3.1 Why is there no charge inside a conductor?

There can be charge inside a conductor, but only if the conductor is excited at an extremely high frequency. A charge density ρ inside the conductor implies an electric field E according to div $\mathbf{E} = \rho/\varepsilon$. This **E** field then causes a current $\mathbf{J} = \sigma\mathbf{E}$ which attempts to drain the charge off to the surface.

Here is a simple way to estimate the time constant for this process. Imagine a one dimensional conductor (dimension x) with some charge density ρ inside. Then from div $E = \rho/\varepsilon_0$ we get

$$\partial_x E_x = \rho/\varepsilon_0 \ . \tag{3.1.1}$$

The electric field so generated causes a current $J_x = \sigma E_x$ so apply ∂_x to get

$$\partial_x J_x = \sigma \partial_x E_x \quad . \tag{3.1.2}$$

But this current drains the charge away according to div $\mathbf{J} = -\partial_t\rho$, which says

$$\partial_x J_x = -\partial_t\rho \ . \tag{3.1.3}$$

Combining these three equations we get

$$\partial_t\rho = -(\sigma/\varepsilon_0)\,\rho \ . \tag{3.1.4}$$

This implies that ρ decays exponentially with time constant

$$T = \varepsilon_0/\sigma \ . \tag{3.1.5}$$

For copper, $\sigma = 5.81 \times 10^7$ mho/m, and $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, so $T = 1.52 \times 10^{-19}$ sec. Thus, any process which occurs inside a conductor at a frequency much less than 10^{19} Hz always allows plenty of time for any interior charge to move to the surface. This is 10^{10} GHz, far beyond the operating frequency of a transmission line. We may therefore conclude that:

Fact 1: In a transmission line, charge exists only on the surface of conductors.