Power Transmission Lines PhL 1.26.14

I added a short section on this subject after the twin lead example of Chapter 4.

I thought it would be good to gather a few facts on this subject.

This is from Rajput's google book on Power System Engineering

13.1. INTRODUCTION

The performance of a transmission line as an element of a power system is affected by the following *four parameters* :

(*i*) Inductance (*L*) (*ii*) Resistance (*R*)

(*iii*) Capacitance (*C*) (*iv*) Conductance (*G*).

- ***Inductance* is by far the most dominant line parameter. It is the inductive reactance which limits the transmission capacity of a line.**
- **The *capacitance* together with *conductance* forms the shunt admittance of a transmission line. Shunt conductance, which is mostly due to leakage over the surface of insulators, is almost always neglected in power transmission lines.**

So we speak the same language. The claim is you can neglect G. One would compare ωC with G and you would think that with such low ω, G might matter. On the other hand,

$$C/G = \varepsilon/\sigma \ . \qquad (4.4.10)$$

$$\approx \varepsilon_0/\sigma \approx 10^{-11}/10^{-15} = 10^4 \qquad C \text{ (farads/m)} = 10^4 \ G \text{ (mhos/m)}$$

So compare the capacitative "admittance" to the conductive admittance

$$Y_C = \omega C = 2\pi f C = 377\ 10^4\ G \approx 10^7\ G$$

So this just says a lot more current flows across due to C than G, so you neglect G, even at 60 Hz. So you would be very wrong to think G + jωC ≈ G at "low frequency" of 60 Hz.

In overhead lines it may be assumed without appreciable error that the current uniformly distributed.

$$\lambda_{int.} = \frac{\mu_0 I}{8\pi} \text{ Wb-T/m} \qquad \text{...(13.3)}$$

In my notation this says flux = L_i I and $L_i = (\mu_0/8\pi)$. Raiput gets this result using some weird flux that I do not understand. It is through a thin math loop of width dr with long direction along z, so it actually captures some azimuthal B flux, but this guy says it all wrong. But I could verify that calculation in my Appendix C.

Raiput then does lots of multi-conductor geometries.

crease. The increase in resistance should not be neglected in very large conductors at commercial power frequencies and in most conductors at carrier and radio frequencies. *Since skin effect reduces the amount of current carried by the centre portion of the conductor, hollow conductors are sometimes employed to use them more effectively.* Aluminium cable steel inforced (ACSR) is used even more than hollow conductors. *This cable has steel core which strengthens the cable, and an outer covering of aluminium which, because of skin effect and high resistivity of the steel, carries most of the current.* These conductors also have the advantage of a *large surface area available for cooling.*

So how can that be? Let's compute δ for 60 Hz system:

$\delta \equiv \sqrt{2/(\omega\mu\sigma)} \quad = \delta \equiv \sqrt{2/(377\mu_0\sigma)}$

$\delta^2 = 2/[\ 377 \times 4\pi\ 10^{-7}\ *\ 5.8\ 10^{7}\ ]$

$= 2000/377 * 10^{-3} * 10^{3} \approx 5.3\ m^2$

$\delta = 2.3$ meters!!!! wrong it is 1 cm!!! Do the math!

Who would ever have a line with a diameter in meters?? There might be some slight effect.

14.7.4. Surge Impedance and Surge Impedance Loading of Transmission Lines.

Surge Impedance (Z_0). It is defined as the *square root of the ratio of line impedance (Z) and shunt admittance (Y).*

i.e. $$Z_0 = \sqrt{\frac{Z}{Y}} \qquad ...(14.51)$$

where, $Z = R + jX$ and $Y = G + jB$

(*Surge impedance is also sometimes called "Natural impedance"*).

It may be seen that the characteristic impedance was also defined in the same way. Actually characteristic impedance is more correct nomenclature.

Surge impedance is the characteristic impedance of a loss-free line

In case of a line having negligible resistance (such will be the case when conductors are of a large cross-section) and no shunt leakage (*i.e.* $G = O$),

$$Z_0 = \sqrt{\frac{L}{C}}, \text{ which is a pure resistance} \qquad ...(14.52)$$

Its value varies as given below :

Overhead transmission lines ..... 400 to 600 Ω

Under ground cables ..... 40 to 60 Ω

The *phase value of* Z_C for transmission lines is usually between **0° and – 15°.**

A line terminated in its characteristic impedance is called a **flat line** or *an* **infinite line.** The latter term arises from the fact that a line of infinite length cannot have a reflected wave.

So they are claiming maybe 500 ohms, similar to twin lead which is 300 ohms. I guess even at 60 Hz we are still in the large ω limit so we have

$$Z_0 = \sqrt{\frac{L_e}{C}} = \sqrt{\frac{(\mu/4\pi)K}{4\pi\varepsilon/K}} = (1/4\pi)\ K\ \sqrt{\mu/\varepsilon} \quad = \quad (1/4\pi)\ K\ Z_m \quad = K * 30\Omega$$

So power lines don't have crazy Z_0 impedances. I have not yet seen how R and ωL compare.

The *resistance* of transmission line conductors is the *main source of line power loss* (although the contribution of line resistance of series line impedance can be neglected in most cases). Thus,

So can I verify this claim quickly? $R = \sigma/A = \sigma/(\pi a^2)$, whereas

$\omega L = 2\pi f\,(\mu_0/4\pi)K = 2\pi f\,(10^{-7})K = 377 \times 10^{-7}\ K = 0.4 \times 10^{-4}\ K$

But what is R for a power line wire? I see products from southwire where resistance varys from .02 to .06 ohms per 1000 feet, amps ranging from 100 to 900. So lets go with

$R = .02$ ohms/1000ft * 3 ft/m $= (.06/1000)$ ohms/m $= .06 \times 10^{-3} = 6 \times 10^{-5}\ \Omega/m$

OK, now compare this to

$\omega L = 0.4 \times 10^{-4}\ K = 4 \times 10^{-5}\ K\ \Omega/m$

Now for twinlead style,

$K = 4 \ln(b/a) = 4 \ln(1m/1") = 4 \ln(39) = 14.6$ so then

$\omega L = 4 \times 10^{-5}\ 15\ \Omega/m = 6 \times 10^{-4}\ \Omega/m$

So in my little ball park example, I find that $\omega L = 10R$ so you could roughly neglect R in this case.

So then we are justifying the idea that $Z_0 = \sqrt{L/C}$ even at 60 Hz in power systems!!!

In my example you would have

$Z_0 = K * 30\Omega = 14.6 * 30 = 438\Omega$, which is in the range 400-600Ω quoted above!

So I think I have grokked the basic power line parameters. The surprise is that Z_0 is still in the high frequency limit!

Now suppose you put 12,000 volts on this 400Ω line which assume was properly terminated. Then

$I = V/Z = 12000/400 = 30$ amps = not very much, but

power $= V^2/R = I^2R = 900 * 400 = 360$ KW = 1/3 megawatt, could serve 180 houses say.

I think big lines are 230 to 800 KV so then

$I = 230{,}000/400 = 575$ amps (Southwire had some up to 1000 amps)

power = $I^2R = 575^2\ 400 = 132$ megawatts.

Let's go back to the first example of 30 amps and 12000 volts. Then E = 12000 volts/m, that is an easy computation. What is B ?

$2\pi rH = I_{enclosed}$ r = 1 cm say = 10^{-2}m

H = I/2πr = 30amps/[$2\pi\ 10^{-2}$m] = 500 amp/m at the wire surface just due to one wire

So

E = 12000 volts/m
H = 500 amp/m ballpark numbers

curl **H** = (jωε + σ)**E**

So let's now think about the right angles argument. Assume σ = 0 for dielectric (air) then

curl **H** = $j\omega\varepsilon_0$**E** = C **E** C = 377 x 10^{-11} = 4 x 10^{-9}

Then here is the argument:

$\mathbf{E} \bullet \mathbf{H} = [\ \text{curl}\ \mathbf{H} / j\omega\varepsilon_0] \bullet \mathbf{H} \approx C^{-1}\ \text{curl}\ \mathbf{H} \bullet \mathbf{H}$

$\approx 10^9 \{ (\partial_x H_y - \partial_y H_x)H_z + (\partial_y H_z - \partial_z H_y)H_x + (\partial_z H_x - \partial_x H_z)H_y . \}$

$\approx 10^9\ H_y^2\ \partial_z\ (H_x/H_y)$ if we assume $H_z \equiv 0$

= 0 since expect H_x/H_y = constant along transmission line

So the only way this baby can fail is if there is some small H_z which throws it off. Suppose there were some small H_z . then it is hard to know that **E** • **H** is. In terms of angle:

$\cos\theta = 10^9 \{ (\partial_x H_y - \partial_y H_x)H_z + (\partial_y H_z - \partial_z H_y)H_x + (\partial_z H_x - \partial_x H_z)H_y . \} / \sqrt{H_x^2 + H_y^2}\ E$

Now suppose you ran a power line at 0.6 Hz instead of 60 Hz. Then since you are down ω by 100, the R would dominate in the R+jωL. The other thing G + jωC I don't know.

If the 10^9 were 1 instead, then your assumption would be roughly $H_z << H_x$ or the like.

But if it is 10^9, then you have to assume that $10^9 H_z << H_x$ which is probably not going to fly!