This is the Title PhL 3.26.05

An aborted older version of 6.5 with an interesting plot including a disk.

(d) The Proximity Effect At Low Frequencies

Above we have shown plots of the proximity effect (J_z asymmetry) in the conductors of a transmission line made of two round cylinders. One wonders what happens to this proximity effect as $\omega \to 0$. As frequency drops, the fact that the two cylinders form a capacitor does not go away, so the asymmetric surface charge $n(\theta)$ persists and continues to have moments η_m which are non-zero. The resulting asymmetric current density is then given by (D.10.18) which says

$$J_z(r,\theta) = \sigma\,(1/4)\,I\,R_{dc}\,[\,4 + 4\,\Sigma_{m=1}^{\infty}\,(r/a)^m\,(m+1)\,\eta_m\,\cos(m\theta)\,]$$

$$= J_0\,[\,1 + \Sigma_{m=1}^{\infty}\,(r/a)^m\,(m+1)\,\eta_m\,\cos(m\theta)\,]$$

where $J_0 \equiv \sigma\,I\,R_{dc} = I/(\pi a^2)$ is the uniform current one expects in a round wire carrying a DC current I. Since the η_m are given by (6.5.4) as $\eta_m = (-1)^m\,e^{-|m\xi_1|}$, we have this explicit expression for the transmission line conductor current in the $\omega \to 0$ limit

$$J_z(r,\theta) = J_0\,[\,1 + \Sigma_{m=1}^{\infty}\,(m+1)\,(-1)^m\,e^{-|m\xi_1|}(r/a)^m\cos(m\theta)\,]$$

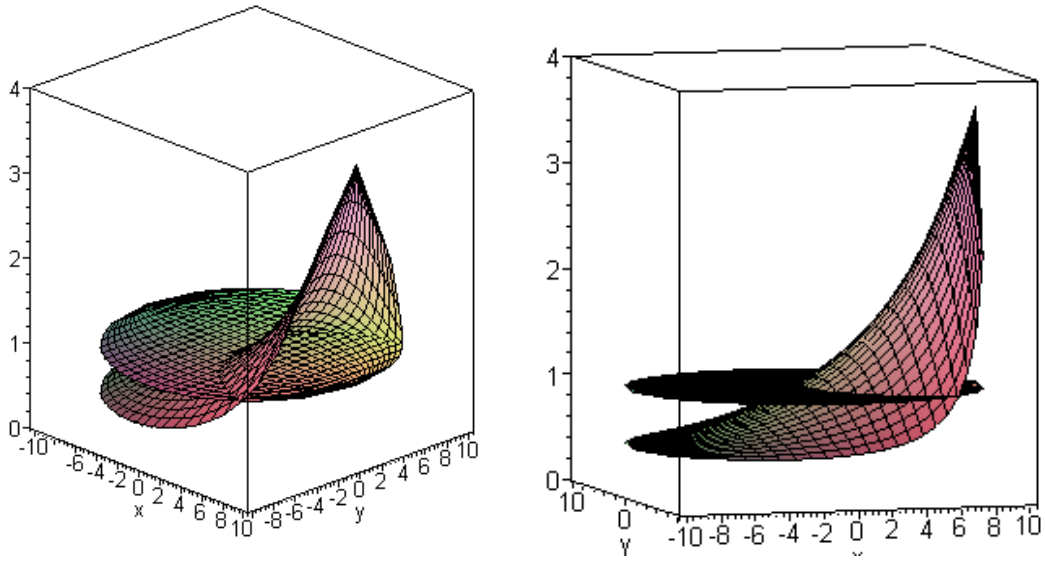
Maple plots this function for $\xi_1 = -0.75$ and a = 10:

```
Jz := 1 + Sum((-1)^m*exp(-m*abs(xi))*(r/a)^m*(m+1)*cos(m*theta),m=1..20);
```

$$Jz := 1 + \left(\sum_{m=1}^{20} (-1)^m\, e^{(-m\,|\xi|)} \left(\frac{r}{a}\right)^m (m+1)\cos(m\,\theta) \right)$$

```
a := 10: xi := - 0.75:
r := sqrt(x^2+y^2):
theta := arctan(y,-x):
plot3d({Jz,1},x=-a..a,y=-sqrt(a^2-x^2)..sqrt(a^2-x^2),axes=boxed, view =0..4 );
```

and here are two views of the result:



This strong asymmetry as $\omega \rightarrow 0$ is rather disturbing, since we know there is no asymmetry in a DC current flowing in an isolated round wire. The asymmetry shown arises from the "charge pumping boundary condition"

$$J_{\mathbf{r}}(a,\theta) = j\omega\, n(\theta) \quad . \qquad \textit{(D.2.23)}$$

which reflects charge conservation at the conductor surface, assuming $G = 0$ ($\sigma = 0$) for the dielectric.