Matick Section 4.5 PhL 1.9.14

Since I mention this section in my original lines doc, I might as well read it (probably for the first time).

[p 108] The stripline is drawn, width W and separation S with W >> S. What I usually call the z direction (wave direction) he has as his x direction.

[p 109] This is all statement of Maxwell's equations.

[p 110] He obtains the vector Helmholtz equation for the **E** field, same as mine. His E points only across the gap, while B is in the W direction. His equation (4.50) is strange since the LHS is a vector which is $\nabla^2\mathbf{E}$ whereas the rest of the equation is scalar. I have fixed this up.

[p 111-113] He assumes the form (4-52) for his longitudinal E field where γ and Γ_1 are TBD. He then just goes on and on for several pages. He is matching E_z values at the conductor surface. He then gets a transcendental equation for constant γ in (4-66).

[p 114-116] He discovers the skin effect δ value. He computes inductance L internal and external.

The following is now in lines doc Chapter 5:

In the 12-page Section 4.5 of his book Matick studies a lossy-transmission line in the simplest possible case which is a stripline geometry whose gap S is small compared to the width and whose metal strips are much thicker than the skin depth δ. His parameter γ is related to our parameter k by $\gamma = jk$, and his longitudinal direction is x instead of our z. He ends up with a transcendental "eigenvalue equation" (4-66) for γ, but if loss is very small, he can approximately solve for γ with these results

$$\mathrm{Im}(\gamma) = \beta(1+\delta/2S) \qquad \mathrm{Re}(\gamma) = \beta\,(\delta/2S) \qquad \text{// Matick (4-75,76,77)}$$

which with $\gamma = jk$ we translate to

$$\mathrm{Im}(k) = -\,\mathrm{Re}(\gamma) = -\,\beta\,(\delta/2S)$$
$$\mathrm{Re}(k) = \mathrm{Im}(\gamma) = \beta(1+\delta/2S)$$

$$k = \beta(1+\delta/2S) - j\,\beta\,(\delta/2S) = \beta - [-(\delta/2S) + j(\delta/2S)]$$

so

$$\Delta = -(\delta/2S) + j(\delta/2S)\ .$$

Thus, for such a stripline transmission line, the longitudinal dependence of all functions has this form,

$$q(z,t) = e^{j(\omega t-[\beta-\mathrm{Re}(\Delta)]z)}\ e^{-\mathrm{Im}(\Delta)z}$$

$$= e^{j(\omega t-[\beta+\delta/2S]z)}\ e^{-(\delta/2S)z}$$

which shows the exponential loss factor and an increased wavenumber $\beta+\delta/2S$ which corresponds to a decreased wavelength λ and a decreased wave velocity $v = \omega/k = \omega\lambda/2\pi = f\lambda$, the "drag effect".

Matick has an errata in this section which is a bit confusing, so we repair it right here. His equation (4-50) should read

$$\nabla^2\mathbf{E} = \left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial z^2}\right)\hat{\mathbf{x}} + \left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial z^2}\right)\hat{\mathbf{z}} = (j\omega\mu\sigma - \omega^2\mu\varepsilon)(E_x\hat{\mathbf{x}} + E_z\hat{\mathbf{z}}) \qquad \text{Matick (4-50)}$$