This is all installed in lines doc in "the capacitor problem" section of Chapter 5.

$$\varphi_t(x,y) = -\oint_{C1} ds'\, K_1\, \partial_n g(x,y|x',y') - \oint_{C2} ds'\, K_2\, \partial_n g(x,y|x',y') \;=\; K_1\, F_1(x,y) + K_2 F_2(x,y)$$

$$E_n(x,y) = -\partial_{n1}\varphi = -\frac{q(z)}{4\pi\varepsilon}\,\partial_{n1}\varphi_t(x,y) = \frac{q(z)}{4\pi\varepsilon}\,\{\, K_1\, G_{11}(\mathbf{x}) + K_2\, G_{12}(\mathbf{x})\,\} \qquad \mathbf{x} \text{ on } C_1$$

$$E_n(x,y) = -\partial_{n1}\varphi = -\frac{q(z)}{4\pi\varepsilon}\,\partial_{n2}\varphi_t(x,y) = \frac{q(z)}{4\pi\varepsilon}\,\{K_1\, G_{21}(\mathbf{x}) + K_2\, G_{22}(\mathbf{x})\,\} \qquad \mathbf{x} \text{ on } C_2$$

$$n_1(x,y) = \varepsilon E_n(x,y) = \frac{q(z)}{4\pi\varepsilon}\,\varepsilon K_1\, G_{11}(\mathbf{x}) + \frac{q(z)}{4\pi\varepsilon}\,\varepsilon K_2\, G_{12}(\mathbf{x}) \qquad \mathbf{x} \text{ on } C_1$$

$$n_2(x,y) = \varepsilon E_n(x,y) = \frac{q(z)}{4\pi\varepsilon}\,\varepsilon K_1\, G_{21}(\mathbf{x}) + \frac{q(z)}{4\pi\varepsilon}\,\varepsilon K_2\, G_{22}(\mathbf{x}) \qquad \mathbf{x} \text{ on } C_2$$

$$q_1 = \oint_{C1} ds'\, n_1(x',y') = \frac{q(z)}{4\pi\varepsilon}\,[\varepsilon K_1 H_{11} + \varepsilon K_2 H_{22}] = \frac{q(z)}{4\pi\varepsilon}\,[\, c_{11}V_1 + c_{12}V_2\,]$$

$$q_2 = \oint_{C2} ds'\, n_2(x',y') = \frac{q(z)}{4\pi\varepsilon}\,[\varepsilon K_1 H_{21} + \varepsilon K_2 H_{22}] = \frac{q(z)}{4\pi\varepsilon}\,[\, c_{21}V_1 + c_{22}V_2\,]$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{q(z)}{4\pi\varepsilon}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \qquad \text{or} \qquad \mathbf{q} = \frac{q(z)}{4\pi\varepsilon}\, c\, \mathbf{K} \quad .$$

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = \frac{4\pi\varepsilon}{q(z)}\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \qquad \text{or} \qquad \mathbf{K} = \frac{4\pi\varepsilon}{q(z)}\, s\mathbf{q}$$

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = \frac{4\pi\varepsilon}{q(z)}\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}\begin{pmatrix} q(z) \\ -q(z) \end{pmatrix} = 4\pi\varepsilon\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$K_1 = 4\pi\varepsilon\,(s_{11} - s_{12})$
$K_2 = 4\pi\varepsilon\,(s_{21} - s_{22})$

$K = K_1 - K_2 = 4\pi\varepsilon\,[s_{11} + s_{22} - 2s_{12}]$. // since $s_{12} = s_{21}$ as noted above

$$= 4\pi\varepsilon\,\frac{c_{11} + c_{22} + 2c_{12}}{c_{11}c_{22} - c_{12}^2}$$

Then the same capacitance shown in *** is recovered from.

$$C = 4\pi\varepsilon/K = \frac{c_{11}c_{22} - c_{12}^2}{c_{11} + c_{22} + 2c_{12}}$$

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$$\varphi_t(x,y) = -\oint_{C1} ds'\, K_1\, \partial_n g_K(x,y|x',y') - \oint_{C2} ds'\, K_2\, \partial_n g_K(x,y|x',y')$$

$$= K_1 F_1(x,y,K) + K_2 F_2(x,y,K)$$

From this point on, every function and constant acquires and argument K: $G_{ij}(\mathbf{x},K)$, $H_{ij}(K)$ and then $c_{ij}(K)$. We end up then with

$$K = 4\pi\varepsilon \frac{c_{11}(K) + c_{22}(K) + 2c_{12}(K)}{c_{11}(K)c_{22}(K) - [c_{12}(K)]^2}$$

$$E_n(x,y) = -\partial_{n1}\varphi = -\frac{q(z)}{4\pi\varepsilon}\partial_{n1}\varphi_t(x,y) = \frac{q(z)}{4\pi\varepsilon}\{K_1 G_{11}(\mathbf{x}) + K_2 G_{12}(\mathbf{x})\} \qquad \mathbf{x} \text{ on } C_1$$

$$E_n(x,y) = -\partial_{n1}\varphi = -\frac{q(z)}{4\pi\varepsilon}\partial_{n2}\varphi_t(x,y) = \frac{q(z)}{4\pi\varepsilon}\{K_1 G_{21}(\mathbf{x}) + K_2 G_{22}(\mathbf{x})\} \qquad \mathbf{x} \text{ on } C_2$$

$$n_1(x,y) = \varepsilon E_n(x,y) = \frac{q(z)}{4\pi\varepsilon}\varepsilon K_1 G_{11}(\mathbf{x}) + \frac{q(z)}{4\pi\varepsilon}\varepsilon K_2 G_{12}(\mathbf{x}) \qquad \mathbf{x} \text{ on } C_1$$

$$n_2(x,y) = \varepsilon E_n(x,y) = \frac{q(z)}{4\pi\varepsilon}\varepsilon K_1 G_{21}(\mathbf{x}) + \frac{q(z)}{4\pi\varepsilon}\varepsilon K_2 G_{22}(\mathbf{x}) \qquad \mathbf{x} \text{ on } C_2$$

$$q_1 = \oint_{C1} ds'\, n_1(x',y') = \frac{q(z)}{4\pi\varepsilon}[\varepsilon K_1 H_{11} + \varepsilon K_2 H_{22}] = \frac{q(z)}{4\pi\varepsilon}[c_{11}V_1 + c_{12}V_2]$$

$$q_2 = \oint_{C2} ds'\, n_2(x',y') = \frac{q(z)}{4\pi\varepsilon}[\varepsilon K_1 H_{21} + \varepsilon K_2 H_{22}] = \frac{q(z)}{4\pi\varepsilon}[c_{21}V_1 + c_{22}V_2]$$

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \frac{q(z)}{4\pi\varepsilon}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \qquad \text{or} \qquad \mathbf{q} = \frac{q(z)}{4\pi\varepsilon} c\, \mathbf{K} \quad .$$

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = \frac{4\pi\varepsilon}{q(z)}\begin{pmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{pmatrix}\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \qquad \text{or} \qquad \mathbf{K} = \frac{4\pi\varepsilon}{q(z)} s\mathbf{q}$$

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