This is the Title PhL 3.26.05

This is all installed into lines doc, here is where I first wrote it up. Older notes at the bottom.

4.12 Modifications of Chapter 4 to account for $\mu \neq \mu_1 \neq \mu_2$.

First of all, the first part of Chapter 4 involving potentials φ and V(z) is completely unaffected. Thus, there is no modification to the first six sections of Chapter 4.

If the equality $\mu = \mu_1 = \mu_2$ assumed in Section 4.7 is broken, the result is that surface magnetization currents appear on one or both of the conductor surfaces and these cause an alteration of the theory. Thanks to the "J_m Theorem" proven in Appendix B, this alteration can be carried through with a very minimal impact, as we now show.

In Appendix B conductor magnetization surface currents are studied in some detail. The reader interested in how the magnetic modification is carried out would do well to read Appendix B at this point. A reader less interested can accept the Appendix B results and then learn below that basically nothing changes!

The first modification arises in Section 4.7 where now we no longer assume that μ_1 of conductor C_1 is the same as μ of the dielectric. As described in Appendix B.6, the modified version of (4.7.2) is this,

$$A_{z1}(\mathbf{x}) = \frac{\mu_1}{4\pi} \int_{C_1} [\, J_{z1}(\mathbf{x}') + \frac{\mu_0}{\mu_1} J_{zm1}(\mathbf{x}') \,] \frac{e^{-j\beta R}}{R} dx'dy'dz' \quad . \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad (4.7.2)'$$

where J_{zm1} includes only the surface component of the magnetization current on conductor C_1. Appendix B.6 shows how this J_{zm1} adder term in effect adds a certain homogeneous solution to the particular solution (first term above) of the A_z Helmholtz equation such that the A_z boundary conditions are duly satisfied at the magnetic conductor C_1 boundary.

We maintain the next two equations of Section 4.7 as is,

$$J_{z1}(x,y,z) = b_1(x,y)\, i_1(z)$$
$$A/m^2 \qquad 1/m^2 \quad A \qquad (4.7.3)$$

where i_1 is scaled such that

$$\int_{C_1} dx\, dy\, b_1(x,y) = 1 \; . \qquad (4.7.4)$$

This $i_1(z)$ is still the total conduction current in C_1. But we now add two new equations,

$$J_{z1m}(x,y,z) = b_{1m}(x,y)\, i_{1m}(z)$$
$$A/m^2 \qquad 1/m^2 \quad A \qquad (4.12.1)$$

where i_{1m} is scaled such that

$$\int_{C_1} dx\, dy\, b_{1m}(x,y) = 1 \ . \tag{4.12.2}$$

It is understood here that $b_{1m}(x,y)$ is a distribution which is restricted to the surface of C_1, but we continue to write it as if it existed at all points in the cross section of C_1. The integration in (4.12.2) is of course meant to include this surface distribution.

We know from (B.1.11) and (B.1.12) that, for an arbitrarily shaped conductor C_1,

$$i_{1m}(z) \equiv -\left(\frac{\mu_1}{\mu_0} - \frac{\mu}{\mu_0}\right) i(z) \qquad [\ \mu_1 = \text{conductor } C_1,\ \mu = \text{dielectric }], \tag{4.12.3}$$

and that the current ratio is therefore given by,

$$f_{1m} \equiv i_{1m}(z)/\, i(z) = -\left(\frac{\mu_1}{\mu_0} - \frac{\mu}{\mu_0}\right) . \tag{4.12.4}$$

With the above definitions, our modified (4.7.6) becomes

$$A_{z1}(x,y,z) = \frac{\mu_1}{4\pi} \int_{-\infty}^{\infty} dz'\, i(z') \int_{C_1} dx'\, dy' \left[\, b_1(x',y') + \frac{\mu_0}{\mu_1} f_{1m}\, b_{1m}(x',y') \,\right] \frac{e^{-j\beta R}}{R} \ .$$

$$= \frac{\mu}{4\pi} \int_{-\infty}^{\infty} dz'\, i(z') \int_{C_1} dx'\, dy' \left[\, \frac{\mu_1}{\mu} b_1(x',y') + \frac{\mu_0}{\mu} f_{1m}\, b_{1m}(x',y') \,\right] \frac{e^{-j\beta R}}{R} \ . \tag{4.12.5}$$

This leads us to define a new effective transverse current density,

$$b'_1(x,y) \equiv \frac{\mu_1}{\mu} b_1(x',y') + \frac{\mu_0}{\mu} f_{1m}\, b_{1m}(x',y')$$

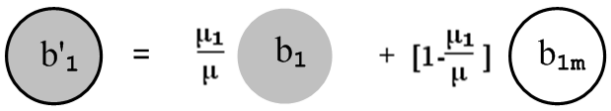
$$= \frac{\mu_1}{\mu}\, b_1(x',y') + \left[1-\frac{\mu_1}{\mu}\right] b_{1m}(x',y') \ . \tag{4.12.6}$$

This new transverse density b'_1 is still normalized to unity,

$$\int_{C_1} dx\, dy\, b'_1(x,y) = \frac{\mu_1}{\mu} \int_{C_1} dx\, dy\, b_1(x,y) + \left[1-\frac{\mu_1}{\mu}\right] \int_{C_1} dx\, dy\, b'_{1m}(x,y)$$

$$= \frac{\mu_1}{\mu} * 1 + \left[1-\frac{\mu_1}{\mu}\right] * 1 = 1 \ . \tag{4.12.7}$$

How does b'_1 differ from b_1? The difference is that b_1 does not include a surface current and b'_1 does. We can represent equation (4.12.6) in this symbolic graphic manner:



(4.12.6)

Thus, we have this new version of (4.7.6),

$$A_{z1}(x,y,z) = \frac{\mu}{4\pi} \int_{-\infty}^{\infty} dz'\, i(z') \int_{C_1} dx'\, dy'\, b'_1(x,y)\, \frac{e^{-j\beta R}}{R} \quad . \tag{4.7.6)'}$$

The differences are that the leading factor is μ instead of μ_1, and b_1 is replaced by b'_1.

Moving into Section 4.8 we find this new version of (4.8.1),

$$A_{z12}(\mathbf{x}) = A_{z1}(\mathbf{x}) + A_{z2}(\mathbf{x}) \;=$$

$$\frac{\mu}{4\pi} \int_{-\infty}^{\infty} dz'\, i(z') \left\{ \int_{C_1} dx_1'\, dy_1'\, b'_1(x_1',y_1') \frac{e^{-j\beta R_1}}{R_1} - \int_{C_2} dx_2'\, dy_2'\, b'_2(x_2',y_2') \frac{e^{-j\beta R_2}}{R_2} \right\} \tag{4.8.1)'}$$

which is identical to (4.8.1) except $b_i \rightarrow b'_i$. Then in the transmission line limit, we get this new version of (4.9.1),

$$A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi}\, i(z) \int_{-\infty}^{\infty} dz' \left\{ \int_{C_1} dx_1'\, dy_1'\, b'_1(x_1',y_1') \frac{1}{R_1} - \int_{C_2} dx_2'\, dy_2'\, b'_2(x_2',y_2') \frac{1}{R_2} \right\} \tag{4.9.1)'}$$

From this point onward, all equations are the same apart from $b_i \rightarrow b'_i$. Here are some of those equations after modification:

$$W(z) \equiv A_{z12}(\mathbf{x}_1) - A_{z12}(\mathbf{x}_2) \tag{4.10.1)'}$$

$$= \frac{\mu}{4\pi}\, i(z) \int_{-\infty}^{\infty} dz' \left\{ \int_{C_1} dx_1'\, dy_1'\, b'_1(x_1',y_1') \frac{1}{R_{11}} - \int_{C_2} dx_2'\, dy_2'\, b'_2(x_2',y_2') \frac{1}{R_{12}} \right\}$$

$$- \frac{\mu}{4\pi}\, i(z) \int_{-\infty}^{\infty} dz' \left\{ \int_{C_1} dx_1'\, dy_1'\, b'_1(x_1',y_1') \frac{1}{R_{21}} - \int_{C_2} dx_2'\, dy_2'\, b'_2(x_2',y_2') \frac{1}{R_{22}} \right\}$$

$$W(z) \tag{4.10.3)'}$$

$$= \frac{\mu}{4\pi}\, i(z) \int_{-\infty}^{\infty} dz' \left\{ \int_{C_1} dx_1'\, dy_1'\, b'_1(x_1',y_1') \left(\frac{1}{R_{11}} - \frac{1}{R_{21}} \right) - \int_{C_2} dx_2'\, dy_2'\, b'_2(x_2',y_2') \left(\frac{1}{R_{12}} - \frac{1}{R_{22}} \right) \right\} .$$

$$W(z) = i(z)\frac{\mu}{4\pi}\ \{\ \int_{C_1} dx_1'\ dy_1'\ b'_1(x_1',y_1')\ \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2'\ dy_2'\ b'_2(x_2',y_2')\ \ln(s_{22}^2/s_{12}^2)\ \}$$

(4.10.4)'

$$K_L \equiv \int_{C_1} dx_1'\ dy_1'\ b'_1(x_1',y_1')\ \ln(s_{21}^2/s_{11}^2) - \int_{C_2} dx_2'\ dy_2'\ b'_2(x_2',y_2')\ \ln(s_{22}^2/s_{12}^2) \qquad (4.10.9)'$$

$$L_e = \frac{W(z)}{i(z)} = \frac{\mu}{4\pi}\ K_L \qquad \text{// no change} \qquad \textit{(4.10.8)}$$

We then enter Section 4.11. The derivation of the transmission line equations (4.11.11) is *unaffected* by the above modifications; the only change is that the b'_i appear in the integral K_L in place of the b_i. The derivation of the fact that $K = K_L$ ending in (4.11.26) is also unchanged! This at first seems strange since K has not changed, but we have apparently altered K_L by the replacements $b_i \rightarrow b'_i$. But K_L is not an evaluation -- it is an integral equation relating K_L to the b'_i. In the self-consistent solution, the new functions (distributions) b'_i adjust themselves so that K_L does not change. K_L cannot change because (4.11.26) says it must remain equal to K which is determined by the electrostatic side of the problem. It is perhaps helpful to look at (4.11.25) which says $L_e = \frac{\mu}{4\pi}\ K_L$. We know that if the dielectric μ value does not change, the *external* inductance L_e of the transmission line cannot change so K_L stays fixed. Changing μ_1 and/or μ_2 away from the value μ will of course change the *internal* inductances of the conductors, as noted below.

Finally, if we look at the example associated with Fig 4.13, we still find explicitly that $K_L = K$ because the calculation leading to (4.11.29) is unchanged when b_i are replaced with b'_i, since the b'_i are still normalized as shown in (4.12.7).

The happy bottom line is that all of summary box (4.11.30) is unchanged except $b_i \rightarrow b'_i$ in the K_L integral. The constant K can still be evaluated using the "capacitor problem" of Section 5.5 and it is unaffected.

Having said this, let us now consider what happens to an operating transmission line which starts off with $\mu_1 = \mu_2 = \mu = \mu_0$ and we then gradually turn a magic "permeability knob" so that μ_1 gradually increases from μ_0 to some value $\mu_1 > \mu_0$. That is to say, we gradually cause conductor C_1 to become magnetic. The constant K (and therefore $K_L = K$) does not change at all. This K determined by the potential φ part of the problem in Section 4.4 and does not even know about the magnetic modification. Thus, looking at (4.11.30), C', C, G and L_e do not change. In particular, L_e does not change because we have not altered μ of the dielectric. The following two items shown in box (4.11.30) *do* change :

$$R = \mathrm{Re}(Z_{s1} + Z_{s2}) \qquad L = L_e + (1/\omega)\ \mathrm{Im}(Z_{s1} + Z_{s2})\ .$$

They change because Z_{s1} changes if we change μ_1. This is so because Z_{s1} is always a function of the skin depth δ_1, and $\delta_1 \equiv \sqrt{2/(\omega\mu_1\sigma_1)}$ from (2.2.20). In the special case that C_1 is a round wire of radius a_1 with an axially symmetric current distribution (such as the center wire of a coaxial cable), we showed in (2.4.11) that the surface impedance is given by

$$Z_{1s}(\omega) = \frac{+j\omega\mu_1}{2\pi a_1(\sqrt{2}/\delta_1)} \; \frac{ber_0[\sqrt{2}(a_1/\delta_1)] + j\, bei_0[\sqrt{2}(a_1/\delta_1)]}{ber_0'[\sqrt{2}(a_1/\delta_1)] + j\, bei_0'[\sqrt{2}(a_1/\delta_1)]} \,, \qquad (2.4.11)$$

so certainly this $Z_s(\omega)$ is a function of μ_1 both due to the leading constant and through the five occurrences of δ_1. Both the real and imaginary parts of $Z_{1s}(\omega)$ will change as μ_1 changes, so the transmission line parameters R and L both change. In the high frequency limit ,

$$Z_{1s}(\omega) \approx \frac{1}{\sigma_1(2\pi a_1)\delta_1}\,(1+j) \qquad \delta << 16a \;. \qquad (2.4.16)$$

so now the variation with μ_1 is through the single δ_1 factor shown. Again, both real and imaginary parts of $Z_{1s}(\omega)$ vary with μ_1.

Since R and L change as noted above, the transmission line characteristic impedance will also change,

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{z}{y}} \qquad (K.11)$$

This means, for example, if we drive a semi-infinite transmission line with some fixed voltage V(z), the driving current i(z) will vary in amplitude and phase as we turn our "permeability knob" for conductor C_1. This is simply because $i(z) = V(z)/Z_0$.

So **the good news** is that the theory of Chapter 4 is easily extended to allow for magnetic conductors and or dielectric. Once again, the summary box (4.11.30) is unchanged when $\mu_1 = \mu_2 = \mu$ is broken except for the appearance of b'_i in the K_L integral, and except for the fact that Z_{s1} and Z_{s2} change as noted above, causing changes in R, L and Z_0. At very high frequency, one will have $Z_0 = \sqrt{(L_e/C)}$ and in this case Z_0 is not altered, see (4.11.32).

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Paradox: The above stuff seems to say that since L_e and W(z) don't change, then K_L should not change when one "turns on" μ_1 inside C_1, say. But maybe the impedance changes, so if you keep the driving V(z) fixed, then i(z) will in fact change!

Pause: What happens to "An example of $K = K_L$" ???? Look at (4.11.29) and replace b_i with b'_i and we get down to

$$K_L = 2\pi\,[\ln[(b-a_1)^2/a_1^2]\;\int_0^{a_1} r_1 dr_1\, b'_1(r_1) - 2\pi\,[\ln[(b-a_2)^2/a_2^2]\;\int_0^{a_2} r_2 dr_2\, b'_2(r_2).$$

But since we are using "widely spaced round wires", we know that J_z is uniform and thus we know from *** that $\int_{C_1}$ dx dy $b'_1(x,y)$ = 1 and so $\int_0^{a_1} r_1 dr_1\ b'_1(r_1) = 2\pi$ just as before, and we end up with the same result

$$K_L = [\ln[(b-a_1)^2/a_1^2] - [\ln[(b-a_2)^2/a_2^2] = K$$

so in this example, even with magnetic media, we still get K_L= K !!!!

Assume W(z) is still just a function of z but it has those b'_i functions in there. This is a "new" W(z) by the way since it is a difference of "new" A_z objects.

4.10.1 = OK with b'_i
4.10.3 = OK with b'_i
4.10.4 = OK with b'_i and with μ on the outside as shown.

What about the flux and L_e stuff??? Our "modification" changes only the μ_i inside the conductors, the μ in the dielectric is unchanged. I think such μ_i changes only change B *inside* the conductors, and H is the same both inside and out. The B field in the dielectric is the same, so flux is same, so L_e is same!!

Note: From (2.2.30) the claim is that H(a) at the surface does not change (symmetric current case).

4.10.7. If L_e is "the same", and W(z) is "new", and i(z) is unchanged, how can this equation be true? It suggests that somehow W(z) does not change!

I think (4.10.8) is OK with the μ shown, but I think K_L has been modified so L_e is then not the same. This is all as we expect due to the b'_i functions inside K_L.

4.11.1 = OK as stated except for the b'_i in K_L.

thru (4.11.3) = OK

(4.11.4) = OK, but we have new A_{z12} objects now

Now look at the second of (4.11.5). I guess we evaluate each A_{z12} potential at x_1+ and x_2+ just outside the magnetization current? We have *really modified* the A_z potential because we have added in the mag current as if it were a real current!!!! But I think it is OK to evaluate at x_1+ and x_2+. These are not normal derivatives, so I think $\partial_z A_{z12}(\mathbf{x_1})$ does not have a jump at the surface.

I think all is OK through (4.11.7) now.

Surface impedance still exists as in (4.11.8). Here i_1 is the conduction current in the conductor, something not affected by surface current. But is E_{z1} affected by surface current? I think the answer is yes! This is a real current at the surface in our model here. Surface impedance is not geared up to handle having such a current I suspect. But only conduction current enters Ohm's law stuff! So

$$E_{z1} = Z_{s1}\ i_1(z) \qquad E_{z2} = Z_{s2}\ i_2(z)\ . \qquad (4.11.8)$$

Maybe these are still OK with i_1 being the conduction current only. A sticky wicket. Let's assume these equations are OK and continue.

Note: The Chapter 2 round wire shows that Z_{s1} is a complicated function of μ_1 so changing μ_1 away from μ *will change* Z_{s1}.

What about (4.11.9? It is the same as before, but W(z) is a new W(z) somehow. So these are now different pair of equations. Now in (4.11.1) we still have $W(z) = L_e i(z)$ and both L_e and W are changed, but this equation is still true! So

4.11..10 = OK
4.11.11 = OK
4.11.12 = OK BUT the parameters are may be different!
4.11.13 = OK, but L_e is different so L and z are both "new" as well. There has been a change.
4.11.14 = OK.

4.11.14 BUT: how can G and C be the same, but L_e is different???? We have not changed μ in the dielectric, so we have not changed β^2. Something is amiss here!

What about the King gauge? Does it get modified?? I carefully took different μ_i into account doing that gauge development, I think it is exactly right.

Whiteboard Theorem: A battery does not supply magnetization current along with conduction current! I convinced myself of this fact.

Question. Go back to

$$A_{z1}(\mathbf{x}) = \frac{\mu_1}{4\pi}\int_{C_1}\left[\ J_{z1}(\mathbf{x}') + \frac{\mu_0}{\mu_1}\ J_{m1}(\mathbf{x}')\ \right]\frac{e^{-j\beta R}}{R}\,dx'dy'dz'\ . \qquad R = |\mathbf{x} - \mathbf{x}'| \qquad (4.7.2)'$$

Does the addition of the J_m term change the value of $A_{z1}(x)$? I can answer this question for the round wire since I computed both terms in Appendix B