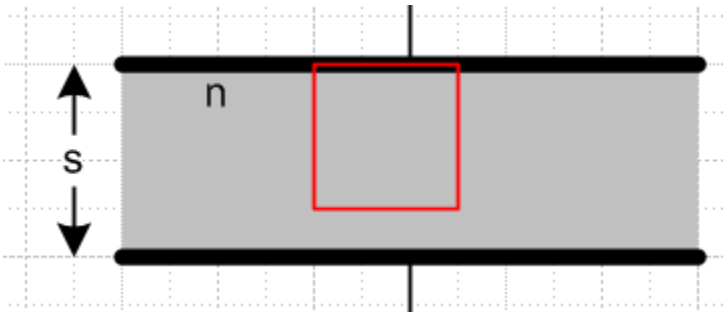
Capacitor with conducting dielectric PhL 10.16.13

I suddenly realize that I have never solved this problem, and one must solve it cleanly first before attempting a transmission line problem!

1. Parallel plate cap with large area A 1
2. Cap made of long cylinder-like conductors 2

1. Parallel plate cap with large area A



Put a gaussian box which includes the charge on the positive plate. Apply this law

$$\text{div } \mathbf{D} = \rho \quad \Leftrightarrow \quad \int_V \rho \, dV = \int_S \mathbf{D} \bullet d\mathbf{A} \qquad (1.1.13)$$

The left side will be ndA. The right side we assume only contribution is the far surface of the box, and since $\mathbf{D} = \varepsilon\mathbf{E}$ have the RHS = ε E dA, and therefore

$$n = \varepsilon E$$

and also E is independent of z falls out of this analysis, since z appears nowhere in this result. The total charge on the plate is

$$Q = nA.$$

2. If the voltmeter voltage is the line integral of E between the plates, then

$$V = Es = (n/\varepsilon)s = ns/\varepsilon = Qs/\varepsilon A$$

Since $Q = CV$, we say

$$C = \varepsilon(A/s)$$

and there is the capacitance, boosted by ε as expected. Function of geometry and ε.

But something is missing here! Where is the leakage voltage drop? Well I think we know

$R = (1/\sigma)s/A = s/(\sigma A)$

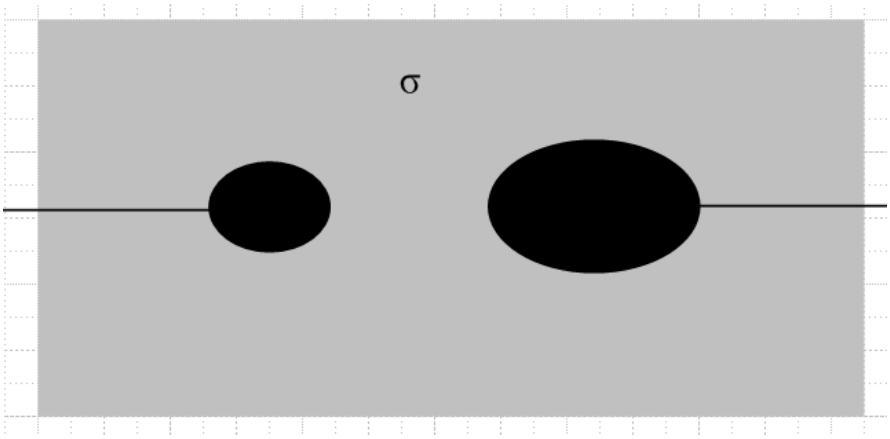
$I = V/R = V\sigma A/s$

$J = V\sigma/s$

$E = V/s$

and this is consistent with that found above. This is a DC situation. So we just have our capacitor in which we have a leakage current, all is well.

2. Cap made of long cylinder-like conductors



Imagine we solve this potential theory problem? Is Laplace valid with σ ? Very good question. If we use the potential with the Lorenz gauge, our potential equation here is

$$(\nabla^2 + \beta^2)\,\varphi(\mathbf{x},\omega) = -(1/\varepsilon)\,\rho(\mathbf{x},\omega) \qquad (1.5.3)$$

where $\beta^2 = \omega^2\varepsilon\mu$ does not see σ ! I don't think ρ is at all affected by σ, it is not in the simpler problem above. Maybe φ is the voltmeter voltage after all. Suppose yes. Then solve the above for φ(x) and we find

$V = Q/C$

where C is some function of the geometry. This result is not affected by σ. The new feature is this

$\mathbf{E}(x) = -\nabla\varphi(x) + \partial_t\mathbf{A}(x)$

$\mathbf{J}(\mathbf{x}) = \sigma\mathbf{E}(\mathbf{x})$

So you have already solved for **E**(x), the new feature is that there now exists some **J**.

What is the total current? Put a surface around conductor 1 and integrate over it. E = n/ε just outside the surface. so the surface integral of E•dA is Q/ε.

$$\int \mathbf{E}\bullet d\mathbf{A} = \int (n(x)/\varepsilon) dA = Q/\varepsilon$$

where A is the area of conductor C_1 say. Then

$$I_{leak} = \int \mathbf{J}\bullet d\mathbf{A} = \int \sigma \mathbf{E}\bullet d\mathbf{A} = \sigma\, Q/\varepsilon$$

Then it must be that

$$R = V/I_{leak} = V\varepsilon/(\sigma Q) = \varepsilon/(\sigma C)$$

The impedance of the capacitor is then

$$Z = R + jX_c = R + j\,(1/\omega C)$$

$$= \varepsilon/(\sigma C) + j\,(1/\omega C) = (1/C)(\varepsilon/\sigma + j/\omega)$$

The admittance is then

$$Y = 1/Z = C\,(\varepsilon/\sigma + j/\omega)^{-1} = C\varepsilon\sigma\,/\,(\varepsilon^2\sigma^2 + 1/\omega^2) - (Cj/\omega)\,/\,(\varepsilon^2\sigma^2 + 1/\omega^2)$$

Why is this so messy? Maybe I will say

$$G = 1/R = \text{conductance}$$

$$Y = G + j\omega C = (\sigma C/\varepsilon) + j\omega C = j\omega C\,[\,1 + \sigma/j\omega\varepsilon]$$

$$= j\omega C[\,\varepsilon + \sigma/j\omega]/\varepsilon = j\omega C(\xi/\varepsilon) = j\omega C'$$

$$\Rightarrow \qquad C' = C + G/j\omega$$

The idea here is this: Once you know the capacitance C, you also know the total Y without having to do any more work. [This fact should be installed somewhere in lines doc]

Thus, I am able to get the conductance to appear at least in my Case 2 capacitor situation.