Bug in Section 4.7 PhL 3.26.05

Statement of the Problem

4.7 BUG below (4.7.7) talking about μ: Why do I say there is no free surface current on a TL conductor? Later I say that there *is* in fact a Debye Surface current which is caused by the motion of the "free" surface charge. When I wrote Section 4.7, I was probably unaware of the Debye current. This could be a major problem!

Ouch ouch ouch! Time to digress! I start with

$$A_{z1}(x,y,z) = \frac{\mu_1}{4\pi} \int_{-\infty}^{\infty} dz'\, i(z') \int_{C_1} dx'\, dy'\, b_1(x',y') \frac{e^{-j\beta_d R}}{R} \quad . \tag{4.7.6}$$

I claim that there is no "magnetic boundary", so this function and its derivatives are continuous through the conductor boundary. King I think says this as well. *But* if there is a Debye surface current, then the first derivative is NOT continuous. Both facts cannot be true!! Yeouch!

Let's look at the picture

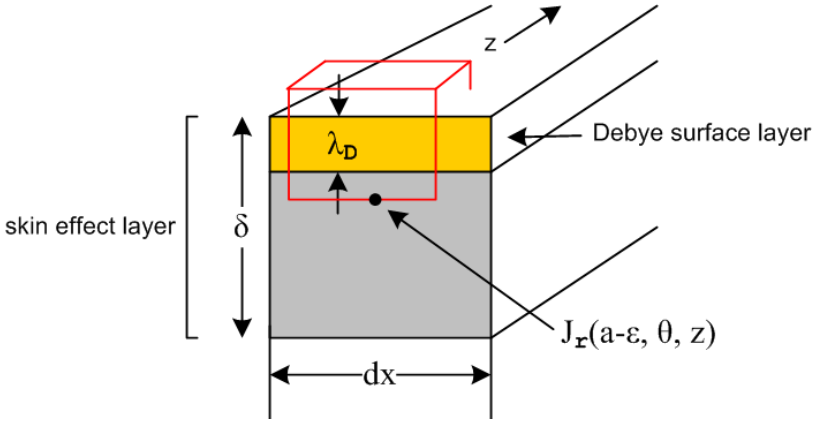


Fig D.6

Suppose I regard the "boundary" as being the upper surface. Then on that surface there is no free surface current, so $K_z = 0$, and then my argument is OK if + and - refer to the two sides of this *top* surface. I am then putting the Debye surface current inside the conductor along with the regular conduction current.

Comments regarding μ

This is a subtle subject and is not discussed in King's transmission line theory book.

If $\mu_1 = \mu_d$, and if there is no surface current on the C_1 conductor boundary, then in fact there is no "magnetic boundary" at the conductor surface. The solution (4.7.2) is then smooth at this boundary, and so $A_{z1}(x,y,z)$ "naturally" satisfies these two boundary conditions,

$$A_{z1}(\mathbf{x}+) = A_{z1}(\mathbf{x}-)$$
$$(1/\mu_d)\partial_n A_{z1}(\mathbf{x}+) = (1/\mu_1)\,\partial_n A_{z1}(\mathbf{x}-) \qquad (4.7.7)$$

where $\mathbf{x}+$ is just outside the conductor surface and $\mathbf{x}-$ is just inside. The second equation here is just (1.1.46) in the case there is no free surface current K_z^{free}. In fact there is a Debye surface current as discussed in Section D.9, but it is totally negligible compared to the bulk conductor current and we therefore just ignore it. Since we have assumed that $\mu_d = \mu_1$, this second boundary condition just says $\partial_n A_{z1}(\mathbf{x}+) = \partial_n A_{z1}(\mathbf{x}+)$. Since there is no magnetic boundary at the conductor/dielectric interface, the solution (4.7.2) is continuous and all its derivatives are also continuous at the boundary, since nothing special happens at that boundary. Thus, the Helmholtz integral solution provides the whole solution for A_{z1} since it meets both "boundary conditions" at this pseudo boundary.

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Question: Later I say this:

$$\varphi_1(\mathbf{x}+) = \varphi_1(\mathbf{x}-)$$
$$[\varepsilon_1\partial_n\varphi_1(\mathbf{x}+) - \varepsilon\partial_n\varphi_1(\mathbf{x}-)] = n_{free}(\mathbf{x}). \qquad (4.7.10)$$

where n_{free} cannot be just neglected. Suppose both C_1 and dielectric had ε_0. Then this would say

$$[\varepsilon_0\partial_n\varphi_1(\mathbf{x}+) - \varepsilon_0\partial_n\varphi_1(\mathbf{x}-)] = n_{free}(\mathbf{x}). \qquad (4.7.10)$$

which says

$$\partial_n\varphi_1(\mathbf{x}+) - \partial_n\varphi_1(\mathbf{x}-) \neq 0$$

But at the same time I argue that the Helm integral provides the complete solution!

$$\varphi_1(\mathbf{x},\omega) = \frac{1}{4\pi\xi_d}\int_{C_1}\rho_1(x',y',z',\omega)\,\frac{e^{-j\beta_d R}}{R}\,dx'dy'dz' \quad . \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad (4.1.1)$$

If this is a complete and smooth solution, I would argue that all derivatives match on both sides of the boundary and that includes this one: $\partial_n\varphi_1(\mathbf{x}+) - \partial_n\varphi_1(\mathbf{x}-) = 0$.

Ouch again. This is really a surface layer integral,

$$\varphi_1(\mathbf{x},\omega) = \frac{1}{4\pi\varepsilon_d} \int_{C1} n_s(\mathbf{x}',\omega) \frac{e^{-j\beta_d R}}{R} dS' \qquad R = |\mathbf{x} - \mathbf{x}'| \tag{1.5.13}$$

Stak tells us what happens when we approach this thing from the two sides.

$$u(x) = \int_\sigma dS_\xi \, a(\xi) \, E(x|\xi)$$

$$u(s) = \int_\sigma dS_\xi \, a(\xi) \, E(s|\xi) \tag{B.6.2}$$

$$\partial_\nu u(x) = \int_\sigma dS_\xi \, a(\xi) \, \partial_\nu E(x|\xi)$$

$$\partial_\nu u(s) \;=\; [\partial_n u(x)]^{x \to s^\pm} = \int_\sigma dS_\xi \, a(\xi) \, \partial_\nu E(s|\xi) \;\mp a(s)/2 \qquad \text{// extra term !} \tag{B.6.3}$$

and this is why n_s appears in the jump condition! So the φ Helm integral is NOT so smooth as I was saying.

For A_z I guess we could consider the integral as two terms, where one is for the bulk current, and the other is for the Debye Surface current. This second contribution would then have a jump. I better then not make any comments about all derivatives being continuous.

Comments regarding μ

This is a subtle subject and is not discussed in King's transmission line theory book. In this section we regard conductor C_1 as having parameters ε_1, μ_1 while the dielectric outside the conductor has ε_d and μ_d.

Before dealing with A_z and μ, it is useful to start with φ and ε. Our Helmholtz integral solution for φ at a point **x** in the dielectric, expressed for a single conductor C_1, can be written as follows based on (1.5.13),

$$\varphi(\mathbf{x},\omega) = \frac{1}{4\pi\varepsilon_d} \int_{C_1} n_s(\mathbf{x}',\omega) \frac{e^{-j\beta_d R}}{R} dS' \;. \qquad R = |\mathbf{x} - \mathbf{x}'| \tag{1.5.13}$$

The potential φ is continuous at the boundary, but $\partial_n\varphi$ is not continuous, having a jump there, though this fact is not obvious since we only have φ stated for **x** in the dielectric. We do know from (1.1.47) that,

$$[\varepsilon_a E_{an} - \varepsilon_b E_{2b}] \;=\; n_{free} \;. \tag{1.1.47}$$

Letting a = dielectric and b = conductor, and with $\hat{\mathbf{n}}$ pointing out from the conductor, we translate this equation to read,

$$\varepsilon_d E_{n,d} - \varepsilon_1 E_{n,c} = n_s \tag{4.7.7}$$

where n_s is the free surface charge appearing in (1.5.13). According to (4.7.1) we can set transverse A_i components to zero, so then (1.3.1) which says $\mathbf{E} = -\text{ grad } \varphi - j\omega\mathbf{A}$ tells us that $E_n = -\partial_n\varphi$. We then have

$$\varepsilon_d\ (-\partial_n\varphi(\mathbf{x}+)) - \varepsilon_1\ (-\partial_n\varphi(\mathbf{x}-))\ =\ n_s(\mathbf{x}) \tag{4.7.8}$$

where $\mathbf{x}+$ is just outside the conductor surface and $\mathbf{x}-$ is just inside. From this result one can compute the discontinuity or jump in $\partial_n\varphi$ at the conductor boundary. If $\varepsilon_d = \varepsilon_1 = \varepsilon_0$, then $\partial_n\varphi(\mathbf{x}+) - \partial_n\varphi(\mathbf{x}-) = -n_s/\varepsilon_0$, for example.

Normally, however, $E_{n,c} \approx 0$ so (4.7.7) reads $\varepsilon_d E_{n,d} = n_s$ and then $E_{n,d} = n_s/\varepsilon_d$ is the normal E field in the dielectric just outside the conductor. And since $E_{n,c} \approx 0$, we have $\varphi(\mathbf{x},\omega) \approx$ constant inside the conductor, so we are generally not interested in finding a version of (1.5.13) that is valid inside the conductor. We just evaluate (1.5.13) at the surface for some $\mathbf{x}+$ and that gives φ inside the conductor. Thus, the Helmholtz integral (1.5.13) provides our full solution of interest, and we have no homogeneous adder terms to worry about of the type discussed below.

With this as warm up, we now consider the case of A_z and μ. Our Helmholtz integral solution for A_z at a point in the dielectric, expressed for a single conductor C_1, can be written as follows based on (4.7.2),

$$A_{z1}(\mathbf{x},\omega) = \frac{\mu_1}{4\pi}\int_{C_1} J_{z1}(x',y',z',\omega)\,\frac{e^{-j\beta_d R}}{R}\,dx'dy'dz'\ . \qquad R = |\mathbf{x} - \mathbf{x}'| \tag{4.7.2}$$

where μ_1 is for the conductor C_1. The potential A_{z1} is continuous at the boundary but $\partial_n A_{z1}$ is not continuous, having a jump.

Appendix D.9 shows that there is a miniscule free surface current K_z^D which flows on the surface of a transmission line conductor, which we call the Debye surface current. It is n_s being moved slightly by E_z at the surface. Equation (D.9.3) shows that the Debye current contribution to the above integral is totally negligible relative to bulk current contribution, so the left side of (4.7.2) is unchanged if we completely ignore this Debye current contribution to J_z. Thus, we are in effect setting $K_z^D = 0$ in this well justified approximation. We may then apply (1.1.46) to find that

$$(1/\mu_d)\,(\partial_n A_{z1}(\mathbf{x}+)) - (1/\mu_1)\,(\partial_n A_{z1}(\mathbf{x}-))\ =\ 0\ . \tag{1.1.46}$$

Thus we arrive at these boundary conditions on A_{z1} produced by conductor C_1 :

$$\begin{aligned} &A_{z1}(\mathbf{x}+)\ = A_{z1}(\mathbf{x}-) \\ &(1/\mu_d)\,\partial_n A_{z1}(\mathbf{x}+) = (1/\mu_1)\,\partial_n A_{z1}(\mathbf{x}-) \end{aligned} \tag{4.7.9}$$

where again $\mathbf{x}+$ is just outside the conductor surface and $\mathbf{x}-$ is just inside.

If $\mu_1 = \mu_d$, there is no "magnetic boundary" at the surface, and (4.7.9) says $\partial_n A_{z1}(\mathbf{x}+) = \partial_n A_{z1}(\mathbf{x}-)$, so both the function A_{z1} and its normal derivative are continuous through the boundary -- nothing special is happening there. Thus, the Helmholtz integral solution (4.7.2) provides the whole solution for A_{z1} in both the dielectric and conductor since it meets both "boundary conditions" at this pseudo boundary.

If on the other hand we have $\mu_1 \neq \mu_d$, then there *is* a magnetic boundary between conductor and dielectric which we have to worry about. In this case, (4.7.2) applied in both dielectric and conductor cannot possibly satisfy the second boundary condition of (4.7.9) since, as already noted, the A_{z1} of (4.7.2) satisfies $\partial_n A_{z1}(\mathbf{x}+) = \partial_n A_{z1}(\mathbf{x}+)$. Thus, in this case (4.7.2) is not the full solution for A_{z1}. One must add a homogeneous Helmholtz equation solution to (4.7.2) in order to have a proper solution for A_{z1} that satisfies both equations in (4.7.9).

It *turns out* that the correct total A_{z1} solution can be generated by adding a certain fictitious surface current term to $\mu_1 J_{z1}$ in (4.7.2). Since such a surface current vanishes on both sides of the boundary between μ_d and μ_1, the Helmholtz solution due just to this surface current term is in fact a homogeneous solution to the Helmholtz equation in both the conductor and dielectric regions, away from that boundary. It turns out moreover that the correct fictitious surface current to add is in fact the magnetization surface current J_m which is created at the boundary between $\mu_d \neq \mu_1$. Adding this surface current is just a "trick" in order to generate the correct homogeneous adder solution so that the resulting total A_{z1} satisfies both boundary conditions in (4.7.9). Formally speaking, the $\mathbf{J}_i$ appearing in (1.5.4) and then J_{z1} in (4.7.2) should *not* include such magnetization currents since this **J** is really the **J** in Maxwell's equation curl **H** = $\partial_t \mathbf{D}$ + **J,** and this **J** does not include magnetization currents -- it includes only normal conduction currents.

In our current Chapter 4, we *want* (4.7.2) to represent the complete solution for A_{z1} and for that reason we must restrict our analysis to the situation where dielectric and all conductors have the same permeability which we shall just call μ_d. In practice, one normally has $\mu_d = \mu_1 = \mu_0$. In order to handle the more general case of $\mu_d \neq \mu_1$, we have to deal with the inhomogeneous adder solutions or equivalently with the abovementioned fictitious surface current, and this complicates our analysis which is already quite complicated. So, for the moment, we now make the same assumption made by King and other authors:

Fact: From now on, conductors and dielectric must have the same permeability μ_d. (4.7.10)

After fully developing this special case, we shall then extend the theory in Section 4.13 to allow for $\mu_d \neq \mu_1$.

There are several Appendices which relate to this subject.

Appendix G shows for the round wire how the inhomogeneous adder solution is found and how it then causes the boundary conditions (4.7.9) to be met when $\mu_1 \neq \mu_d$. However, in Appendix G the notation is different: 1 = dielectric and 2 = conductor, and $\mu_2 \neq \mu_1$ is studied.

Appendix B shows how the addition of a fictitious surface current term $\mu_0 J_m$ provides an alternate and simpler solution to the same problem of meeting boundary conditions (4.7.9) when $\mu_1 \neq \mu_d$. It then shows exactly how this works in the special case of a round wire (same notation as Appendix G).