Old Jr argument PhL 10.12.13

This doc is now replaced by Section 3.7 (c) and (d). *In retrospect*, the J_r argument presented here is exactly right!! The only thing wrong is that the "sine" graph is reversed, but this does not affect anything since I just use the average of sine over a quarter wave which is invariant under flipping. You can identify the blue Gaussian box below with the left half of the green one in the Section 3.7(c) figure!

This doc contains two other random items:

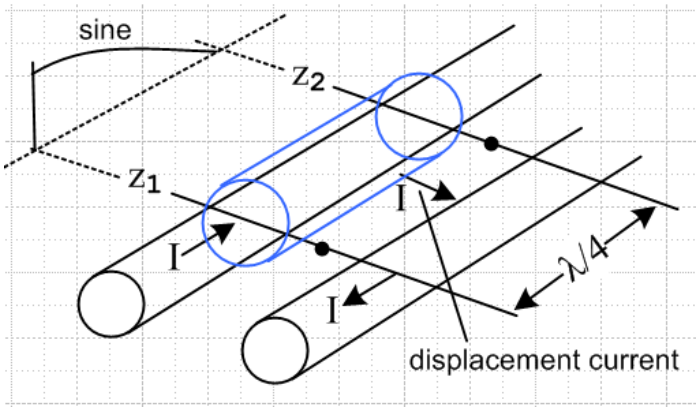
Original Table 3 in Section 3.6 which I got rid of and stored here.

Old cross section ratty fields picture from original doc

(d) J_z and J_r in the conductor.

In the table above for region 2 we have set J_z "large" and J_r "small". In this section, we show that the ratio J_r/J_z is quite small, that that is why J_r is small in the table.

Consider the following drawing of a piece of a transmission line:



In this picture the wavelength λ is highly distorted; it is intended to be much larger than the transverse dimensions of the transmission line. The picture is drawn at an instant in time when the total longitudinal current in the left conductor has its maximum value I at $z = z_1$ and vanishes at $z = z_2$. Thus, a total current of I is entering the interior of the left conductor between z_1 and z_2. This current has to go somewhere, and one can regard it as charging the capacitance of the quarter wave transmission line section between z_1 and z_2. In other words, this total current I is equal to the integral of the dielectric displacement current density over some area which divides the two conductors, such as the blue cylinder shown. As discussed above, the displacement current is fed by the radial current density J_r just inside the conductor. As a rough estimate, if the active perimeter of the left conductor is p, then

$$p * \lambda/4 * J_r * (2/\pi) \approx I \quad \Rightarrow \quad p\lambda J_r \approx 2\pi I \quad \Rightarrow \quad J_r = 2\pi I/(p\lambda) \qquad (3.6.4)$$

Here $(2/\pi)$ represents the average value of the sine-shaped displacement current curve over the z region of interest, and p would be $2\pi a$ for a wire of radius a if the two conductors were widely separated. For an

arbitrary conductor shape and position, p is some effective distance associated with the transverse geometry; it is the "active" perimeter discussed earlier.

On the other hand, for a round conductor operating in the skin effect regime where $\delta < a$,

$$J_z \approx I/(p\delta) \tag{3.6.5}$$

where p is the same active perimeter just mentioned. So

$$J_z \approx I/(p\delta)$$
$$J_r \approx 2\pi I/(p\lambda)$$

$$J_r/J_z \approx 2\pi\,(\delta/\lambda)\,. \tag{3.6.6}$$

For δ we had

$$\delta \equiv \sqrt{2/\omega\mu\sigma} \tag{2.1.20}$$

For λ one may write,

$$\lambda = v/f = 2\pi v/\omega \tag{3.6.7}$$

where v is the wave velocity. Then

$$(\delta/\lambda) = \sqrt{\frac{2}{\omega\mu\sigma}} \; * \; = \sqrt{\frac{2\omega}{\mu\sigma}} \; \frac{1}{2\pi v} = \sqrt{\frac{4\pi f}{\mu\sigma}} \; \frac{1}{2\pi v}\;. \tag{3.6.8}$$

Setting $v \approx c$ and $\mu = \mu_0 = 4\pi \times 10^{-7}$ and $\sigma = 5.81 \times 10^{7}$ (copper) and $f = 10^{9} f(\text{Ghz})$ we get

$$(\delta/\lambda) \approx \sqrt{\frac{4\pi f}{\mu\sigma}} \; \frac{1}{2\pi v} = \sqrt{\frac{4\pi\, 10^{9}\, f(\text{GHz})}{4\pi \times 10^{-7} \times 5.81 \times 10^{7}}} \; \frac{1}{2\pi} \; \frac{1}{3 \times 10^{8}}$$

$$= \sqrt{\frac{10^{10}\, f(\text{GHz})}{58.1}} \; \frac{1}{2\pi} \; \frac{1}{3 \times 10^{8}} = \sqrt{\frac{f(\text{GHz})}{58.1}} \; \frac{1}{6\pi} \; 10^{-3} = 7 \times 10^{-6} \sqrt{f(\text{GHz})}$$

and so

$$J_r/J_z \approx (2\pi)\,(\delta/\lambda) \approx 4.4 \times 10^{-5} \sqrt{f(\text{GHz})} \tag{3.6.9}$$

For $f \leq 10$ GHz we then find

$$J_r/J_z \leq 1.4 \times 10^{-4}\,. \qquad f \leq 10 \text{ GHz} \tag{3.6.10}$$

A round conductor is in the low-frequency non-skin-effect regime when $\delta > a$, where a is the wire radius. This means

$$\sqrt{2/\omega\mu\sigma} > a \qquad \Rightarrow \qquad \omega < 2/(\mu\sigma a^2) \qquad \text{or} \qquad \omega a/2 < 1/(\mu\sigma a) \qquad (3.6.11)$$

In this low frequency regime we must replace (3.6.5) by

$$J_z \approx I/(\pi a^2) \qquad (3.6.12)$$

Since (3.6.4) is still valid, we find now that

$$J_z \approx I/(\pi a^2)$$
$$J_r \approx 2\pi I/(p\lambda) \approx 2\pi I/(2\pi a\lambda) \approx I/(a\lambda)$$

$$J_r/J_z \approx \pi(a/\lambda) \approx (\pi a)(\omega/2\pi v) \approx \omega a/2v = (\omega a/2)(1/v) \qquad (3.6.13)$$

Using ** this says

$$J_r/J_z < 1/(\mu\sigma a v) \qquad (3.6.14)$$

With $\mu = \mu_0 = 4\pi \times 10^{-7}$, $\sigma = 5.81 \times 10^7$ (copper) and $v = c = 3 \times 10^8$ we find for a wire of radius 1 mm,

$$J_r/J_z < \frac{1}{4\pi * 5.81 * 10^{-3} * 3 * 10^8} = \frac{10^{-5}}{4\pi * 5.81 * 3} = 4.6 \times 10^{-8} \qquad (3.8.15)$$

Therefore, our high frequency result of 1.4×10^{-4} for $f \leq 10$ GHz represents a much worse case, so we will stick with that number in our work below.

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We now display once again the table for a real transmission line, showing relative estimates of the sizes of things . A few primes have been added to remove ambiguities. [huh?]

Table 3: E, B and J for a real transmission line

Region 2. In the conductor, just under the surface charge layer, and in the current layer.

$E_r' \leq 10^{-6}\ E_r$ (c)	$B_r = 0$	$J_r \leq 2 \times 10^{-5}\ J_z$ (see later)
$E_\varphi = 0$	$B_\varphi \sim 20\ \mu T$	$J_\varphi = 0$
$E_z \leq 2\times10^{-4}\ E_r$ (a)	$B_z \sim 0.25\mu T$	J_z = large [$\sim 10^7$ A/m^2]

Region 3. In the dielectric, just outside the super-thin surface charge layer.

E_r = large [1500 v/m]	$B_r = 0$	$J'_r \leq 2\times10^{-4}\ J_r$ (d)

$E_\varphi = 0$	$B_\varphi \sim 20\ \mu T$	$J_\varphi = 0$
$E_z \leq 2x10^{-4}\ E_r$ (a)	$B_z \sim 0.25 \mu T$	$J_z' \approx 10^{-12}\ J_z$ (b)

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