perimeter p in the transverse plane. As shown in Figure 2 below, over a distance of $\lambda/2$, we expect the total displacement current to be 2I, where I is the peak current. Since we are doing ballpark estimates, we omit factors of 2 and make this estimate for the average radial current,

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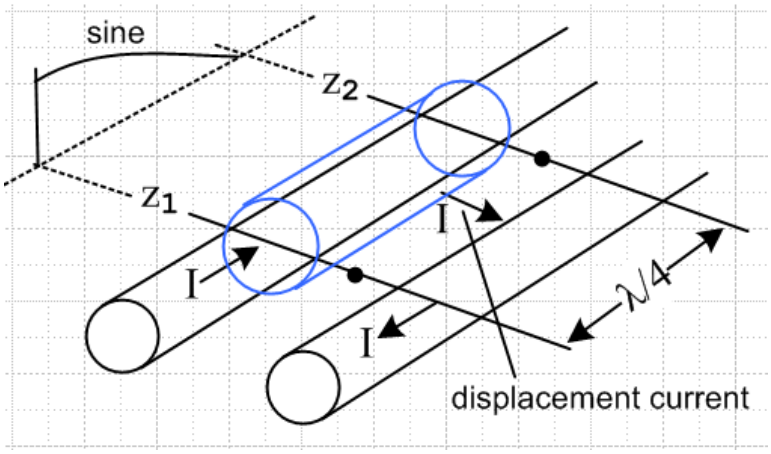
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2. Reestablishing the connection between I and $I_{transverse}$ (hence J_z and J_r)

Oct 10, 2013. HOWEVER, I maybe can rejuvenate my argument discarded above! Consider this picture

The relationship between J_r and J_z in Region 2 .

Consider the following drawing of a piece of a transmission line:



In this picture the wavelength λ is highly distorted; it is intended to be much larger than the transverse dimensions of the transmission line. The picture is drawn at an instant in time when the total longitudinal current in the left conductor has its maximum value I at $z = z_1$ and vanishes at $z = z_2$. Thus, a total current of I is entering the interior of the left conductor between z_1 and z_2. This current has to go somewhere, and one can regard it as charging the capacitance of the quarter wave transmission line section between z_1 and z_2. In other words, this total current I is equal to the integral of the dielectric displacement current density over some area which divides the two conductors, such as the blue cylinder shown. As discussed above, the displacement current is fed by the radial current density J_r just inside the conductor. As a rough estimate, if the active perimeter of the left conductor is p, then

$$p * \lambda/4 * J_r * (2/\pi) \approx I \qquad \Rightarrow \qquad p\lambda J_r \approx 2\pi I \qquad \Rightarrow \qquad J_r = 2\pi I/(p\lambda)$$

Here $(2/\pi)$ represents the average value of the sine-shaped displacement current curve over the z region of interest, and p would be $2\pi a$ for a wire of radius a if the two conductors were widely separated. For an arbitrary conductor shape and position, p is some effective distance associated with the transverse geometry; it is the "active" perimeter discussed earlier.

On the other hand, for a round conductor operating in the skin effect regime where $\delta < a$,

$$J_z \approx I/(p\delta)$$

where p is the same active perimeter just mentioned. So

$$J_z \approx I/(p\delta)$$

$J_r \approx 2\pi I/(p\lambda)$

$J_r/J_z \approx 2\pi\,(\delta/\lambda)$.

For δ we had

$$\delta \equiv \sqrt{2/\omega\mu\sigma} \qquad (2.1.20)$$

For λ one may write,

$\lambda = v/f = 2\pi v/\omega$

where v is the wave velocity. Then

$$(\delta/\lambda) = \sqrt{\frac{2}{\omega\mu\sigma}} * \frac{\omega}{2\pi v} = \sqrt{\frac{2\omega}{\mu\sigma}}\ \frac{1}{2\pi v} = \sqrt{\frac{4\pi f}{\mu\sigma}}\ \frac{1}{2\pi v}\ .$$

Setting $v \approx c$ and $\mu = \mu_0 = 4\pi \times 10^{-7}$ and $\sigma = 5.81 \times 10^7$ (copper) and $f = 10^9 f(\text{Ghz})$ we get

$$(\delta/\lambda) \approx \sqrt{\frac{4\pi f}{\mu\sigma}}\ \frac{1}{2\pi v} = \sqrt{\frac{4\pi\ 10^9\ f(\text{GHz})}{4\pi \times 10^{-7} \times 5.81 \times 10^7}}\ \frac{1}{2\pi}\ \frac{1}{3 \times 10^8}$$

$$= \sqrt{\frac{10^{10}\ f(\text{GHz})}{58.1}}\ \frac{1}{2\pi}\ \frac{1}{3 \times 10^8} = \sqrt{\frac{f(\text{GHz})}{58.1}}\ \frac{1}{6\pi}\ 10^{-3} = 7 \times 10^{-6}\ \sqrt{f(\text{GHz})}$$

and so

$J_r/J_z \approx (2\pi)\,(\delta/\lambda) \approx 4.4 \times 10^{-5}\ \sqrt{f(\text{GHz})}$

For $f \leq 10$ GHz we then find $J_r/J_z \leq 1.4 \times 10^{-4}$.

For a conductor at low frequency, things are different

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So I will go with the above, and remove my hedging section from Section 3.5:

Estimate of J_r under the surface. The claim we want to make is that just below the conductor surface $|E_r| << |E_z|$ for a transmission line operating in the "transmission line limit". This limit means the transverse dimensions of the line are small compared to the wavelength of the wave going down the line which is $\lambda = 2\pi/\beta_d$ where β_d is the wavenumber in the dielectric. We appeal to the case study in Appendix D of a round wire (radius a) which carries a certain longitudinal wave equation solution described in that Appendix. The solution is given in box (D.4.6) which shows that E_r and E_φ have a factor $(a\beta_d)$ which E_z does not have, and the transmission line limit means $(a\beta_d) << 1$. Thus, $|E_r| << |E_z|$ both inside and just

below the surface of a conductor. If the conductor has conductivity σ, then it follows that $|J_r| << |J_z|$ as well inside the conductor. The fact that $|E_r| << |E_z|$ is difficult to prove at this point for a general transmission line geometry, so we lean on this round wire result until we have more tools available. Right now we are just outlining qualitative aspects of the transmission line so this is not a critical assumption.

3. (10/13/13) What about those charge moments η_m in the round wire?

"With the round wire in App D, you could have a very large J_r if it happened that the charge moments η_m were very large."

This statement requires some kind of response. In App D I solve for the fields inside and outside of a round wire which carries a longitudinal wave. The only equations I make use of are these:

Helmholtz equation for **E**
div **E** = 0
charge conservation for box straddling the surface

In (D.4.6) I come up with a complete solution for the E fields inside the wire (and outside two). In these solutions the surface charge in each partial wave is represented by η_m . I was able to relate η_0 directly to the wire current I, but the higher moments η_m are all arbitrary!

When this round wire is made part of a transmission line, I think the η_m then get nailed down and are no longer free parameters. There is some computable variation in J_z inside the wire, and this corresponds somehow to a set of η_m values. We see this happening on our formal transmission line solution of Chapters 4 and 5. Having "the other wire" present creates a more specific problem and it has boundary conditions and those force the η_m values on the original round wire.