Is anything Real ? PhL 9.22.14

I was having trouble seeing which things are real and which are not. The gist of this note is now installed in lines doc as a Technical Note at the end of Section 4.1. Basically everything floats against everything else. I added into lines doc some other comments on this subject. I think it is all resolved now. In particular, I show that even if something is complex, it can still be normalized to 1 in a certain sense explained in that Tech notes. That was the concern I guess. This little doc mainly relates to sections 4.1 and 4.7 so they will be stored in the Chapter 4 folder. This review done 9.23.14.

Examine Chapter 2. We find there that

$$E(r) = E(a)\,\frac{J_0(\beta r)}{J_0(\beta a)} \qquad \beta = e^{j3\pi/4}\,(\sqrt{2}/\delta)$$

Even if E(a) were real for some reason, E(r) is in general very complex! Same for B(r).

Examine Chapter 4. Opener is this

$$\varphi_1(\mathbf{x},\omega) = \frac{1}{4\pi\xi_d}\int_{C_1}\rho_1(x',y',z',\omega)\,\frac{e^{-j\beta_d R}}{R}\,dx'dy'dz' \;. \qquad R = |\mathbf{x}-\mathbf{x}'| \qquad (4.1.1)$$

Here

$$\rho_1(x,y,z,\omega) = \rho_1(x,y,z)\,2\pi\delta(\omega-\omega_1) \qquad \rho_1(x,y,z,t) = \rho_1(x,y,z)\,\exp(j\omega_1 t)$$

Can we say that $\rho_1(x,y,z)$ is real ? Maybe the answer is that we could insist this be real. Then

$$\rho_1(x,y,z,t)_{physical} = \mathrm{Re}[\rho_1(x,y,z,t)] = \rho_1(x,y,z)\cos(\omega_1 t) = \text{real}$$

Well then what about $J_{z1}(x,y,z)$ in (4.7.3) ? This is the current inside conductor C_1 and I know from Chapter 2 that in general its phase varies with radius, so I cannot just say that it is real.

In general, you have to say something like this for some general quantity Q

$$Q(x,y,z,t) = Q(x,y,z)\,e^{j\omega_1 t} = |Q(x,y,z)|\,e^{j\varphi_Q(x,y,z)}\,e^{j\omega_1 t}$$

and then

$$Q_{physical}(x,y,z,t) = \mathrm{Re}(Q) = |Q(x,y,z)|\cos[\,\omega_1 t + \varphi_Q(x,y,z)\,]$$

So I guess in general it might be that $\rho_1(x,y,z,\omega)$ on conductor C_1 might have some complicated phase which is a function of position on the surface. Certainly it varies with z in Appendix D as e^{jkz}.

Conclusion: Assume that nothing is real!

Now look at Chapter 4 in this light.

$$\varphi_1(\mathbf{x},\omega) = \frac{1}{4\pi\xi_d}\int_{C_1}\rho_1(x',y',z',\omega)\frac{e^{-j\beta_d R}}{R}\,dx'dy'dz' \quad . \qquad R = |\mathbf{x} - \mathbf{x}'| \tag{4.1.1}$$

Both ρ_1 and φ_1 are assumed to have position-dependent phase.

$$\rho_1(x,y,z) = \alpha_1(x,y)\;\; q_1(z) \quad . \tag{4.1.2}$$

Here can we assume that $\alpha_1(x,y)$ has the same phase at all points on a conductor perimeter? In the time domain, we would ask if the charge peaks at all points on the perimeter at the same instant. How would you answer such a question? This might be true at the driving end of a conductor. But maybe things change going down the line. For example, does n(θ) have a θ-dependent phase? I write elsewhere

$$n(z,y,z,t) = e^{-jkz}\, e^{j\omega t}\, n(\theta)$$

where I always think of n(θ) as having the same phase at all θ. What can be said about the E field in some plane z = constant in a TL?

This time-coincidence around a cross section of surface charge is a hidden assumption of my theory. It comes from the electrostatic capacitor problem idea.

Go back to these equations in Ch 4:

$$\rho_1(x,y,z) = \alpha_1(x,y)\;\; q_1(z) \quad . \tag{4.1.2}$$
$$C/m^3 \qquad 1/m^2 \quad C/m$$

$$\int_{C_1} dx\, dy\, \alpha_1(x,y) = 1\ . \tag{4.1.3}$$

I think I am really making an ansatz here that $\alpha_1(x,y)$ is real, and the integral is real. If it had some weird phase around the perimeter, then perhaps the integral could not be made real.

Big Problem. In (4.7.3) I write

$$J_{z1}(x,y,z) = b_1(x,y)\, i_1(z)$$
$$A/m^2 \qquad 1/m^2 \quad A \tag{4.7.3}$$

where i_1 is scaled such that

$$\int_{C_1} dx\, dy\, b_1(x,y) = 1\ . \tag{4.7.4}$$

I am pretty sure that I cannot assume $b_1(x,y)$ is real as I assumed $\alpha_1(x,y)$ was real. I just showed that inside the round wire, E_z varies with r, so E_z varies with (x,y).

How then can you impose the condition $\int_{C_1} dx\ dy\ b_1(x,y) = 1$ on a complex function? You are requiring that

$$\int_{C_1} dx\ dy\ b_1(x,y) = \int_{C_1} dx\ dy\ |b_1(x,y,)|\ e^{j\varphi_{b1}(x,y)} = 1$$

Maybe I can bail this out. I think I could require that. Suppose we make our separation above and we find that

$$\int_{C_1} dx\ dy\ b_1(x,y) = C = \text{a complex number}$$

Define these new functions

$$b'_1(x,y) \equiv (1/C)\ b_1(x,y)$$
$$i'_1(z) \equiv C\ i'(z)$$

We then have

$$J_{z1}(x,y,z) = b_1(x,y)\ i_1(z) = b'_1(x,y)\ i'_1(z)$$

with

$$\int_{C_1} dx\ dy\ b'_1(x,y) = 1$$

$$\rho_1(x,y,z) = \alpha_1(x,y)\ q_1(z)$$

Then we have

$$\int_{C_1} dx\ dy\ b'_1(x,y) = \int_{C_1} dx\ dy\ b_1(x,y)/C = C/C = 1$$

Then rewrite

$$b_1(x,y)\ i_1(z) = b'_1(x,y)\ i'_1(z)$$

$$\int_{C_1} dx\, dy\, b_1(x,y) = e^{j\kappa}$$

with κ = real. What this really says is that

$$\int_{C_1} dx\, dy\, |b_1(x,y)| = 1$$

Now $i_1(z)$ has some unknown phase. Write

$$\int_{C_1} dx\, dy\, J_{z1}(x,y,z) = \int_{C_1} dx\, dy\, b_1(x,y)\, i_1(z)$$

$$= i_1(z) \int_{C_1} dx\, dy\, b_1(x,y) = i_1(z)\, e^{j\kappa}$$

Suppose we now make a new partition

$$J_{z1}(x,y,z) = b'_1(x,y)\, i'_1(z)$$

where

$$b'_1(x,y) = e^{-jk}\, b_1(x,y)$$

$$i'_1(z) = e^{j\kappa}\, i_1(z)$$

Then for this new partition we will have

$$\int_{C_1} dx\, dy\, b'_1(x,y) = \int_{C_1} dx\, dy\, e^{-jk}\, b_1(x,y) = e^{-jk} \int_{C_1} dx\, dy\, b_1(x,y) = 1$$

So yes, this does bail things out for normalization.

So why not just assume that both $\alpha_1(x,y)$ and $b_1(x,y)$ have phases which vary with (x,y). In each case, the partition allows the integral to be 1. I like it. I added a comment for $\alpha_1(x,y)$ but I will now remove it and allow these things to be complex.

Actions: I added a Tech Note at the end of Section 4.1 and a small paragraph after (4.7.4) to explain why the integral of a complex function like $\alpha_1(x,y)$ or $b_1(x,y)$ can be set to the real number 1. I have not required that either of these functions be real!

I scan down now through Chapter 4 looking for "phase and reality" issues.

4.1 as already noted, and surely $\varphi_1(x,y,z)$ has some phase which varies with x,y,z though I don't explicitly state that fact.

4.2 nothing need be done

4.3 nothing need be done

4.4 Here I assume that φ = constant on a slice. That suddenly says φ has a constant phase on that perimeter, but it does not say φ is real. We then get down to this:

$$\frac{1}{C'} = \frac{V(z)}{q(z)} = \frac{1}{4\pi\xi_d} \{ \int_{C_1} dx_1' \, dy_1' \, \alpha_1(x_1',y_1') \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2' \, dy_2' \, \alpha_2(x_2',y_2') \ln(s_{22}^2/s_{12}^2) \}$$

$$= \frac{1}{4\pi\xi_d} K \qquad \text{and} \qquad V(z) = q(z) \frac{1}{4\pi\xi_d} K \tag{4.4.7}$$

Now something must be said! I kind of want K to be real here, but $\alpha_1(x_1',y_1')$ is complex, so that is a problem Houston. Maybe I should just avoid stating that K is real! But I think I really have to say something here that $\alpha_1(x_1',y_1')$ is taken to be real. Otherwise equations like

$C = 4\pi\varepsilon_d/K$ capacitance per unit length of the transmission line

$G = 4\pi\sigma_d/K$ conductance per unit length of the transmission line

so

$$C/G = \varepsilon_d/\sigma_d \ . \tag{4.4.10}$$

don't make any sense. I could say

$$4\pi\xi_d/K = C' = C + G/j\omega$$

and allow K to still be complex. Then for example C would be C = Re $[4\pi\xi_d/K]$ and we are still OK. But I really want to see $C = 4\pi\varepsilon_d/K$ because that is one of my main results!

Eventually in Chapter 4 we assume that a z=constant perimeter has a constant φ and a constant A_z. In this case, we can take $\alpha_1(x_1',y_1')$ = real and also $b_1(x_1',y_1')$ = real. Then we get all the usual claims.

Now how does φ, A_z = constant on perimeter say something about $\alpha_1(x_1',y_1')$? Well, these two conditions tell us that $E_\theta = 0$ and then we cannot have $\rho_1(x,y,z) = \alpha_1(x,y) \ q_1(z)$ peaking at different points on the perimeter at different times, because that would imply $E_\theta \neq 0$.

Action: I added to the Tech note to say that $\varphi \approx$ constant allows us to take $\alpha_1(x_1',y_1')$ = real.

In this case K comes out real, and we get all our desired stuff above.

Action: Cleaned up statement that K is real near equation (4.4.8).

I now continue the scan through Ch 4.

I have now to hold off on (4.4.11). We then skip through the two huge examples and we end up starting into Section 4.7.

I have my little comment about $b_1(x,y)$ after (4.7.4), but I don't claim it is real!!! Since this $b_1(x,y)$ is a current inside the wire, I suspect it canNOT be taken real. I say nothing at this point about b_1 being real. Nothing happens for the rest of Section 4.7 so

We come to Section 4.8. It is OK as is.

Section 4.9 also seems OK, not saying much. Now we come to

Section 4.10. Here there will be issues. Equation (4.10.4) shows K_L but I don't define it yet. I then come to my little red loop in Fig 4.11. I end up with

$$W(z) = L_e \; i(z) \; . \qquad (4.10.7)$$

Do W(z) and i(z) have the same phase so L_e = real ??? Don't know at this point. But then we come to

$$L_e = \frac{W(z)}{i(z)} = \frac{\mu_d}{4\pi} K_L \qquad (4.10.8)$$

where K_L is the following dimensionless real number,

$$K_L \equiv \int_{C_1} dx_1' \, dy_1' \, b_1(x_1',y_1') \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2' \, dy_2' \, b_2(x_2',y_2') \ln(s_{22}^2/s_{12}^2) \; . \qquad (4.10.9)$$

I want to say K_L is real, but the same argument does not work here! I know that $b_1(x_1',y_1')$ has a phase which varies especially with radius r, probably also with θ.

Suppose we just assume L_e is in general complex and K_L is in general complex. I would then delete the word real appearing below (4.10.8) and that would take me to

Section 4.11.

opening text OK

(a) OK

(b) on repair. OK through (4.11.13). OK through (4.11.14a) .

OK thru (4.11.16). OK through :

$$\mathbf{z} = R + j\omega L = Z_{s1} + Z_{s2} + j\omega L_e \qquad (4.11.17)$$

$$\mathbf{y} = G + j\omega C = j\beta_d^2/(\omega L_e) \qquad (4.11.18)$$

Comments: If L_e were completely real, then since $\beta_d = \omega/v_d$ we would have **y** = completely imaginary. So I have already assumed that L_e is complex!!! I did not realize that!

OK through (4.11.18). I might delete text shown in red. I continue:

I think after (4.11.18) things get muddled, something is just not right here. Way back in Section 4.4 I already say this

For example, in analogy to what we did with a parallel plate capacitor in (1.5.19), we may define the complex capacitance C' per unit length of our transmission line using (4.4.6) as follows:

$$\frac{1}{C'} = \frac{V(z)}{q(z)} = \frac{1}{4\pi\xi_d} \{ \int_{C_1} dx_1' \, dy_1' \, \alpha_1(x_1',y_1') \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2' \, dy_2' \, \alpha_2(x_2',y_2') \ln(s_{22}^2/s_{12}^2) \}$$

$$= \frac{1}{4\pi\xi_d} K \qquad \text{and} \qquad V(z) = q(z) \frac{1}{4\pi\xi_d} K \tag{4.4.7}$$

so I already have the symbol C' introduced. Maybe this is where my Digression on C' should go. I go on right away to say this

Recall from (1.5.20) that (C', C and G are discussed further in Section 4.11 below)

$$C' = C + 1/(j\omega R) = C + G/(j\omega) \tag{4.4.9}$$

So I already have C' involved and defined and active. No need to start it again.

I am going to start a scratch Section 4.11 doc so I have a sandbox to play in without wrecking lines doc.

TBC below this red text post feeding. // Resuming.

I now look in more detail into Fig 4.13 and surrounding discussion. I assume G = 0 in this little discussion, very similar to the CP BC. If we really have k = ω/v then I guess it is really true

$$i(z) = q(z)\, v_{\mathbf{d}} \qquad (4.11.19a)$$

Do I really believe this result? Why is there no capacitive current ? Because we are talking here free charge and real conduction current, I should clarify that both here and at CPBC. So no free charge can flow off the surface radially if G = 0 and even if there is C > 0.

I guess this is OK as long as you remember that G = 0. I guess OK thru (4.11.19a).

OK thru (4.11.20). But I am confused here. I already know that

$$\mathbf{y} = G + j\omega C = j\beta_{\mathbf{d}}^{2}/(\omega L_{\mathbf{e}}) = j\omega C'$$

so (4.11.21) is already known??
Why not just say this:

$$\mathbf{z} = R + j\omega L = Z_{\mathbf{s1}} + Z_{\mathbf{s2}} + j\omega L_{\mathbf{e}} \qquad (4.11.17)$$

$$\mathbf{y} = G + j\omega C = j\beta_{\mathbf{d}}^{2}/(\omega L_{\mathbf{e}}) = j\omega C' \qquad (4.11.18)$$

where $Z_{\mathbf{s1}}$, $Z_{\mathbf{s2}}$ and $L_{\mathbf{e}}$ are the averages shown in (4.11.9) and (4.11.13). In the last line we *define* C' to be the complex capacitance C' = C + G/(jω), in analogy with