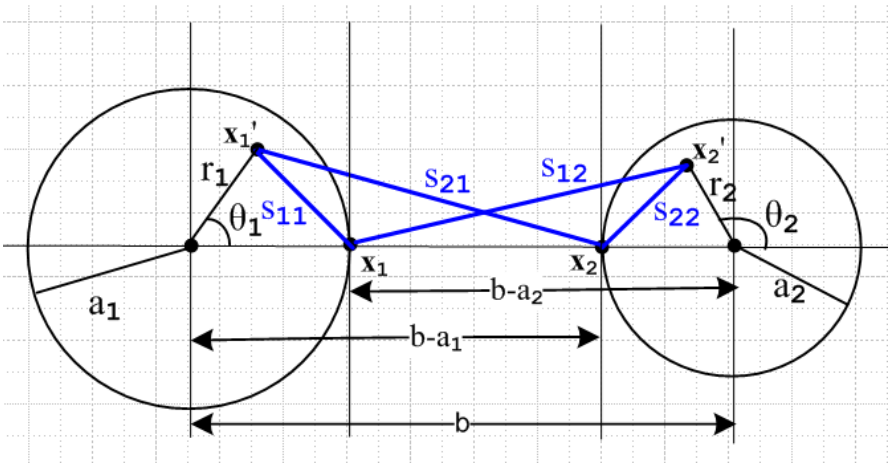
K calculation for symmetric PhL 1.8.14

This example is installed.

We are supposed to have K = K_L in general, but I don't see how that will work in simple symmetric cases. Here I will try some examples.

1. The widely space unequal round wire transmission line.

Need a new picture, so



$s_{21}^2 = r_1^2 + (b-a_1)^2 - 2\, r_1(b-a_1)\cos(\theta_1)$

$s_{11}^2 = r_1^2 + a_1^2 - 2\, r_1\, a_1 \cos(\theta_1)$

$s_{22}^2 = r_2^2 + a_2^2 + 2\, r_2\, a_2 \cos(\theta_2)$

$s_{12}^2 = r_2^2 + (b-a_2)^2 + 2\, r_2(b-a_2)\cos(\theta_2)$ (4.5.4)

The general rule is

$$\int_0^{2\pi} d\theta \ln(A \pm B\cos\theta) = 2\pi \ln[(1/2)(A + \sqrt{A^2-B^2}\ )] \ . \quad (4.5.5)$$

The θ integrals of interest are:

$$\int_0^{2\pi} d\theta_1 \ln(s_{21}^2) = \int_0^{2\pi} d\theta_1 \ln([r_1^2 + (b-a_1)^2 - 2\, r_1(b-a_1)\cos(\theta_1)]$$

$A = r_1^2 + (b-a_1)^2$ $B = 2\, r_1(b-a_1)$

$A^2-B^2 = [r_1^2 + (b-a_1)^2]^2 - 4\, r_1^2(b-a_1)^2 = [r_1^2 - (b-a_1)^2]^2 \Rightarrow \sqrt{A^2-B^2} = (b-a_1)^2 - r_1^2 > 0$ $b >> a_1$

$$\Rightarrow \quad \int_0^{2\pi} d\theta_1 \ln(s_{21}^2) = 2\pi \ln[(1/2)(r_1^2 + (b-a_1)^2 + (b-a_1)^2 - r_1^2) = 2\pi \ln[(b-a_1)^2] \quad // \text{ same}$$

The fourth integral is the same with $1 \leftrightarrow 2$, and the different sign of the second term in s_{12}^2 makes no difference,

$$\int_0^{2\pi} d\theta_2 \ln(s_{12}^2) = 2\pi \ln[(b-a_2)^2] \ .$$

The second integral is the first with $b-a_1 \rightarrow a_1$ so

$$\int_0^{2\pi} d\theta_1 \ln(s_{11}^2) = 2\pi \ln(a_1^2)$$

$$\int_0^{2\pi} d\theta_2 \ln(s_{22}^2) = 2\pi \ln(a_2^2)$$

So all four integrals are exactly this same! I was not expecting that result.

Now let's look at the K_L calculation

$$K_L \equiv \int_{C_1} dx_1' \, dy_1' \, b_1(x_1',y_1') \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2' \, dy_2' \, b_2(x_2',y_2') \ln(s_{22}^2/s_{12}^2) \qquad (4.10.9)$$

In a case where each b_i is axially symmetric for its round wire we have

$$b_1(r_1,\theta_1) = b_1(r_1)$$

where

$$\int_0^{2\pi} d\theta_1 \int_0^{a_1} r_1 dr_1 \, b_1(r_1) = 1 \qquad \Rightarrow \qquad \int_0^{a_1} r_1 dr_1 \, b_1(r_1) = 1/2\pi$$

Then,

$$K_L = \int_0^{2\pi} d\theta_1 \int_0^{a_1} r_1 dr_1 \, b_1(r_1) \ln(s_{21}^2/s_{11}^2) - \int_0^{2\pi} d\theta_2 \int_0^{a_2} r_2 dr_2 b_2(r_2) \ln(s_{22}^2/s_{12}^2)$$

$$= \int_0^{a_1} r_1 dr_1 \, b_1(r_1) \int_0^{2\pi} d\theta_1 \ln(s_{21}^2/s_{11}^2) - \int_0^{a_2} r_2 dr_2 b_2(r_2) \int_0^{2\pi} d\theta_2 \ln(s_{22}^2/s_{12}^2)$$

$$= 2\pi \int_0^{a_1} r_1 dr_1 \, b_1(r_1) [\ln[(b-a_1)^2] - \ln(a_1^2)] - \int_0^{a_2} r_2 dr_2 b_2(r_2) [\ln[(b-a_2)^2] - \ln(a_2^2)]$$

$= 2\pi \left[\ln[(b-a_1)^2/a_1^2]\right] \int_0^{a_1} r_1 dr_1\, b_1(r_1) - 2\pi \left[\ln[(b-a_2)^2/a_2^2]\right] \int_0^{a_2} r_2 dr_2\, b_2(r_2)$

$= \left[\ln[(b-a_1)^2/a_1^2]\right] - \left[\ln[(b-a_2)^2/a_2^2]\right]$

$= 2\pi \ln\left[\frac{(b-a_1)^2(b-a_2)^2}{a_1^2 a_2^2}\right]$

$= K$