Chapter 4 Problems PhL 3.26.05

I found a problem of some sort as noted below.

Here I am just tracing through section 4.5 looking for trouble"

There are two equations from Chapter 1 which we now wish to press into service:

$$\mathbf{E} = -\,\text{grad}\,\varphi - \partial\mathbf{A}/\partial t \qquad \text{div}\,\mathbf{A} = -\,\frac{\mu\varepsilon}{c^2}\frac{\partial\varphi}{\partial t} \tag{4.5.2}$$

If we regard **B** = curl **A** as the definition of **A**, then the first equation above can be regarded as the definition of φ. The second equation above is our Lorenz gauge condition. In the frequency domain these equations become

$$\mathbf{E} = -\,\text{grad}\,\varphi - j\omega\mathbf{A} \qquad \text{div}\,\mathbf{A} = -\,j\,(\beta^2/\omega)\,\varphi \tag{4.5.3}$$

Since **A** has only component A_z, these equations become,

$$E_z = -\,\partial\varphi/\partial z - j\omega A_z$$

$$\partial A_z/\partial z = -\,j\,(\beta^2/\omega)\,\varphi \tag{4.5.4}$$

Letting φ be $\varphi_{12}(\mathbf{x}) = \varphi_1(\mathbf{x}) - \varphi_2(\mathbf{x})$ and similarly for A and E, we find

$$E_{12,z}(\mathbf{x}) = -\partial_z\,\varphi_{12}(\mathbf{x}) - j\omega\,A_{12,z}(\mathbf{x})$$

$$\partial_z\,A_{12,z}(\mathbf{x}) = -\,j\,(\beta^2/\omega)\,\varphi_{12}(\mathbf{x})$$

Evaluating at x_1 on C_1 and x_2 on C_2 (same z plane) and subtracting gives

$$E_{12,z}(\mathbf{x_1}) - E_{12,z}(\mathbf{x_2}) = -\partial_z\,[\varphi_{12}(\mathbf{x_1}) - \varphi_{12}(\mathbf{x_2})] - j\omega\,[A_{12,z}(\mathbf{x_1}) - A_{12,z}(\mathbf{x_2})]$$

$$\partial_z\,[A_{12,z}(\mathbf{x_1}) - A_{12,z}(\mathbf{x_2})] = -\,j\,(\beta^2/\omega)\,[\varphi_{12}(\mathbf{x_1}) - \varphi_{12}(\mathbf{x_2})]$$

or

$$E_{12,z}(\mathbf{x_1}) - E_{12,z}(\mathbf{x_2}) = -\partial_z\,V(z) - j\omega\,W(z)$$

$$\partial_z\,W(z) = -\,j\,(\beta^2/\omega)\,V(z)$$

Now we found that

$$W(z) = L_e\,i(z) \tag{4.4.7}$$

so the second equation says

$\partial_z\, i(z) = - j\,(\beta^2/\omega L_e)\, V(z)$

STOP! Already this is a problem because we are supposed to get

$$\frac{\partial i(z)}{\partial z} = - \mathbf{y}\, V(z) \qquad \mathbf{y} = G + j\omega C$$

but we have G = 0. So forget the first equation for now, something is wrong with the second!

It is just the gauge condition. Go backwards,

$$\text{div}\, \mathbf{A} = - j\,(\beta^2/\omega)\, \varphi$$

$$\partial A_z/\partial z = - j\,(\beta^2/\omega)\, \varphi$$

Here I have thrown out transverse derivatives in div**A**. But if there is leakage current, then there is transverse current which I have ignored, and it will produce transverse **A** !

$$\mathbf{A}_t(\mathbf{x}) = \frac{1}{4\pi\varepsilon} \int_{die} \mathbf{J}_t(\mathbf{x}')\, \frac{e^{-j\beta R}}{R}\, dx'dy'dz' \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad (4.1.1)$$

$$\mathbf{A}_t(\mathbf{x}) = \frac{\sigma}{4\pi\varepsilon} \int_{die} \mathbf{E}_t(\mathbf{x}')\, \frac{e^{-j\beta R}}{R}\, dx'dy'dz' \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad (4.1.1)$$

Now maybe approximate $E_t = -\nabla_t \varphi_{12}$ and you have

$$\mathbf{A}_t(\mathbf{x}) = - \frac{\sigma}{4\pi\varepsilon} \int_{die} -\nabla_t' \varphi_{12}(\mathbf{x}')\, \frac{e^{-j\beta R}}{R}\, dx'dy'dz' \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad (4.1.1)$$

Then I don't have a plan! Somehow that King gauge just takes care of all this stuff! I would just have to add the conductance in by hand.

Why does the King method get it in there automatically? Because the King wave equation accounts for loss, that's why.

So OK, let's go back to