This is the Title PhL 3.26.05

This has been installed into lines doc.

(f) Time domain equations

The results above are all stated in the frequency domain, but it is a simple matter to convert them to the time domain using $j\omega \leftrightarrow \partial_t$. One then makes these replacements

$$\begin{aligned} \mathbf{z} &= R+j\omega L \quad \rightarrow \quad L\partial_t + R \\ \mathbf{y} &= G+j\omega C \quad \rightarrow \quad C\partial_t + G \\ \mathbf{zy} &= (R+j\omega L)(G+j\omega C) \quad \rightarrow \quad (R + L\partial_t)(G + C\partial_t) = LC\partial_t^2 + (LG+RC)\partial_t + RG \end{aligned} \tag{4.11.38}$$

Here then are selected equations and their translations to the time domain:

Transmission Line Equations (4.11.14b) : [coupled first order PDE's]

$$\begin{aligned} \partial_z V &= -\mathbf{z}\, i \quad \Rightarrow \quad \partial_z V(z,t) = -L\partial_t i(z,t) - LR\, i(z,t) \\ \partial_z i &= -\mathbf{y}\, V \quad \Rightarrow \quad \partial_z i(z,t) = -C\partial_t V(z,t) - CGV(z,t) \end{aligned} \tag{4.11.39}$$

Transmission Line Wave Equations (4.11.15) [damped wave equations]

$$\begin{aligned} (\partial_z^2 - \mathbf{zy})\, V(z) = 0 \quad &\Rightarrow \quad [\partial_z^2 - LC\,\partial_t^2 - (LG+RC)\partial_t - RG]\, V(z,t) = 0 \\ (\partial_z^2 - \mathbf{zy})\, i(z) = 0 \quad &\Rightarrow \quad [\partial_z^2 - LC\,\partial_t^2 - (LG+RC)\partial_t - RG]\, i(z,t) = 0 \end{aligned} \tag{4.11.40}$$

If we set the loss factors R and G both to 0 these equations become

$$\begin{aligned} \partial_z V &= -L\partial_t i \quad & [\partial_z^2 - LC\,\partial_t^2]\, V(z,t) = 0 \quad & \text{[undamped wave equations]} \\ \partial_z i &= -C\partial_t V \quad & [\partial_z^2 - LC\,\partial_t^2]\, i(z,t) = 0 \quad & \text{// lossless} \end{aligned} \tag{4.11.41}$$

At large ω one has $L \approx L_e$ (note 1 above) and since (4.11.28) says $L_eC = \mu\varepsilon = 1/v^2$ we conclude that the factor LC appearing in the above wave equations is $1/v^2$ where v is the dielectric wave velocity.

The various transmission line equations shown above in the time domain are often referred to as the **telegraph** (telegrapher, telegrapher's) **equations**.