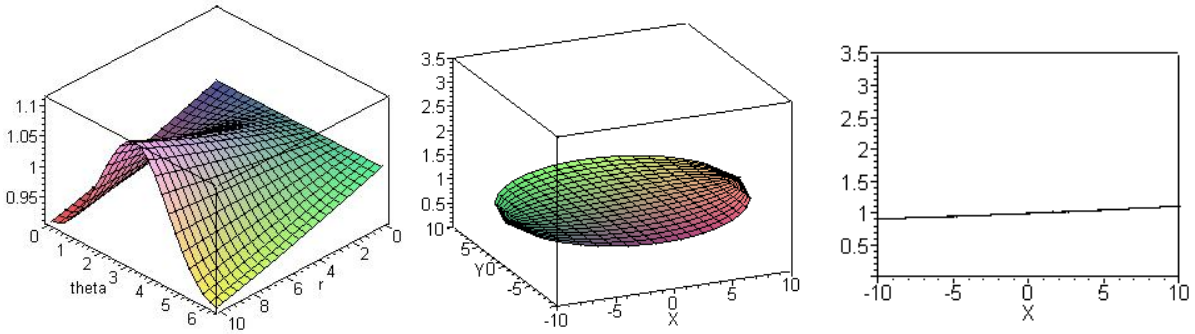
Old Section 6.5 (e) PhL 3.26.05

(e) The Proximity Effect At Low Frequencies

In various places in the above analysis, it was assumed that ω is high enough to put the transmission line in either the strong or extreme skin effect regime of (3.7.3) which roughly means $\delta < a/10$. In order to base the analysis on "the capacitor problem" as we have done (the electro-quasi-static model), we had to avoid the low frequency range which, expressed in terms of skin depth δ, is roughly $a/10 < \delta < \infty$. Only then could we be assured that φ = constant *and* A_z = constant on the conductor surfaces as declared in (3.7.4) and (3.7.20), and only then are the transmission line equations and parameter evaluations of Chapter 4 valid. Our analysis of the proximity effect presented above inherits all these assumptions (in addition to the requirement of being in the transmission line limit so $\lambda >> a$).

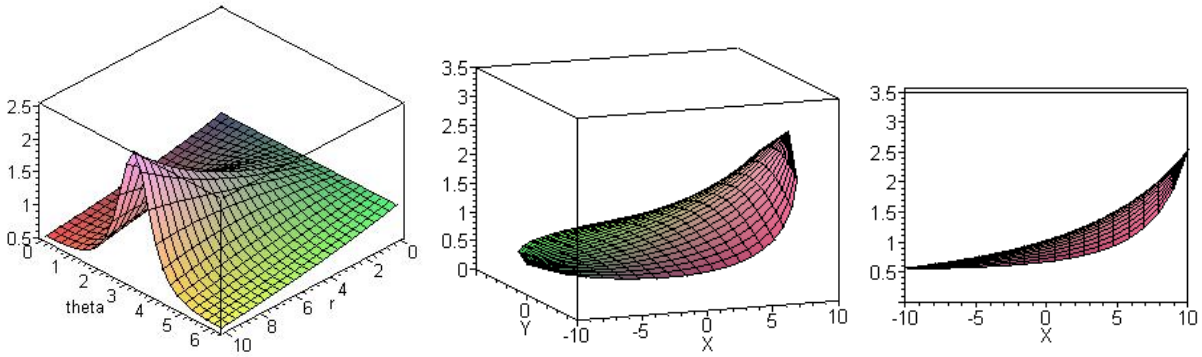
As described in the text section below (3.7.21), we might blindly assume that our transmission line theory is "ballpark valid" in the low frequency range, and then the proximity effect calculation as given above would also apply approximately. For example, if we use our program above with a = 10 and δ = 10, which is deep into the low frequency range, the plots look like this for $\xi_1 = -3$:



$|J_z|$ Distribution in left round wire for a = 10, δ = 20 and $\xi_1 = -3.00$ Fig 6.16

The current distribution $|J_z|$ shown on the right is a flat disc which is almost horizontal but still has a very slight proximity effect visible in the right view. Since $\xi_1 = -3$ in this example, the conductors are "far apart", and we certainly expect intuitively to see a nearly flat $|J_z|$ distribution as we would see at DC (ω= 0, since no eddy currents).

However, if we instead use $\xi_1 = -1.00$, the program gives a strong proximity effect even if we lower the frequency way down to δ = 200 (and a = 10),



$|J_z|$ Distribution in left round wire for a = 10, δ = 200 and ξ_1 = - 1.00 Fig 6.17

Even at a very low frequency, the strongly peaked capacitor's n(θ) is maintained and "gets into" the $|J_z|$ distribution. This result seems incorrect based on one's expectation that J_z is "uniform at DC", so for the last example, our model blindly stretched to low ω seems to be giving a wrong result.

However, there is an escape hatch which explains this paradoxical limit as ω→ 0. In the eddy current analysis of Section P.8, the two parallel cylinders considered there carry current I and -I. But our proximity effect described above is for a properly terminated active transmission line, not two parallel wires that are shorted together at one end. A properly terminated transmission line, which we equate with the infinite transmission line this document addresses, must be terminated by Z_0. If we naively apply our theory as if it were valid at very low ω, we have

$$Z_0 \equiv V/I = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad . \qquad (4.11.16)$$

Appendix D (in particular, the charge pumping boundary condition (D.2.24)) assumes a vacuum dielectric, so conductance G = 0. Thus, in the limit ω → 0 we find that $Z_0 = \infty$. If we drive this transmission line with V = 10 volts at f = 0.1Hz, we find that I ≈ 0 and there is no current at all, so the fact that J_z is asymmetric is not relevant.

Our suspicion is that the entire theory is only "ballpark accurate" at low ω, and therefore we have no trustworthy conclusions regarding the proximity effect at low frequencies. However, if we incorporate the loss model of Appendix D.11 and go ahead and assume "ballpark accuracy", here is a statement of our conclusions regarding J_z asymmetry:

1. For a well-conducting $\mu=\mu_0$ round wire which is part of a 2-conductor transmission line carrying a traveling wave of the form $e^{j(\omega t-\beta_d z)}$ through a vacuum dielectric, the E fields inside the wire are completely determined by the relative moments η_m of the surface charge distribution n(θ) as obtained from an electrostatic calculation based on the shape of the line cross section. From (D.2.33) the fields are:

Second summary of the E field solutions : $R_{dc} = \frac{1}{\sigma\pi a^2}$ $\beta'^2 = \beta^2 - \beta_d^2$ *(D.2.33)*

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta')\, f_m \qquad f_m = \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] \qquad x = \beta' r$$

$$E_r(r,m) = (j/4)\,\eta_m\, I\, R_{dc}\ (a\beta_d)\, g_m \qquad g_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} + \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right] \qquad x_a = \beta' a$$

$$E_\theta(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\ (a\beta_d)\ h_m \qquad h_m = \left[\frac{J_{m+1}(x)}{J_{m+1}(x_a)} - \frac{J_{m-1}(x)}{J_{m-1}(x_a)}\right]$$

Parameters in the E field expressions are:

a	radius of the wire
σ	conductivity of the round wire
R_{dc}	$= 1/(\pi a^2\sigma)$ = DC resistance of the round wire per unit length
I	current magnitude in the wire
β_d	wavenumber of the dielectric wave
β	wavenumber inside the conductor
β'	$\beta'^2 = \beta^2 - \beta_d^2$
η_m	relative moments of $n(\theta)$, $\eta_m = N_m/N_0$ as in (D.1.5)

where (see App D.10 for the soft cutoff frequency ω_c) ,

$\beta_d = (\omega - j\omega_c)/v_d$ $\qquad \omega_c = (v_d^2/2)\,(R'_{dc}C)$ $\qquad v_d = c$ = the speed of light
ω = angular frequency of the monochromatic wave
R'_{dc} = total resistance per unit length of both conductors.
C = capacitance per unit length of the two conductors
$\beta = e^{j3\pi/4}\,(\sqrt{2}/\delta) = (j-1)/\delta$ $\qquad \delta \equiv \sqrt{2/\omega\mu_0\sigma}$ = skin depth (6.5.21)

2. At all frequencies, including the limit $\omega \to 0$ (where $Z_0 \to \infty$), the current distribution J_z in the wire is non-uniform, and is determined from $J_z = \sigma E_z$. The current is non-uniform because the amplitudes of the partial-wave fields $E_z(r,m)$ for $m \neq 0$ do not in general vanish -- see for example (D.11.18). The one exception occurs if the round wire is the center conductor of a coaxial cable (see Chapter 2). See Appendix D.10 and D.11 for discussion of the high and low frequency limits of the boxed E fields above.

3. These conclusions apply to an infinite transmission line, or to a truncated transmission line which is properly terminated by $Z_t = Z_0$ where Z_0 is given by (4.4.12) with G = 0 (since vacuum dielectric). The conclusions do not apply to a truncated transmission line which is terminated with some fixed impedance at all ω, such as $Z_t = 0$.

We know that for the problem of two parallel cylindrical conductors (or any uniform parallel conductors) which carry I and -I and are shorted at one end, J_z is uniform at DC. We know this because at $\omega = 0$ there are no eddy currents induced by one wire into the other. The DC B field of wire #2 has no influence on the current density J_z in wire #1, though it does induce a tiny Hall charge onto the surface of wire #1 and a corresponding transverse Hall E field since the B field of wire #2 temporarily deflects electrons in wire #1 (see Appendix N for various Hall examples). This deflection effect is mentioned in the text below Fig

P.12 in Appendix P. In any event, if the DC B field of wire #2 does not affect J_z in wire #1, then J_z in wire #1 doesn't even know that wire #2 is present, so wire #2 could just as well be removed. The isolated wire #1 then if round (Chapter 2) would have a uniform J_z certainly with no proximity effect.

It would be reassuring if our transmission line model could account for the gradual transition from low ω to ω = 0 where the asymmetric J_z smoothly approaches a uniform a J_z. Unfortunately, our model only applies to an infinite (or properly terminated) transmission line, and thus cannot expose the ω→0 limit for parallel conductors terminated in a fixed load.

In a proper treatment of the fixed-load problem, as ω→0 one would see the eddy currents gradually decrease, one would arrive at a DC current I, and one would have J_z → uniform. The problem with our transmission line model is that, for the infinite line, or one in which the termination is constantly increased to equal Z_0, as ω→0 the eddy currents *do* gradually vanish, but *so does* the J_z associated with the current I, so the ratio of these currents maintains the asymmetric pattern of Fig 6.17.

A solution of the fixed load two-cylinder problem could be based on the eddy current methods outlined qualitatively in Appendix P and is no doubt available somewhere in the literature. This solution would then show the proximity and skin effects gradually vanishing as ω→ 0, leaving a uniform J_z.