Comments regarding μ

This is a subtle subject and is not discussed in King's transmission line theory book.

If the conductor C_1 and dielectric have the same permeability so that $\mu_1 = \mu_d$, then there exists no "magnetic boundary" between the conductor and dielectric. The solution (4.7.2) is then smooth at this boundary, and so $A_{z1}(x,y,z)$ "naturally" satisfies these two boundary conditions,

$$A_{z1}(\mathbf{x}+) = A_{z1}(\mathbf{x}-)$$
$$(1/\mu_d)\partial_n A_{z1}(\mathbf{x}+) = (1/\mu_1)\,\partial_n A_{z1}(\mathbf{x}-) \qquad (4.7.7)$$

where $\mathbf{x}+$ is just outside the conductor surface and $\mathbf{x}-$ is just inside. The second equation here is just (1.1.46) in the case that there is no free surface current K_{free} flowing on the boundary, and indeed in our example at hand there is no such free surface current. Since we have assumed that $\mu_d = \mu_1$, this second boundary condition just says $\partial_n A_{z1}(\mathbf{x}+) = \partial_n A_{z1}(\mathbf{x}+)$. Since there is no magnetic boundary at the conductor/dielectric interface, the solution (4.7.2) is continuous and all its derivatives are also continuous at the boundary, since nothing special happens at that boundary. Thus, the Helmholtz integral solution provides the whole solution for A_{z1} since it meets both "boundary conditions" at this pseudo boundary.

If on the other hand we have $\mu_1 \neq \mu_d$, then there *is* a magnetic boundary between conductor and dielectric which we have to worry about. In this case, (4.7.2) cannot possibly satisfy the second boundary condition of (4.7.7) since, as already noted, the A_{z1} of (4.7.2) satisfies $\partial_n A_{z1}(\mathbf{x}+) = \partial_n A_{z1}(\mathbf{x}+)$. Thus, in this case (4.7.2) is not the full solution for A_{z1}. One must add a homogeneous Helmholtz equation solution to (4.7.2) in order to have a proper solution for A_{z1} that satisfies both equations in (4.7.7).

It *turns out* that the correct total A_{z1} solution can be generated by adding a certain fictitious surface current term to $\mu_1 J_{z1}$ in (4.7.2). Since such a surface current vanishes on both sides of the boundary between μ and μ_1, the Helmholtz solution due just to this surface current term is in fact a homogeneous solution to the Helmholtz equation in both the conductor and dielectric regions, away from that boundary. It turns out moreover that the correct fictitious surface current to add is in fact the magnetization surface current J_m which is created at the boundary between $\mu_d \neq \mu_1$. Adding this surface current is just a "trick" in order to generate the correct homogeneous adder solution so that the resulting total A_{z1} satisfies both boundary conditions in (4.7.7). Formally speaking, the $\mathbf{J}_i$ appearing in (1.5.4) and then J_{z1} in (4.7.2) should *not* include such magnetization currents since this **J** is really the **J** in Maxwell's equation curl **H** = $\partial_t \mathbf{D}$ + **J,** and this **J** does not include magnetization currents -- it includes only normal conduction currents.

In our current Chapter 4, we *want* (4.7.2) to represent the complete solution for A_{z1} and for that reason we must restrict our analysis to the situation where dielectric and all conductors have the same permeability which we shall just call μ. In practice, one normally has $\mu_d = \mu_1 = \mu_0$. In order to handle the more general case of $\mu_d \neq \mu_1$, we have to deal with the inhomogeneous adder solutions or equivalently with the abovementioned fictitious surface current, and this complicates our analysis which is already quite complicated. So, for the moment, we now make the same assumption made by King and other authors:

Fact: From now on, conductors and dielectric must have the same permeability μ_d. (4.7.8)

After fully developing this special case, we shall then extend the theory in Section 4.13 to allow for $\mu_d \neq \mu_1$.

Appendix G shows for the round wire how the inhomogeneous adder solution is found and how it then causes the boundary conditions (4.7.7) to be met when $\mu_1 \neq \mu_d$.

Appendix B shows how the addition of a fictitious surface current term $\mu_0 J_m$ provides an alternate and simpler solution to the same problem of meeting boundary conditions (4.7.7) when $\mu_1 \neq \mu_d$. It then shows exactly how this works in the special case of a round wire.

Having now mentioned that the Helmholtz integral might not provide a total solution, the reader might fairly ask why it is that the Helmholtz integral solution $\varphi_1(\mathbf{x},\omega)$ of (4.1.1) provides a complete and viable solution to the φ Helmholtz equation, given that in general the conductor (ε_1) and dielectric (ε) have different ε values, so there should be an "electric boundary" where ε meets ε_1. The reason is that, according to (1.1.47), the boundary condition corresponding to the second line of (4.7.7) reads

$$[\varepsilon_1 E_n(\mathbf{x}+) - \varepsilon E_n(\mathbf{x}+)] = n_{free}(\mathbf{x}) \ .$$

Since we are neglecting transverse **A** components as stated in (4.7.1), and since our notation ∂_n indicates a normal conductor derivative which is transverse (to z), we have

$$\mathbf{E} = -\text{ grad } \varphi - \partial_t \mathbf{A} \qquad \Rightarrow \qquad E_n = -\partial_n \varphi \tag{4.7.9}$$

so we have then this set of boundary conditions for φ_1,

$$\begin{aligned} &\varphi_1(\mathbf{x}+) = \varphi_1(\mathbf{x}-) \\ &[\varepsilon_1 \partial_n \varphi_1(\mathbf{x}+) - \varepsilon \partial_n \varphi_1(\mathbf{x}-)] = n_{free}(\mathbf{x}). \end{aligned} \tag{4.7.10}$$

These look a bit like (4.7.7) for A_{z1}. The big difference is that in this case there *does* exist a free surface charge n_{free} and it simply adjusts itself to make (4.7.10) be true. Thus, the Helmholtz integral (4.1.1) does in fact meet the required electrical boundary conditions without the need for a homogeneous solution adder term. A less formal way to state this is that, in the electrical case, we can regard the surface charge as in fact lying on the dielectric side of the boundary, and then the boundary is of no interest in our problem of analyzing fields in the dielectric.