Disaster Repair Log 2 PhL 11.17.13

On 11.21.13 6 PM I finished a complete review of this document. Holy Cow.

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1. Background for the Paradox

Here is my A wave equation derivation: [reviewed]

curl $\mathbf{H} = \partial_t \mathbf{D} + \mathbf{J}$ // Maxwell (1.1.1)

$(1/\mu)$ curl curl $\mathbf{A} = \mu\varepsilon\, \partial_t \mathbf{E} + \mathbf{J}$ // $\mathbf{H} = \mathbf{B}/\mu$, $\mathbf{B}$ = curl $\mathbf{A}$ from (1.3.1), and $\mathbf{D} = \varepsilon\mathbf{E}$

grad div$\mathbf{A}$ - $\nabla^2\mathbf{A}$ = $\mu\varepsilon\, \partial_t[-$ grad $\varphi - \partial_t \mathbf{A}] + \mu\mathbf{J}$ // vector identity and $\mathbf{E} = -$ grad $\varphi - \partial_t\mathbf{A}$

$(\nabla^2 - \mu\varepsilon\, \partial_t^2)\, \mathbf{A}$ = grad $[\mu\varepsilon\, \partial_t\varphi$ + div$\mathbf{A}$] - $\mu\mathbf{J}$ [= Jackson (6.11)] (1.3.3)

Here I have not used a gauge yet, nor have I talked about "regions". But what I do know is that the $\mathbf{J}$ shown here does NOT include $\mathbf{J}_m$ mag currents, because $\mathbf{H}$ does not "see" such currents so these currents don't exist in (1.1.1) [correct] . Thus, there is no equation of the form (1.3.3) in any "region" which is going to include such $\mathbf{J}_m$ currents [correct] . Fine. I think in the way I already did things, here is how I end up.

$(\nabla^2 - \mu_1\varepsilon_1\, \partial_t^2 - \mu_1\sigma_1\partial_t\,)\, \mathbf{A} = 0$ region 1
$(\nabla^2 - \mu_1\varepsilon_1\, \partial_t^2 - \mu_1\sigma_1\partial_t\,)\, \mathbf{A} = -\, \mu_2\mathbf{J}_2$ region 2 [both correct]

This is the King gauge for region 1 applied in both region 1 and region 2, so I end up with unified result [be careful when you say that!]

$(\nabla^2 - \mu_1\varepsilon_1\, \partial_t^2 - \mu_1\sigma_1\partial_t\,)\, \mathbf{A} = -\, \mu_2\mathbf{J}_2$ region R [correct and viable]

where $\mathbf{J}_2$ is the conduction current in the conductor. Going to ω space, and then going to DC, this says

$(\nabla^2 + \beta^2)\mathbf{A}(\mathbf{x},\omega) = -\, \mu_2\mathbf{J}_2(x,\omega)$ [correct]

What could be simpler! We know that at $\omega = 0$ $\beta^2 = 0$ so this says in 2D

$\mathbf{A}(\mathbf{x},\omega{=}0) = +(\mu_2/4\pi)\int dV'\, \mathbf{J}_2(\mathbf{x}',\omega{=}0)\, (1/R)$ $R = |\, \mathbf{x} - \mathbf{x}'\, |$ (*) [particular sol]

Now, I think $\mathbf{J}_2$ has a meaning independent of μ_2 of the conductor. It is amps/m^2 inside. Since this formula is supposed to be valid for all of region R, it is valid inside and outside of the conductor. [yes]

2. So here is the Paradox:

On the one hand:

(1) $\mathbf{A}(\mathbf{x},0)$ shown above (at $\omega = 0$) does not depend on μ_1 in any way (the dielectric's perm. value)

(2) **B** = curl **A,** therefore **B** cannot depend on μ_1.

On the other hand:

Outside the conductor, I know from the Maxwell curl **H(x,ω)** = **J(x,ω)** in ω space I know that

$$H(r,\omega) = I(\omega)/2\pi r \qquad r > a \qquad I(\omega) = \int dx'dy' \ \mathbf{J}_2(x',\omega)$$

and therefore

$$B(r,\omega) = \mu_1 H(r,\omega) = \mu_1 I/2\pi r \qquad r > a$$

and we see that in fact B *is* in fact a function of μ_1 for r > a.

So doing things the first way gives B *not* a function of μ_1, while doing things the second way gives a B which *is* a function of μ_1. This is the Paradox.

Both these ways cannot be correct, and I know the second way is correct, so there is something wrong with the first way. [correct]

Fact: Since **B** = curl **A**, in order for **B** to depend on μ_1, we must have **A** depend on μ_1, and that is true no matter what gauge you use, and no matter what ω you work at in ω-space. [correct]

Paradox Resolution: It is true that the particular solution (*) does not depend on μ_1. But that does not say that possibly required homogeneous adder solutions won't depend on μ_1, and in fact they do! These adders might be needed to satisfy boundary conditions.

3. Look Harder now at the Region 1 and Region 1 wave equation derivations:

Region 1: [reviewed]

We first obtain the wave equation for **A** in **region 1**. Start with (1.3.3) which gives the wave equation for **A** in region 1 before any gauge choice is made:

$$(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2)\,\mathbf{A} = \text{grad}\,[\mu_1\varepsilon_1 \partial_t\varphi + \text{div}\mathbf{A}\,] - \mu_1\mathbf{J} \quad . \qquad \text{// region 1} \qquad (1.3.3)$$

Now insert the King gauge (1.3.18) to get

$$(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2)\,\mathbf{A} = \text{grad}\,[\mu_1\varepsilon_1 \partial_t\varphi + (-\mu_1\varepsilon_1 \partial_t\varphi - \mu_1\sigma_1\varphi)\,] - \mu_1\mathbf{J} \qquad \text{// note cancelling of terms!}$$

$$= -\mu_1\sigma_1 \,\text{grad}\,\varphi - \mu_1\mathbf{J}$$

$$= -\mu_1\sigma_1 (-\mathbf{E} - \partial_t\mathbf{A}) - \mu_1\mathbf{J} \ . \qquad \text{// from (1.3.1)}$$

Now in region 1 we have relation $\mathbf{J} = \sigma_1 \mathbf{E}$ where $\mathbf{J}$ is the conduction current in region 1, and this is exactly the $\mathbf{J}$ which appears in the above equation. Therefore,

$$= - \mu_1\sigma_1 (-\mathbf{J}/\sigma_1 - \partial_t \mathbf{A}) - \mu_1 \mathbf{J} = + \mu_1 \mathbf{J} + \mu_1\sigma_1 \partial_t \mathbf{A} - \mu_1 \mathbf{J} = \mu_1\sigma_1 \partial_t \mathbf{A}$$

$$(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2) \mathbf{A} = \mu_1\sigma_1 \partial_t \mathbf{A}$$

which says

$$(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2 - \mu_1\sigma_1\partial_t) \mathbf{A}(\mathbf{x},t) = 0 \qquad \text{x in region 1}$$

I just do NOT see anything wrong with the above region 1 derivation. [there IS nothing wrong]

Region 2:

Start with are pre-Gauge wave equation now in region 2

$$(\nabla^2 - \mu_2\varepsilon_2 \partial_t^2) \mathbf{A} = \text{grad} [\mu_2\varepsilon_2 \partial_t\varphi + \text{div}\mathbf{A}] - \mu_2\mathbf{J} \quad . \qquad \text{// region 2} \qquad (1.3.3)$$

So far all I have done is insert μ,ε,σ appropriate for this new region.

Next, I use the exact same gauge used for region .

$$\text{div } \mathbf{A} = - \mu_1\varepsilon_1 \partial_t\varphi - \mu_1\sigma_1\varphi$$

Is there anything wrong with deciding to do this? [no] It is the same gauge used in region 1, but now I apply it to x values in region 2. I really think this is OK to do. We then get

$$(\nabla^2 - \mu_2\varepsilon_2 \partial_t^2) \mathbf{A} = \text{grad} [\mu_2\varepsilon_2 \partial_t\varphi + (- \mu_1\varepsilon_1 \partial_t\varphi - \mu_1\sigma_1\varphi)] - \mu_2\mathbf{J} \quad . \qquad \text{// region 2}$$

$$= [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)]\text{grad } \varphi - \mu_2\mathbf{J} \qquad \text{// region 2}$$

$$= [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)] (-\mathbf{E} - \partial_t\mathbf{A}) - \mu_2\mathbf{J} \qquad \text{// region 2}$$

$$= [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)] (-\mathbf{E}) - \mu_2\mathbf{J} \qquad (*)$$
$$+ [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)](-\partial_t\mathbf{A})$$

$$= [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)] (-\mathbf{J}/\sigma_2) - \mu_2\mathbf{J}$$
$$- [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)](\partial_t\mathbf{A})$$

$$= [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)] (-\mathbf{J}/\sigma_2) - \mu_2\mathbf{J}$$
$$- [\mu_2\varepsilon_2 \partial_t^2 + (- \mu_1\varepsilon_1 \partial_t^2 - \mu_1\sigma_1\partial_t)] \mathbf{A}$$

$$= [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)] (-\mathbf{J}/\sigma_2) - \mu_2\mathbf{J}$$

$$+ (-\mu_2\varepsilon_2 \partial_t^2 + \mu_1\varepsilon_1 \partial_t^2 + \mu_1\sigma_1\partial_t)] \mathbf{A}$$

Now look at the second line, which has 3 terms. The first term cancels the corresponding term on the LHS. We then move the two other terms to the LHS to get

$$(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2 - \mu_1\sigma_1\partial_t) \mathbf{A} = [\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)] (-\mathbf{J}/\sigma_2) - \mu_2\mathbf{J}$$

Now we simplify the RHS

$$[\mu_2\varepsilon_2 \partial_t + (- \mu_1\varepsilon_1 \partial_t - \mu_1\sigma_1)] (-\mathbf{J}/\sigma_2) - \mu_2\mathbf{J}$$

$$= [(\mu_2\varepsilon_2-\mu_1\varepsilon_1) \partial_t - \mu_1\sigma_1] (-\mathbf{J}/\sigma_2) - \mu_2\mathbf{J}$$

$$= [- (\mu_2\varepsilon_2-\mu_1\varepsilon_1) \partial_t + \mu_1\sigma_1] (\mathbf{J}/\sigma_2) - \mu_2\mathbf{J}$$

$$= [- \{(\mu_2\varepsilon_2-\mu_1\varepsilon_1)/\sigma_2\} \partial_t + \mu_1\sigma_1/\sigma_2] \mathbf{J} - \mu_2\mathbf{J}$$

$$= [- \{(\mu_2\varepsilon_2-\mu_1\varepsilon_1)/\sigma_2\} \partial_t + \mu_1\sigma_1/\sigma_2 - \mu_2] \mathbf{J}$$

This seems to be something new and a little unexpected. Previously at step (*) I just set $\mathbf{E} = 0$ saying it was inside a conductor. Maybe that was the wrong thing to do. But let's go to ω space

$$(\nabla^2 + \beta_1^2) \mathbf{A} = [- \{(\mu_2\varepsilon_2-\mu_1\varepsilon_1)/\sigma_2\} j\omega + \mu_1\sigma_1/\sigma_2 - \mu_2] \mathbf{J}$$

At this point you could say that σ_2 is very large [correct] and so you then really to get

$$(\nabla^2 + \beta_1^2) \mathbf{A} = - \mu_2 \mathbf{J} \qquad \text{// region 2} \quad [\text{correct}]$$

as long as ω is not TOO high. **J** is the conductors conduction current.

So here then are my results:

$$(\nabla^2 + \beta_1^2) \mathbf{A} = 0 \qquad \text{x in region 1}$$

$$(\nabla^2 + \beta_1^2) \mathbf{A} = - \mu_2 \mathbf{J} \qquad \text{x in region 2}$$

[both correct]

Understanding that J exists only in region 2 of region R, we unify to get

$$(\nabla^2 + \beta_1^2) \mathbf{A} = - \mu_2 \mathbf{J} \qquad \text{x in region R}$$

[unified is correct and viable]

Then we go do DC $\omega = 0$ to get

$$(\nabla^2) \mathbf{A} = - \mu_2 \mathbf{J} \qquad \text{x in region R}$$

and then we are back to the Paradox, because μ_1 does not appear in this equation! [But as noted, μ_1 may appear in homo adder solutions.]

4. Vague Idea for Resolving the Paradox.

[reviewed]

The general idea is this: I treat things like ε,μ,σ as "constants" in space, but in the region of a boundary, they are NOT constants. They have different values on the two sides of the boundary, so you have to be a little careful in such a region. You cannot for example pass them through a curl operator right at the boundary. But if you stay cleanly in one region or the other, you are OK.

[This was just a straw in the wind idea. But really I already am careful which region I am in, and this vague idea is not the source of the paradox.]

curl $\mathbf{H} = \partial_t \mathbf{D} + \mathbf{J}$ // Maxwell (1.1.1) in region 1

$\mathbf{H} = \mathbf{B}/\mu_1$ in region 1

$(1/\mu_1)$ curl curl $\mathbf{A} = \mu_1\varepsilon_1 \partial_t \mathbf{E} + \mathbf{J}$ // $\mathbf{H} = \mathbf{B}/\mu$, $\mathbf{B}$ = curl $\mathbf{A}$ from (1.3.1), and $\mathbf{D} = \varepsilon\mathbf{E}$

grad div$\mathbf{A}$ - $\nabla^2\mathbf{A}$ $= \mu_1\varepsilon_1 \partial_t[-$ grad $\varphi - \partial_t\mathbf{A}] + \mu_1\mathbf{J}$ // vector identity and $\mathbf{E} = -$ grad $\varphi - \partial_t\mathbf{A}$

$(\nabla^2 - \mu_1\varepsilon_1 \partial_t^2)\ \mathbf{A}$ = grad $[\mu_1\varepsilon_1 \partial_t\varphi +$ div$\mathbf{A}\ ] - \mu_1\mathbf{J}$ [= Jackson (6.11)] (1.3.3)

I can do this in region 2 as well. BUT I don't think I can just take the union of these two equations.

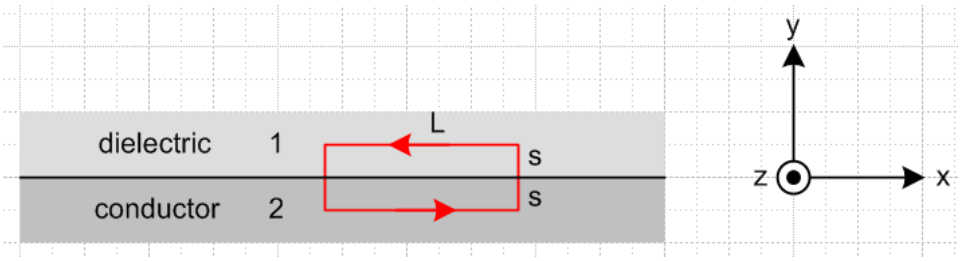
$(\nabla^2 + \beta^2)\ \mathbf{A} = 0$ x in region 1 away from the boundary

[correct as usual]

$(\nabla^2 + \beta^2)\ \mathbf{A} = -\mu_2\ \mathbf{J}$ x in region 2 away from the boundary

But region R includes regions 1 and 2 *and* the boundary region. So that is my vague idea.

Let's roll out this picture



Then I would claim

$(\nabla^2 + \beta^2)\ \mathbf{A} = 0$ for $y > \varepsilon$ region 1

$(\nabla^2 + \beta^2)\,\mathbf{A} = -\mu_2\,\mathbf{J}$ for $y < \varepsilon$ region 2

Now in region 2 I might be able to say $(\omega = 0)$

$\mathbf{A}(\mathbf{x},\omega=0) = +(\mu_2/4\pi)\int_{R2} dV'\,\mathbf{J}_2(\mathbf{x}',\omega=0)\,(1/R)$ for $y < \varepsilon$ region 2

My idea here is this claim: You cannot simply take the union of these two equations, because then you have included the boundary region that you don't know much about. [Again, just a straw. In Stak we do this multi-region stuff all the time, but there are BC's to deal with at the boundary. At this point, I was not very cognizant of what BC's on A_z might look like! I think that came later. Just grasping.]

So suppose go try to solve the problem for region 2. This gives some result and I can take $\varepsilon \to 0$ and approach the surface from below. In fact, if I do this for the round conductor, I will get this: (I can just look at my 2D Section B.6 result!)

$$A_z = -[I/(4\pi^2 a^2)]\,\{\,\mu_2 \int dV'\,\ln(R^2)\,\} \qquad (B.6.4)$$

$\int dV'\,\ln(R^2) = 2\pi\,a^2 \ln a + \pi(r^2 - a^2)$ $r < a$ inside

Then

$A_z(r) = -[I/(4\pi^2 a^2)]\,\mu_2\{\,2\pi\,a^2 \ln a + \pi(r^2 - a^2)\,\} + A_z(r)^{homo2}$ $r \le a$

where I now allow some homogeneous solution inside the wire which is region 2. [I am slowly waking up here, this is the key idea : there will be homo adders].

The limit then gives

$A_z(a-) = -[I/(4\pi^2 a^2)]\,\mu_2\{\,2\pi\,a^2 \ln a\,\} + A_z(a)^{homo}$

Now perhaps the next step would be to examine the boundary condition for A_z at this boundary and maybe for some derivatives of A_z as well. From my newly reworked Section 1.1 I have this result

$(1/\mu_2)\,(\partial_n A_z)_2 - (1/\mu_1)\,(\partial_n A_z)_1 = K_z^{free}$

But I don't think there is any *free* surface current on this boundary [correct] , so this would then read

$(1/\mu_2)\,(\partial_r A_z)_2 = (1/\mu_1)\,(\partial_r A_z)_1$. [correct, and other BC is $A_z^1 = A_z^2$]

Now let's go back to region 2

$$A_z = -[I/(4\pi^2 a^2)]\,\mu_2\{\,2\pi\, a^2 \ln a + \pi(r^2 - a^2)\,\} + A_z(r)^{homo2} \qquad r \le a$$

$$(\partial_r A_z)_2 = -[I/(4\pi^2 a^2)]\,\mu_2 * 2\pi r + (\partial_r A_z^{homo2})_2$$

$$\rightarrow\ -[I/(4\pi^2 a^2)]\,\mu_2 * 2\pi a + (\partial_r A_z^{homo2})|_{r=a} = -[I/(2\pi a)]\,\mu_2 + (\partial_r A_z^{homo2})|_{r=a}.$$

So I then claim that

$$(1/\mu_1)\,(\partial_r A_z)_1 = (1/\mu_2)\ \{ -[I/(2\pi a)]\,\mu_2 + (\partial_r A_z^{homo2})|_{r=a}\,\}$$

$$= -[I/(2\pi a)] + (1/\mu_2)\,(\partial_r A_z^{homo2})|_{r=a}.$$

So maybe what I have for region 1 is a Neumann problem, or at least some kind of BV problem. [Well, it is not a Neumann problem because $\partial_r A_z$ is not specified on a closed boundary, nor is it a mixed problem. But like any Stak potential theory problem with multiple regions, you have to decide what your BC's are at boundaries between regions and then make sure they are met. Title of Stak book !]

Now let's take a look at region 1 again with $\beta_1 = 0$.

$$(\nabla^2 + \beta_1^2)\,\mathbf{A} = 0 \qquad \text{for } y > \varepsilon \qquad \text{region 1}$$

Now I have already done this in App B, so I have

$$A_z(r) = A_z^{homo1}(r)$$

I guess I won't get far without some general atomic form which allows coefficient match at r = a. I now jump to the 2D model of things and write:

$$A_z^{homo1}(r) = A \ln r + Br + Cr^{-1} + D \qquad \text{for example from Stak II p 92}$$

[Error: for azisym function, r and r^{-1} are not atoms, so B = C = 0, as I do later *anyway*.]

Now in region 1 we can go do large r and I think we can set B = 0 at least. So let's just try

$$A_z^{homo1}(r) = A \ln r + Cr^{-1} + D$$

$$A_z^{homo2}(r) = \alpha \ln r + br + cr^{-1} + d = br + d \qquad \text{// I guess}$$

[Not "I guess" but true since r and r^{-1} are not azisym atoms.]

OK, then we have

$$(\partial_r A_z^{homo})_1 = [\,A(1/r) - Cr^{-2}\,]|^{a=r} = A(1/a) - Ca^{-2}$$

and

$(\partial_r A_z)_1 = (\partial_r A_z^{homo})_1 = A(1/a) - Ca^{-2}$

Meanwhile

$(\partial_r A_z^{homo})_2 = b$

and

$A_z(r) = - [I/(4\pi^2 a^2)]\, \mu_2 \{ 2\pi a^2 \ln a + \pi(r^2 - a^2) \} + A_z(r)^{homo2}$ $\qquad r \le a$

$\Rightarrow (\partial_r A_z)_2 = - [I/(4\pi^2 a^2)]\, \mu_2 2\pi r|^{r=a} + (\partial_r A_z^{homo})_2$

$= - [I/(2\pi a)]\, \mu_2 + b$

Then the boundary condition says

$(1/\mu_2)\, (\partial_r A_z)_2 = (1/\mu_1)\, (\partial_r A_z)_1$ $\qquad$ [correct since no free boundary current.]

$(1/\mu_2)\{ - [I/(2\pi a)]\, \mu_2 + b \} = (1/\mu_1)\{ A(1/a) - Ca^{-2}\}$

Meanwhile, I conjecture another BC which is that A_z is continuous so we would then have

$(A_z)_2 = (A_z)_1$ $\qquad$ [correct]

LHS $= (A_z)_2 = - [I/(4\pi^2 a^2)]\, \mu_2 \{ 2\pi a^2 \ln a + \pi(r^2 - a^2) \} + A_z(r)^{homo2}$

$= - [I/(4\pi^2 a^2)]\, \mu_2 2\pi a^2 \ln a + ba + d$

$= - [I/2\pi]\, \mu_2 \ln a + ba + d$

RHS $= (A_z)_1 = A_z^{homo1}(r) = A \ln a + Ca^{-1} + D$

So this BC says

$- [I/2\pi]\, \mu_2 \ln a + ba + d = A \ln a + Ca^{-1} + D$

Here then are the two BC's:

$(1/\mu_2)\{ - [I/(2\pi a)]\, \mu_2 + b \} = (1/\mu_1)\{ A(1/a) - Ca^{-2}\}$

$- [I/2\pi]\, \mu_2 \ln a + ba + d = A \ln a + Ca^{-1} + D$

and we have these unknowns: A, C, b, d, D

I don't think there are any other BC's, so maybe I can set some of these to 0.

Let's just try on the outside that C = D = b = 0 and see what happens

$$-I/(2\pi) = (1/\mu_1)\{ A \} \quad \Rightarrow \quad A = -(1/2\pi)\mu_1 I$$

$$-[I/2\pi]\,\mu_2 \ln a + d = A \ln a \quad \Rightarrow \quad d = +[I/2\pi]\,\mu_2 \ln a + A \ln a = +[I/2\pi]\,\mu_2 \ln a - (1/2\pi)\mu_1 I \ln a$$
$$= [I/2\pi]\,(\mu_2 - \mu_1) \ln a$$

where only A and d are left and I solved for them already on the right. Then our solutions are

$$A_z^{homo1}(r) = A \ln r = -(1/2\pi)\mu_1 I \ln r$$

$$A_z^{homo2}(r) = [I/2\pi]\,(\mu_2 - \mu_1) \ln a$$

And then the full solution is

$$A_z^{1} = A_z^{homo1}(r) = -(1/2\pi)\mu_1 I \ln r$$

$$A_z^{2} = -[I/(4\pi^2 a^2)]\,\mu_2\{ 2\pi a^2 \ln a + \pi(r^2 - a^2) \} + [I/2\pi]\,(\mu_2 - \mu_1) \ln a$$

$$= -[I/(4\pi^2 a^2)]\,\mu_2 2\pi a^2 \ln a - [I/(4\pi^2 a^2)]\,\mu_2\,\pi(r^2 - a^2) + [I/2\pi]\,(\mu_2 - \mu_1) \ln a$$

$$= -[I/(2\pi)]\,\mu_2 \ln a - [I/(4\pi a^2)]\,\mu_2\,(r^2 - a^2) + [I/2\pi]\,(\mu_2 - \mu_1) \ln a$$

$$= -[I/2\pi]\mu_1 \ln a - [I/(4\pi a^2)]\,\mu_2\,(r^2 - a^2)$$

Say it again

$$A_z(r) = -(1/2\pi)\mu_1 I \ln r \qquad r > a$$

$$A_z(r) = -[I/2\pi]\mu_1 \ln a - [I/(4\pi a^2)]\,\mu_2\,(r^2 - a^2) \qquad r < a$$

Now how does this compare with my Appendix B result with the surface current thing? There I got

$$A_z(r>a) = -[I/(2\pi)]\,\{ \mu_1 \ln(r) \} \qquad \text{// which agrees!}$$

$$A_z(r<a) = -[I/(2\pi)]\,\{ \mu_1 \ln(a) + \mu_2(r^2 - a^2)/(2a^2)) \} \qquad \text{// which agrees}$$

Wow! I have resolved the paradox, and now have a lot to ponder. Why does the assumption of a surface magnetization current give the same answer???

[OK, THIS section has the main and correct idea. I just had trouble accepting it. I was hoping those King Helmholtz integrals didn't need adder homo solutions I guess.]

November 19, 2013

Creation of Appendix B Version 2.

I made a copy of Appendix B and am now editing it, making corrections. This results in a slightly different formula for K_z^m.

Section B.1: I now make clear that $K_z^{free} = 0$ and only K_z^m exists at the boundary and that is why you are allowed to say $H_{x2} = H_{x1}$ [true]. Then I use the Maxwell equation version curl **B** $= \mu_0(j\omega \mathbf{D} + \mathbf{J}_c + \mathbf{J}_m)$ in which form we can say that "B sees all currents" and they all have a common μ_0 factor. I derive this equation carefully in Section 1.1 as (1.1.24) [correct] . This is different from what I had before for curl **B**. I then end up with a *different* result equation which is now $K_z^m = [(\mu_2-\mu_1)/\mu_0]\, H_{x2} = -\, [(\mu_2-\mu_1)/\mu_0]\, H_\theta$. My result for the simple round wire (B.1.11) is now different from what it was before. [The new K_z^m expression is the correct one. But I never realized J_m exists everywhere, not just on the boundary.]

[Section B.1 is interesting about the surface mag current, but I am not sure it plays a role in the solution later in the section.]

Section B.2. My derivation of $\mathbf{H}(x,y) = -\frac{1}{4\pi}\int d^2\mathbf{x}'\, \ln(R^2)\ \text{curl}'\, \mathbf{J}(\mathbf{x}')$ which is unchanged.

Section B.3. Unchanged except the formula for K_z^m now has the new form which still includes the factor $(\mu_2-\mu_2)$. [I think my General Method fails because including J_m in the wave equation is wrong in the first place, so why have a fancy General Method to find the surface part of J_m ?]

Section B.4: This is the simple-method way to compute H_θ for round wire and then I use the new K_z^m formula and I restate K_z^m for the round wire. Only thing changed here is formula for K_z^m. [but does anybody care about this surface current?]

Section B.5. This is my very long and detailed calculation of H_θ for the round wire with uniform current which makes use of my fancy formula $\mathbf{H}(x,y) = \int d^2\mathbf{x}'\, [\, \frac{1}{2\pi} \ln(1/R)\,]\ \text{curl}'\, \mathbf{J}(\mathbf{x}')$. I compute the curl J thing for the wire and insert it. I renamed the integral C instead of K to avoid confusion. This section makes no mention of surface current and is therefore completely unchanged except for name C.
[again, I don't think anybody really cares about this surface current computation method. Perhaps it is interesting that you can compute the surface current even if we have no use for it!]

Section B.6: The big change here is that I assume without justification that

$$\nabla^2{}_{2D}\mathbf{A}(\mathbf{x}) = -\mu_0\mathbf{J}_m(\mathbf{x}) - \mu_2\mathbf{J}_c(\mathbf{x})\ . \qquad [\text{ wrong and unjustifiable starting point ! }]$$

where I have μ_0 in the first term whereas I used to have μ_1. Later I will try to justify this equation! [I never did] There are two issues: (1) why μ_0 for J_m ? (2) why should you be including J_m in the first place? Given this new equation, I use Appendix I to write down the particular solution (B.6.1), nothing new here. But now when I insert the new form for K_z^m and account for the new presence of μ_0, these two changes cancel and I get the exact same result for A_z that I got before. This then yields the expected H and B results for the round wire, both inside and outside! So luckily, most of this Section B.6 is the same including the Maple plot.

Section B.7 is my reader exercise. Only change I made is $\mu_1 \rightarrow \mu_0$ at the start.

This concludes my major edit of Appendix B in its separate Version 2 doc.

[This Appendix B has to be completely redone, even its purpose now is not clear.]

Here then is the major issue: I want to justify this fact

$$\nabla^2 \mathbf{A}(\mathbf{x}) = -\mu_0 \mathbf{J}_m(\mathbf{x}) - \mu_2 \mathbf{J}_c(\mathbf{x}) \qquad \text{for all of region R}$$

Somehow this is the proper "union" of region 1 and region 2. What it says is that A "sees" both J_c and J_m and that is why B will see both as well. [call it coincidence.]

I think a similar situation arises in my Jackson ε problem, but I have not cleaned that up yet.

So here is the Theorem I want to somehow derive:

Theorem: We know that separately in our two regions we have

$$(\nabla^2 + \beta_1^2)\, \mathbf{A} = 0 \qquad \text{x in region 1}$$

$$(\nabla^2 + \beta_1^2)\, \mathbf{A} = -\mu_2 \mathbf{J}_{c2} \qquad \text{x in region 2}$$

We know that there is a magnetization current $\mathbf{J}_m$ on the boundary where [ignoring other J_m]

$$K_z^m = -[(\mu_2-\mu_1)/\mu_0]\ H_\theta = -[(\mu_2-\mu_1)/\mu_0]\ I/(2\pi a) \qquad \text{amp/m} \qquad \text{(B.4.4)}$$

$$\mathbf{K}_z^m = K_z^m\, \hat{\mathbf{z}} \qquad \text{(B.4.5)}$$

$$\mathbf{J}_m = K_z^m \delta(r-a)\hat{\mathbf{z}} \qquad \text{amp/m}^2 \ . \qquad \text{(B.4.6)}$$

We know we can find the solution in the two regions by using Smythian forms in each region and matching boundary conditions to find the unknown assumed Smythian form constants, and we know that in this method we have a homo solution in each region as part of the Smythian form. In fact, in region 1 the entire solution is obviously a homo solution since $(\nabla^2 + \beta_1^2)\, \mathbf{A} = 0$. [all correct]

The claim of the Theorem is that there is an alternate solution method [yes, that is the claim] . In that alternate method, you take the union of the two regions to form region R, and you then have this unified equation for region R

$$(\nabla^2 + \beta_1^2)\, \mathbf{A} = -\mu_0 \mathbf{J}_m - \mu_2 \mathbf{J}_{c2} \qquad \text{x in region R}$$

where now **A** is simply the particular integral of the above equation, where we have ADDED the mag current on the right side with a μ_0 multiplier. The resulting **A** is valid everywhere in R, which means it is valid in both region 1 and region 2.

To "prove" the theorem, we have to explain *why* this unified region R problem is the same problem! I think this explanation may involve the use of the word "image". [perhaps]

Viewed from either side, if we add an "image current" $K_z = K_z^m$ on the boundary, we get the same solution A that we get using the Smythian method for both regions outlined above. [I don't think this idea pans out.]

Comparing Problem I and Problem II

[reviewed]

Suppose we claim this Theorem as an ansatz [OK, can do that] , and then see where it leads. Let's call this situation with the image current to be Problem II. We want to somehow show that Problem II is the same as Problem I where there is no image current and where we solve the regions separately using Smythian forms

[I think I am sort of on the right track here in comparing these two problems (ie, with and without the added-in surface charge). But I could not drive it to any useful conclusion.]

(1) First, if we stay away from the boundary, we know that Problem 2 is described by

$(\nabla^2 + \beta_1^2)\, \mathbf{A}^{II} = 0$	x in region 1	PDE set II
$(\nabla^2 + \beta_1^2)\, \mathbf{A}^{II} = -\mu_2\, \mathbf{J}_{c2}$	x in region 2	

And we know that these equations are the same as those for Problem 1 which we can write as

$(\nabla^2 + \beta_1^2)\, \mathbf{A}^{I} = 0$	x in region 1	PDE set I
$(\nabla^2 + \beta_1^2)\, \mathbf{A}^{I} = -\mu_2\, \mathbf{J}_{c2}$	x in region 2	

(2) If we then focus on the boundary region, we know for Problem 2

$(A_z^{II})_2 = (A_z^{II})_1$	BC set II
$(1/\mu_2)\,(\partial_n A_z^{II})_2 - (1/\mu_1)\,(\partial_n A_z^{II})_1 = K_z^{image}$	

whereas for Problem 1 we have instead

$(A_z^{I})_2 = (A_z^{I})_1$	BC set I
$(1/\mu_2)\,(\partial_n A_z^{I})_2 - (1/\mu_1)\,(\partial_n A_z^{I})_1 = 0$	

Summary

Problem I:

$(\nabla^2 + \beta_1^2)\, \mathbf{A}^{I} = 0$	x in region 1	PDE set I
$(\nabla^2 + \beta_1^2)\, \mathbf{A}^{I} = -\mu_2\, \mathbf{J}_{c2}$	x in region 2	
$(A_z^{I})_2 = (A_z^{I})_1$		BC set I
$(1/\mu_2)\,(\partial_n A_z^{I})_2 - (1/\mu_1)\,(\partial_n A_z^{I})_1 = 0$		

Problem II

$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{II} = -\mu_0 \mathbf{J}_m - \mu_2 \mathbf{J}_{c2}$ x in region R // image current present!

$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{II} = 0$ x in region 1 PDE set II

$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{II} = -\mu_2 \mathbf{J}_{c2}$ x in region 2

$(A_z^{II})_2 = (A_z^{II})_1$ BC set II

$(1/\mu_2)(\partial_n A_z^{II})_2 - (1/\mu_1)(\partial_n A_z^{II})_1 = K_z^{image}$

$\mathbf{A}^{II} = \int_R dV' [-\mu_0 \mathbf{J}_m - \mu_2 \mathbf{J}_{c2}]\, e^{-j\beta R}/R$ = the particular solution

(3) Now we might conjecture that, for the entire region R, we can write

$\mathbf{A}^{II} - A^{I} = A^{homo}$

where the following is true

$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{homo} = 0$ x in region 1

$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{homo} = 0$ x in region 2

This would then allow PDE set I and PDE set II to both be satisfied even though $\mathbf{A}^{I} \neq A^{II}$.

(4) Now consider

$(1/\mu_2)(\partial_n A_z^{II})_2 - (1/\mu_1)(\partial_n A_z^{II})_1 = K_z^{image}$

$(1/\mu_2)(\partial_n A_z^{I})_2 - (1/\mu_1)(\partial_n A_z^{I})_1 = 0$

If we subtract these equations (and also the first equations) we find

$(A_z^{homo})_2 = (A_z^{homo})_1$

$(1/\mu_2)(\partial_n A_z^{homo})_2 - (1/\mu_1)(\partial_n A_z^{homo})_1 = K_z^{image}$

So far then we seem to have identified the following description of this homo solution

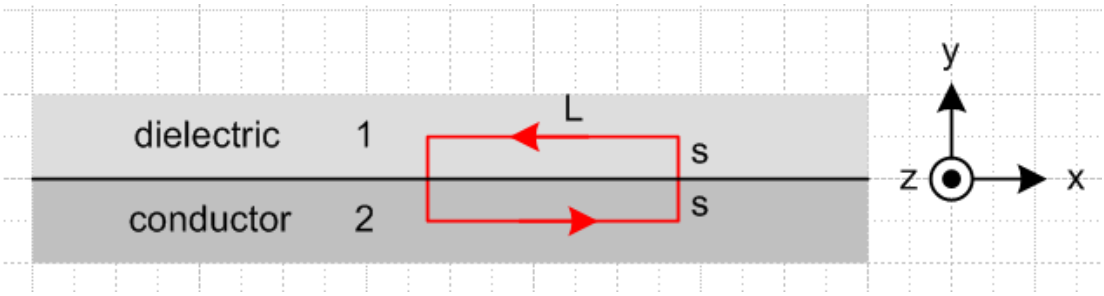
$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{homo} = 0$ x in region 1

$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{homo} = 0$ x in region 2

$(A_z^{homo})_2 = (A_z^{homo})_1$

$(1/\mu_2)(\partial_n A_z^{homo})_2 - (1/\mu_1)(\partial_n A_z^{homo})_1 = K_z^{image} = [(\mu_2-\mu_1)/\mu_0]\, H_{x2} = (B_{x2} - B_{x1})/\mu_0$

Suppose we could *find* a solution A_z^{homo} that solves the above stated boundary value problem. Then this A_z^{homo} expresses the difference between the Problem I and Problem II solutions. If we could show that this homo solution created zero **B** field, then maybe we could say the problems are "the same". But it seems to me that a surface charge K_z^{image} is not going to have a 0 B field! But go back to the picture



If there is a K_z^{image} and if we know that $\mathbf{A} = A_z(y,z)\,\hat{\mathbf{z}}$, then I show in (1.1.46) that

$$\mathbf{B}^{homo} = \hat{\mathbf{x}}\, B_x^{homo} = (\partial_y A_z^{homo})\,\hat{\mathbf{x}}$$

so I conclude that such a K_z^{image} really does create a $\mathbf{B}^{homo}$ field which is non-zero. But then

$$(A_z^{homo})_2 = (A_z^{homo})_1$$
$$(1/\mu_2)\,(\partial_y A_z^{homo})_2 - (1/\mu_1)\,(\partial_y A_z^{homo})_1 = K_z^{image} = [(\mu_2-\mu_1)/\mu_0]\,H_{x2} = (B^{homo}{}_{x2} - B^{homo}{}_{x1})/\mu_0$$

so

$$(A_z^{homo})_2 = (A_z^{homo})_1$$
$$(1/\mu_2)\,B_x^{homo}{}_2 - (1/\mu_1)\,B_x^{homo}{}_1 = (B^{homo}{}_{x2} - B^{homo}{}_{x1})/\mu_0$$

which just says $B_x^{homo}{}_2$ is some multiple of $B_x^{homo}{}_1$ and does not say both are 0.

Conclusion so Far: I wanted to show that Problem I and Problem II were "the same problem", but using the thread above, I end up with Problem I and Problem II having different B fields, so how can they be the same problem?

Idea during Review: Can you show *directly* somehow that $\mathbf{A} = (1/2\pi) \int dV' \,[\mu_0 \mathbf{J}_m(\mathbf{x}') + \mu_2 \mathbf{J}_c(\mathbf{x}')]$ ln(1/R) satisfies the two BC's? That is to say, apply ∂_r to this Helm integral and somehow compute $\partial_r A_z$ on the two sides. If you could show that, then you know that the first J_m term only creates homo solutions on the two sides, so then you could say it is creating *exactly the right* homo terms needed to meet the BC's ! I will pursue this a bit a little later, though it is not necessary./]

Lets look again at the Smythian solution for Problem I found yesterday.

In region 1 we have

$$(\nabla^2 + \beta_1^2)\, A_z^I = 0 \qquad \text{x in region 1}$$

$$A_z^I = A_z(r) = A_z^{homo1}(r) = A \ln r \qquad A = \text{to be determined}$$

In region 2 we have

$$(\nabla^2 + \beta_1^2)\, A_z^I = = -\mu_2 \mathbf{J}_{c2} \qquad \text{x in region 2}$$

$A_z^{I,part} = -[I/(4\pi^2a^2)]\{\mu_2 \int dV' \ln(R^2)\} = -[I/(4\pi^2a^2)]\,\mu_2\{2\pi a^2 \ln a + \pi(r^2-a^2)\}$

$A_z^{I} = A_z^{I,part} + A_z^{I,homo2}$

$A_z^{I,homo2} = d$ d = to be determined

There is no free surface charge at the boundary, so our two boundary conditions are

$(A_z^{I})_2 = (A_z^{I})_1$
$(1/\mu_2)(\partial_n A_z^{I})_2 - (1/\mu_1)(\partial_n A_z^{I})_1 = 0$

From these two conditions we learn that

$A = -(1/2\pi)\mu_1 I$
$d = [I/2\pi](\mu_2 - \mu_1)\ln a$.

Therefore we have

Region 1: $A_z^{I} = A_z(r) = A_z^{homo1}(r) = A \ln r = -(1/2\pi)\mu_1 I \ln r$

Region 2: $A_z^{I} = A_z^{I,part} + A_z^{I,homo2}$

$= -[I/(4\pi^2a^2)]\,\mu_2\{2\pi a^2 \ln a + \pi(r^2-a^2)\} + d$

$= -[I/(4\pi^2a^2)]\,\mu_2\{2\pi a^2 \ln a + \pi(r^2-a^2)\} + [I/2\pi](\mu_2 - \mu_1)\ln a$

$= -[I/2\pi]\mu_1 \ln a - [I/(4\pi a^2)]\,\mu_2 (r^2-a^2)$

and once again our Problem I solution is :

Region 1: $A_z^{I} = -(1/2\pi)\mu_1 I \ln r$

Region 2: $A_z^{I} = -[I/2\pi]\mu_1 \ln a - [I/(4\pi a^2)]\,\mu_2 (r^2-a^2)$

<u>Compare this to the Problem II solution</u>

In Region R = 1 + 2 we have

$(\nabla^2 + \beta_1^2)\,\mathbf{A}^{II} = -\mu_0 \mathbf{J}_m - \mu_2 \mathbf{J}_{c2}$ x in region R // image current present!

$(A_z^{II})_2 = (A_z^{II})_1$ BC set II
$(1/\mu_2)(\partial_n A_z^{II})_2 - (1/\mu_1)(\partial_n A_z^{II})_1 = K_z^{image} = [(\mu_1-\mu_2)/\mu_0]\, I/(2\pi a)$

Now the $-\mu_2 \mathbf{J}_{c2}$ source gives the exact same particular solution as obtained in Problem I, so

$$A_z^{II,parta} = -[I/(4\pi^2a^2)]\{\mu_2 \int dV' \ln(R^2)\} = -[I/(4\pi^2a^2)]\,\mu_2\{2\pi a^2 \ln a + \pi(r^2-a^2)\}$$

But then we have part b of this particular solution which is the $-\mu_0 \mathbf{J}_m$ contribution

$$A_z^{II,partb} = -[I/(4\pi^2a^2)]\{(\mu_1-\mu_2)(a/2)\,4\pi a \begin{cases} \ln(r) & r > a \\ \ln(a) & r < a \end{cases}$$

$$= -[I/(2\pi)](\mu_1-\mu_2) \begin{cases} \ln(r) & r > a \\ \ln(a) & r < a \end{cases} = -[I/(2\pi)](\mu_1-\mu_2)\ln(r_>).$$

Each of these solutions is defined over all R. When we add these solutions we get

$$\mathbf{A}^{II} = A_z^{II,parta} + A_z^{II,partb} = -[I/(4\pi^2a^2)]\,\mu_2\{2\pi a^2 \ln a + \pi(r^2-a^2)\} - [I/(2\pi)](\mu_1-\mu_2)\ln(r_>)$$

$$= -[I/(2\pi)]\,\mu_2\{\ln a + (1/2)(r^2/a^2-1)\} - [I/(2\pi)](\mu_1-\mu_2)\ln(r_>)$$

$$= -[I/(2\pi)]\{\mu_2 \ln a - (\mu_1-\mu_2)\ln(r_>) + (\mu_2/2)(r^2/a^2-1)\}$$

which we can write as

$$A^{II} = -[I/(2\pi)]\{\mu_2 \ln a - (\mu_1-\mu_2)\ln(r) + (\mu_2/2)(r^2/a^2-1)\} \qquad r>a$$

$$A^{II} = -[I/(2\pi)]\{\mu_2 \ln a - (\mu_1-\mu_2)\ln(a) + (\mu_2/2)(r^2/a^2-1)\} \qquad r>a$$

Pause for Back and Fill

I have "carefully" written up the two solution methods for the round wire problem in "Case Study, A for a round wire.doc". I show there all the calculation except the two integrals which are done in Appendix B. I show that the two methods give exactly the same solution. But I don't know WHY they give the same solution.

Plan A: Superposition?

Suppose we had nothing but a *real* surface charge K_z and magically we could set $J_c = 0$ while maintaining that K_z . What would be the solution in that case? It would be the Z integral in my Case Study doc.

$$A^{II,regionR} = -(I/2\pi)(\mu_1-\mu_2) \begin{cases} \ln(r) & r > a \\ \ln(a) & r < a \end{cases}$$

The two media still have μ_1 and μ_2 , and K_z still vanishes when $\mu_1 = \mu_2$. [notice that the solution is a homo solution on both sides!]

Plan B: The Stakgold v function idea.

[Well, this is more a method to compute *true* Green's Functions, it does not really help here, but it was OK to review it. I am only interested in using the free-space Green's Function.]

In this idea, you have some boundaries and a Green's function g which vanishes on same. Then you define v = g - E = full Green's minus free-space Green's. Then you ask: what do I know about the function v? One thing you know about v is that on the surface, its value is given by v = -E ! And you know E, so you then know v = -E on the surface. But then you are in a Dirichlet situation with a known v on the surface.

Let's put more flesh on these bones. We have these Green problems with point charge at x :

$-\nabla^2 g = \delta(x-\xi)$ with g = 0 on σ, g = full Green's inside region R

$-\nabla^2 E = \delta(x-\xi)$ with E = 0 at ∞ and symmetric, E = free-space Green's

Here is the regular Dirichlet solution:

$-\nabla^2 u = q \quad \Rightarrow \quad u = \int_R dV\, g\, q + \int_\sigma dS_\xi\, f\, \partial_n g \qquad f(\xi) = u(\xi) = \text{prescribed}$

Now what do we know about v ?

$v = g - E$

$-\nabla^2 v = 0$ inside region R

$v_x(\xi) = -E(\xi|x)$ and this is known if we know E and we know σ

Now, how do you solve the following problem:

$-\nabla^2 v = 0$ in region R
$v_x(\xi) = -E(\xi|x)$ on boundary σ or region R

It seems to me this is a simple Laplace Dirichlet problem.
To solve such a problem, we need the full Green g such that

$-\nabla^2 g = \delta(x-\xi)$ with g = 0 on σ, g = full Green's inside region R

Then we can solve $-\nabla^2 v = 0$ by saying

$v = \int dV\, g\ (q=0) + \int dS_\xi\, v_x(\xi)\, \partial_n g$

or

$$v_x(x) = \int_\sigma dS_\xi \; v_x(\xi)\, \partial_n g$$

or

$$v_x(x) = \int_\sigma dS_\xi \; [-E(\xi|x)]\, \partial_n g$$

Now in the Green problem

$-\nabla^2 g = \delta(x-\xi)$ with g = 0 on σ, g = full Green's inside region R

we know that $\partial_n g(x|\xi)$ is the charge induced on surface σ.

But I don't know where to go with this idea. I think Stak wanted you to take this system

$-\nabla^2 v = 0$ in region R
$v_x(\xi) = -E(\xi|x)$ on boundary σ or region R

and solve it somehow without using the Green's Function method, to get function v. Then you would have calculated g as g = E + v.

Now with this warmup, go find this in the Stak notes empire. It relates to Section 6.7 (1). I read this a bit, it relates to integral equations for $\partial_n g$ and Stak refers to $-\partial_n g$ as function I and we then get an integral equation of the form (6.97) when you take s→ x. If you could really solve this integral equation for I, then we have a second equation (6.96) which then tells you g. So in theory, this is jus one of Stak's methods of computing g functions. I don't see it having any bearing on what I am trying to do today. But I am grasping at any straw I can remotely think of.

Plan C: Boundary Conditions on A

This "plan" is in the form of a question. I have shown that in Method I and Method II, I get *exactly the same* A fields on the two sides of the boundary. Here are those A fields:

$A^{I,region1}(r) = -\mu_1 (I/2\pi)\ln r$
$A^{I,region2}(r) = -(I/2\pi)\,[\,\mu_1 \ln a + (1/2)\mu_2(r^2/a^2-1)$ Method I

$A^{II,region1}(r) = -\mu_1 (I/2\pi)\ln r$
$A^{II,region2}(r) = -(I/2\pi)[\,\mu_1 \ln a + \mu_2 (1/2)(r^2/a^2-1)\,]$ Method II

Well, if that is the case, how can it be that there is a "jump" of K_z in Method II, but the jump is 0 in Method I??

Answer: Well, as I look back at the Method II solution, I see that although I wrote down a "jump by K_z " statement as a BC, I never used this BC and in fact the BC is wrong. There is in fact no "jump" and that is in agreement with the Method I solution. This strengthens the argument that my surface charge K_z is in fact not a "free" one, because if it were free, we would have to have a jump by K_z. [OK, here I just corrected an error and after correction, things make sense.]

Plan D: Ansatz to simulate correct BC's and Stak surface layer limits

[There is something to this I think, and I may be back on this tack later if I try to directly show that the adder J_m makes the full particular integral satisfy BC's. I ignore this section for the time being.]

Can I say that adding J_m is just an ansatz, which then gives something that "simulates" the right boundary conditions?

$$A_z(r) = (1/2\pi) \int dV' \, [\mu_2 \mathbf{J_c(x')} + \mu_0 \mathbf{J_m(x')}] \ln(1/R) \qquad \text{ansatz adder}$$

$$A_z(r) = (1/2\pi) \int dV' \, \mu_2 J_{cz}\mathbf{(x')} \ln(1/R) + (1/2\pi) \int dA' \, \mu_0 \, K_z(\mathbf{x}')] \ln(1/R)$$

$$A_z(r) = -(1/4\pi) \int dV' \, \mu_2 J_{cz}\mathbf{(x')} \ln(R^2) - (1/4\pi) \int dA' \, \mu_0 \, K_z(\mathbf{x}')] \ln(R^2)$$

In 2D I guess we have

$$R^2 = r^2 + r'^2 - 2rr'\cos(\theta-\theta')$$
$$\partial_r R^2 = 2r - 2r'\cos(\theta-\theta')$$
$$\partial_r \ln R^2 = (1/R^2)\, \partial_r R^2 = 2[r-r'\cos(\theta-\theta')]/[\, r^2 + r'^2 - 2rr'\cos(\theta-\theta') \,] = \text{smooth function}$$

Now suddenly and amazingly, I am reminded of all Stak's painstaking work with those "layers". From my meta meta notes on Ch 6 we have this:

$$u(\mathbf{s}) = \int_\sigma dS_\xi \, a(\boldsymbol{\xi}) \, E(\mathbf{s}|\xi) \qquad (**) \qquad u = Ea \quad \text{Fred 1}$$

$$\partial_\nu u(\mathbf{x}) = \int_\sigma dS_\xi \, a(\xi) \, \partial_\nu E(x|\xi) \qquad \text{value for } \mathbf{x} \text{ not on } \sigma \qquad (***)$$

$$\lim_{\mathbf{x}\to\mathbf{s}+} [\partial_\nu u(\mathbf{x})] = \int_\sigma dS_\xi \, a(\xi) \, \partial_\nu E(x|\xi) - a(\mathbf{s})/2 \qquad \text{"the extra term pops out"}$$

I think moreover that if you approach from the other side you get

$$\lim_{\mathbf{x}\to\mathbf{s}-} [\partial_\nu u(\mathbf{x})] = \int_\sigma dS_\xi \, a(\xi) \, \partial_\nu E(x|\xi) + a(\mathbf{s})/2$$

and that then

$$\text{jump } [\partial_\nu u(\mathbf{x})] = a(s) \qquad \text{ignoring sign}$$

Now look at my $A_z(r)$ equation and write it this way

$$A_z(r) = (1/2\pi) \int dV' \, \mu_2 J_{cz}(\mathbf{x'}) \ln(1/R) + (1/2\pi) \int dA' \, \mu_0 \, K_z(\mathbf{x'}) \ln(1/R)$$

$$A_z(r) = \mu_2 \int dV' \, J_{cz}(\mathbf{x'}) \, E(x|x') + \mu_0 \int_\sigma d\xi \, K_z(\xi) \, E(x|\xi)$$

I suspect that the volume integral here does not create that extra term, so I think we can say

$$\partial_r A_z(r) = \mu_2 \int dV' \, J_{cz}(\mathbf{x'}) \, \partial_r E(x|x') + \mu_0 \int_\sigma d\xi \, K_z(\xi) \, \partial_r E(x|\xi)$$

$$\lim_{r \to a+} [\partial_r A_z(r)] = \mu_2 [\int dV' \, J_{cz}(\mathbf{x'}) \, \partial_r E(x|x')]^{r=a} + \mu_0 [\int_\sigma d\xi \, K_z(\xi) \, \partial_r E(x|\xi)]^{r=a} - \mu_0 K_z(\mathbf{s})/2$$

$$\lim_{r \to a-} [\partial_r A_z(r)] = \mu_2 [\int dV' \, J_{cz}(\mathbf{x'}) \, \partial_r E(x|x')]^{r=a} + \mu_0 [\int_\sigma d\xi \, K_z(\xi) \, \partial_r E(x|\xi)]^{r=a} + \mu_0 K_z(\mathbf{s})/2$$

and then

$$\lim_{r \to a+} [\partial_r A_z(r)] - \lim_{r \to a-} [\partial_r A_z(r)] = \mu_0 K_z(s) = (\mu_1 - \mu_2) \, I/(2\pi a)$$

This looks good but I am missing the $(1/\mu_1)$ and $(1/\mu_2)$ factors. But I think I am definitely ON to something here. The idea is that by doing the ansatz of adding that surface current, you do in fact "simulate" the boundary conditions and therefore that is WHY Method 2 works!

Maybe I have to generalize this Stak idea to a boundary with μ_1 on one side and μ_2 on the other side.

BUT, the boundary condition I want to simulate is THIS:

$$(1/\mu_1) \lim_{r \to a+} [\partial_r A_z(r)] - (1/\mu_2) \lim_{r \to a-} [\partial_r A_z(r)] = 0 \text{ !!!}$$

So not only am I messing the factors, I am getting K_z instead of getting 0 . Oops.

Plan D1: Try again:

$$A_z(r) = \mu_2 \int dV' \, J_{cz}(\mathbf{x'}) \, E(x|x') + \mu_0 \int_\sigma d\xi \, K_z(\xi) \, E(x|\xi)$$

$$A_z(r) = \mu_2 \int dV' \, J_{cz}(\mathbf{x'}) \, E(x|x') + \mu_0 \int_\sigma d\xi \, K_z(\xi) \, E(x|\xi)$$

Suppose you DO get some kickout from the first term as well. Let's just wing it and see what might happen

$$\partial_r A_z(r) = \mu_2 \int dV' J_{cz}(\mathbf{x'})\, \partial_r E(x|x') + \mu_0 \int_\sigma d\xi\, K_z(\xi)\, \partial_r E(x|\xi)$$

$$\lim_{r\to a+} [\partial_r A_z(r)] = \mu_2 \int dV' J_{cz}(\mathbf{x'}) \{\partial_r E(x|x')\}^{r=a} - (1/2)\mu_2 \int dr' J_{cz}(\mathbf{x'})$$
$$+ \mu_0 \int_\sigma d\xi\, K_z(\xi) \{\partial_r E(x|x')\}^{r=a} - (1/2)\mu_0 K_z(a)$$

$$\lim_{r\to a-} [\partial_r A_z(r)] = \mu_2 \int dV' J_{cz}(\mathbf{x'}) \{\partial_r E(x|x')\}^{r=a-} + (1/2)\mu_2 \int dr' J_{cz}(\mathbf{x'})$$
$$+ \mu_0 \int_\sigma d\xi\, K_z(\xi) \{\partial_r E(x|x')\}^{r=a} + (1/2)\mu_0 K_z(a)$$

$$\lim_{r\to a+} [\partial_r A_z(r)] = \mu_2 \int dV' J_{cz}(\mathbf{x'}) \{\partial_r E(x|x')\}^{r=a} - (1/2)\mu_2 \int dr' J_{cz}(\mathbf{x'})$$
$$+ \mu_0 \int_\sigma d\xi\, K_z(\xi) \{\partial_r E(x|x')\}^{r=a} - (1/2)\mu_0 K_z(a)$$

$$= (\text{main stuff}) - (1/2)\mu_2 \int dr' J_{cz}(\mathbf{x'}) - (1/2)\mu_0 K_z(a)$$

$$= (\text{main stuff}) - (1/2)\mu_2\, a\, (I/2\pi a^2) - (1/2)(\mu_1 - \mu_2)\, I/(2\pi a) \qquad ??$$

$$= (\text{main stuff}) - (1/2)\mu_1\, a\, (I/2\pi a^2) \qquad ??$$

And somehow maybe we can arrange

$$\lim_{r\to a-} [\partial_r A_z(r)] = \mu_2 \int dV' J_{cz}(\mathbf{x'}) \{\partial_r E(x|x')\}^{r=a} - (1/2)\mu_2 \int dr' J_{cz}(\mathbf{x'})$$
$$+ \mu_0 \int_\sigma d\xi\, K_z(\xi) \{\partial_r E(x|x')\}^{r=a} + (1/2)\mu_0 K_z(a)$$

where only one of the signs changes for some reason. Then we have..

No, this is never going to give the BC I am seeking which is this:

$$(1/\mu_2)\, (\partial_r A_z)_2 = (1/\mu_1)\, (\partial_r A_z)_1$$

Somehow there must be a way to show that this result can be obtained from

$$A_z(r) = \mu_2 \int dV' J_{cz}(\mathbf{x'})\, E(x|x') + \mu_0 \int_\sigma d\xi\, K_z(\xi)\, E(x|\xi) \qquad K_z = -[(\mu_2-\mu_1)/\mu_0]\, I/(2\pi a)$$

I obtain the result if I compute all the integrals, but I want to show it is true without doing the integrals! Simpler:

$$A_z(r) = \mu_2 (I/\pi a^2) \int dV' \; E(x|x') + \mu_0 (\mu_1-\mu_2) I/(2\pi a) \int_\sigma d\xi \; E(x|\xi)$$

At least it has both μ_1 and μ_2 present somewhere. Maybe it just a coincidence with the round wire that it works.

Status at 7:30 PM 11.19.13. I am beginning to think it is just a coincidence that adding $\mu_0 J_m$ to the particular integral for the full range R just happens to give the exact correct answer. That would be an evil connivance of nature to lead me off on several days of trying to show it was a general rule. After several days, I can think of no "justification" for adding this J_m to the particular integral. If I go ahead and add it, then it just happens to give the exact right answer for the round wire. A huge Red Herring for me.

I am unable to show that adding this as an ansatz "simulates" the correct BC's. This simulation fact only comes out at the very end when everything is evaluated. That was a nice idea, a hopeful one, but it just did not pan out, even with Stak's fancy help.

If this is correct that it is just a coincidence, then only Method I is valid. The reader is then instructed to do a Smythian fit solution to both regions and match the BC's.

Signoff 8 PM. [review complete to here]

11.20.13 Back At It Again: The new μ_0 gauge idea.

[Here I start over doing wave equations with curl **B** $= \mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$ which exposes J_m. When I did this, I was not aware that J_m exists everywhere and not just on the wire surface. I was surprised that I could in fact come up with a full set of A and φ wave equations by this method, using the King μ_0 gauge as I now call it. That is to say, the φ operator is the same as the A operator. In retrospect, this is a lousy gauge because you have to then know all about J_m a priori in order to do anything!]

Plan E1: Question: What do you have to do to cause J_m to honestly appear as a source in a wave equation for A_z ? This is my μ_0 gauge approach.

I have taken this path at least twice, but not in either of these Disaster logs. I see it is in the main edit log, so I will just copy that here. Right now we could be in either region 1 or region 2:

curl **B** $= \mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$ " **B** sees all currents" $(1.1.9g)_m$

curl curl **A** $= \mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$

grad div**A** - ∇^2**A** $= \mu_0(\partial_t \mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$

grad div**A** - ∇^2**A** $= \mu_0(\varepsilon \, \partial_t \mathbf{E} + \mathbf{J_c} + \mathbf{J_m})$

grad div**A** - ∇^2**A** $= \mu_0(\varepsilon \, \partial_t [-$ grad $\varphi - \partial_t \mathbf{A}] + \mathbf{J_c} + \mathbf{J_m})$

grad div**A** - ∇^2**A** $= \mu_0 \varepsilon \, \partial_t [-$ grad $\varphi - \partial_t \mathbf{A}] + \mu_0 \mathbf{J_c} + \mu_0 \mathbf{J_m}$

$$\text{grad div}\mathbf{A} - \nabla^2\mathbf{A} = -\mu_0\varepsilon\,\partial_t\text{grad}\,\varphi - \mu_0\varepsilon\,\partial_t^2\mathbf{A} + \mu_0\mathbf{J_c} + \mu_0\mathbf{J_m}$$

$$\text{grad div}\mathbf{A} - \nabla^2\mathbf{A} = -\mu_0\varepsilon\,\text{grad}\,\partial_t\,\varphi - \mu_0\varepsilon\,\partial_t^2\mathbf{A} + \mu_0\mathbf{J_c} + \mu_0\mathbf{J_m}$$

$$-\text{grad div}\mathbf{A} + \nabla^2\mathbf{A} = +\mu_0\varepsilon\,\text{grad}\,\partial_t\,\varphi + \mu_0\varepsilon\,\partial_t^2\mathbf{A} - \mu_0\mathbf{J_c} - \mu_0\mathbf{J_m}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon\,\partial_t^2\mathbf{A} = \text{grad div}\mathbf{A} + \mu_0\varepsilon\,\text{grad}\,\partial_t\,\varphi - \mu_0\mathbf{J_c} - \mu_0\mathbf{J_m}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon\,\partial_t^2\mathbf{A} = \text{grad}\,[\text{div}\mathbf{A} + \mu_0\varepsilon\,\partial_t\,\varphi] - \mu_0(\mathbf{J_c}+\mathbf{J_m}) \qquad (1.3.3)$$

OK, once "price" you pay with this method is that you get $\mu_0\varepsilon$ on that $\partial_t^2\mathbf{A}$ part of the operator. I accept that as just fine for the moment.

Now before stating the gauge condition, let's apply this to region 1 (dielectric which conducts). As a first possible choice, suppose we decide to absorb the conduction of region 1 into the operator somehow. Then we do these steps:

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} = \text{grad}\,[\text{div}\mathbf{A} + \mu_0\varepsilon_1\,\partial_t\,\varphi] - \mu_0(\mathbf{J_{c1}}+\mathbf{J_{m1}}) \qquad \text{region 1}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} = \text{grad}\,[\text{div}\mathbf{A} + \mu_0\varepsilon_1\,\partial_t\,\varphi] - \mu_0\sigma_1\mathbf{E} - \mu_0\mathbf{J_{m1}}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} = \text{grad}\,[\text{div}\mathbf{A} + \mu_0\varepsilon_1\,\partial_t\,\varphi] - \mu_0\sigma_1[-\,\text{grad}\,\varphi - \partial_t\mathbf{A}] - \mu_0\mathbf{J_{m1}}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} - \mu_0\sigma_1\partial_t\mathbf{A} = \text{grad}\,[\text{div}\mathbf{A} + \mu_0\varepsilon_1\,\partial_t\,\varphi] + \mu_0\sigma_1\text{grad}\,\varphi - \mu_0\mathbf{J_{m1}}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} - \mu_0\sigma_1\partial_t\mathbf{A} = \text{grad}\,[\text{div}\mathbf{A} + \mu_0\varepsilon_1\,\partial_t\,\varphi + \mu_0\sigma_1\varphi] - \mu_0\mathbf{J_{m1}}$$

OK, now this then is our choice of gauge

$$\text{div}\mathbf{A} = -\,\mu_0\varepsilon_1\,\partial_t\,\varphi - \mu_0\sigma_1\varphi$$

so it is a King gauge but with μ_0 in place of the former μ_1. A certain niceness in that since maybe it is more applicable to both the μ_1 and μ_2 regions? Well, just continue on with this gauge to get

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} - \mu_0\sigma_1\partial_t\mathbf{A} = -\,\mu_0\mathbf{J_{m1}}$$

$$(\nabla^2 - \mu_0\varepsilon_1\,\partial_t^2 - \mu_0\sigma_1\partial_t)\mathbf{A} = -\,\mu_0\mathbf{J_{m1}} \qquad \text{region 1}$$

Here I have just assumed there is some surface mag current $\mathbf{J_{m1}}$ which lies in region 1. I will return to that question later. Notice that, in order to have J_{c1} disappear, we needed $\mu_0 J_{c1}$ to appear in the sources.

So that is half the story. Now let's do region 2. Start off with 1.3.3 but in region 2:

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = \text{grad}\,[\text{div}\mathbf{A} + \mu_0\varepsilon_2\,\partial_t\,\varphi] - \mu_0\mathbf{J_{c2}} - \mu_0\mathbf{J_{m2}} \qquad (1.3.3)$$

Now the safest option it seems to me is to use the same gauge here, so that we at least have a CHANCE of having some kind of unified equation for all of R with a uniform gauge. Also, in region 2 I want to leave the current $\mathbf{J}_c$ "as is". Then we can do steps. First step is to insert the above gauge,

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = \text{grad}\,[-\,\mu_0\varepsilon_1\,\partial_t\varphi - \mu_0\sigma_1\varphi + \mu_0\varepsilon_2\,\partial_t\varphi] - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = \text{grad}\,[\,\mu_0(\varepsilon_2-\varepsilon_1)\,\partial_t\varphi - \mu_0\sigma_1\varphi] - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = [\,\mu_0(\varepsilon_2-\varepsilon_1)\,\partial_t - \mu_0\sigma_1]\,\text{grad}\varphi - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = [\,\mu_0(\varepsilon_2-\varepsilon_1)\,\partial_t - \mu_0\sigma_1]\,[-\mathbf{E} - \partial_t\mathbf{A}] - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

Now region 2 is our good conductor so try setting $\mathbf{E} = 0$ to get

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = [\,\mu_0(\varepsilon_2-\varepsilon_1)\,\partial_t - \mu_0\sigma_1]\,[-\partial_t\mathbf{A}] - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = -\,[\,\mu_0(\varepsilon_2-\varepsilon_1)\,\partial_t - \mu_0\sigma_1]\,[\,\partial_t\mathbf{A}] - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = -\,[\,\mu_0(\varepsilon_2-\varepsilon_1)\,\partial_t^2\mathbf{A} - \mu_0\sigma_1\,\partial_t\mathbf{A}] - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_2\,\partial_t^2\mathbf{A} = -\,\mu_0(\varepsilon_2-\varepsilon_1)\,\partial_t^2\mathbf{A} + \mu_0\sigma_1\,\partial_t\mathbf{A}] - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

Now notice that the $\mu_0\varepsilon_2\,\partial_t^2\mathbf{A}$ cancels on both sides which we like to see, and then we can move the $\mu_0\varepsilon_1$ term to the left side, giving

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} = +\,\mu_0\sigma_1\,\partial_t\mathbf{A} - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$\nabla^2\mathbf{A} - \mu_0\varepsilon_1\,\partial_t^2\mathbf{A} - \mu_0\sigma_1\,\partial_t\mathbf{A} = -\,\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$$

$$(\nabla^2 - \mu_0\varepsilon_1\,\partial_t^2 - \mu_0\sigma_1\,\partial_t)\mathbf{A} = -\,\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2} \qquad \text{region 2}$$

Here then are the results for this pathway:

$$(\nabla^2 - \mu_0\varepsilon_1\,\partial_t^2 - \mu_0\sigma_1\partial_t)\mathbf{A} = -\,\mu_0\mathbf{J}_{m1} \qquad \text{region 1}$$
$$(\nabla^2 - \mu_0\varepsilon_1\,\partial_t^2 - \mu_0\sigma_1\,\partial_t)\mathbf{A} = -\,\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2} \qquad \text{region 2}$$

Comments on this approach:

1) the gauge is exactly the same in both regions
2) the wave operator is exactly the same in both regions
3) the surface [part of the] mag current J_m appears as a bona fide source in each region
4) current J_{c2} has a multiplier of μ_0 as does the surface current [not very nice]
5) the gauge is DIFFERENT from that I was using before, since has μ_0

Now it certainly seems a logical step at this point to combine the two regions. Maybe this is wrong, but if we do it, then we can combine the J_m's into a single J_m. We then don't have to decide how much of J_m is in each region, only the total J_m counts. We then get

$$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1 \partial_t)\mathbf{A} = - \mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_m \qquad \text{region R}$$

where it is understood that $\mathbf{J}_m$ only exists on the boundary and $\mathbf{J}_{c2}$ exists only in region 2.

[With the full interpretation of J_m, this is a perfectly fine wave equation for region R!]

Plan E2: φ in the same gauge

Just as an aside, with this gauge, what does the φ wave equation look like? It might be completely different for all I know right now. So go back to the source in lines doc. Start in a generic unspecified region

$\mathbf{E} = -$ grad $\varphi - \partial_t\mathbf{A}$ // (1.3.1) [= Jackson (6.9)]

div $\mathbf{E} = -$ div grad $\varphi - \partial_t$ (div $\mathbf{A}$) // take div of both sides

$$\rho/\varepsilon = - \nabla^2\varphi - \partial_t (\text{div } \mathbf{A})$$

$$\nabla^2\varphi = - \partial_t (\text{div } \mathbf{A}) - \rho/\varepsilon$$

Note: ρ includes both free and polarization charge!!!

Now use the same King gauge we used for the magnetic stuff,

$$\text{div}\mathbf{A} = - \mu_0\varepsilon_1 \partial_t \varphi - \mu_0\sigma_1\varphi$$

and we then get

$$\nabla^2\varphi = - \partial_t (- \mu_0\varepsilon_1 \partial_t \varphi - \mu_0\sigma_1\varphi) - \rho/\varepsilon$$

$$\nabla^2\varphi = \partial_t (\mu_0\varepsilon_1 \partial_t \varphi + \mu_0\sigma_1\varphi) - \rho/\varepsilon$$

$$\nabla^2\varphi = (\mu_0\varepsilon_1 \partial_t^2 \varphi + \mu_0\sigma_1\partial_t \varphi) - \rho/\varepsilon$$

$$\nabla^2\varphi = \mu_0\varepsilon_1 \partial_t^2 \varphi + \mu_0\sigma_1\partial_t \varphi - \rho/\varepsilon$$

$$\nabla^2\varphi - \mu_0\varepsilon_1 \partial_t^2 \varphi - \mu_0\sigma_1\partial_t \varphi = - \rho/\varepsilon$$

$$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t) \varphi = - \rho/\varepsilon$$

Now let's talk regions. But nothing depends on region except ρ and ε! Things are a little different in the electric case, but let's just move forward:

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\,\varphi = -\rho_1/\varepsilon_1$ region 1

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\,\varphi = -\rho_2/\varepsilon_2$ region 2

The very happy news so far is that the wave operator here is the same as for the A equations. Now let's take the step if we like and unify into a single equation

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\,\varphi = -\rho_1/\varepsilon_1 - \rho_2/\varepsilon_2$ region R

Now probably we would say at this point that all the charge density lies on the outside surface of the conductor, which says $\rho_2 = 0$. That is how we always "draw it". Unlike the J_m case, we seem to get some ambiguity here depending on where you imagine ρ to lie.

Where does polarization charge fit into this picture? Let's defer this question for a moment, other than to say both ρ_1 and ρ_2 would include any polarization charges in their respective regions.

Comments on the result:

1) the gauge is exactly the same in both regions
2) the wave operator is exactly the same in both regions and it is the exact same as in the A equation
3) the gauge is DIFFERENT from that I was using before, since has μ_0

So here is a possibly valid summary of our results:

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{m1}$ region 1
$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$ region 2

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\,\varphi = -\rho_1/\varepsilon_1$ region 1
$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\,\varphi = -\rho_2/\varepsilon_2$ region 2

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m}$ region R
$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\,\varphi = -\rho_1/\varepsilon_1 - \rho_2/\varepsilon_2$ region R

This is mildly encouraging in that things are fairly uniform. But I fear there would be problems. First, let's take all these equations to ω space and write

$\beta_{01}^2(\omega) = \mu_0\varepsilon_1\omega^2 - \mu_0\sigma_1 j\omega = \omega^2\mu_0\,(\varepsilon_1 - j\sigma_1/\omega) = \omega^2\mu_0\xi_1$ $\qquad \xi_1 \equiv \varepsilon_1 - j\sigma_1/\omega$

$\beta_{01}^2(\omega) = \mu_0\varepsilon_1\omega^2 - \mu_0\sigma_1 j\omega = \omega(\mu_0\varepsilon_1\omega - j\,\mu_0\sigma_1)$

There is no question but that $\beta_{01}^2(\omega) = 0$ when $\omega = 0$. So we then have these Helmholtz equations.

$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0\mathbf{J}_{m1}$ region 1

$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$ region 2

$(\nabla^2 + \beta_{01}^2)\varphi = -\rho_1/\varepsilon_1$ region 1

$(\nabla^2 + \beta_{01}^2)\varphi = -\rho_2/\varepsilon_2$ region 2

$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m}$ region R

$(\nabla^2 + \beta_{01}^2)\varphi = -\rho_1/\varepsilon_1 - \rho_2/\varepsilon_2$ region R

Now at DC, we have $\beta_{01} = 0$ and then we have in particular this equation

$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m}$ region R

$\text{div}\mathbf{A} = -\mu_0\varepsilon_1 j\omega\varphi - \mu_0\sigma_1\varphi = -\mu_0(\varepsilon_1 - j\sigma_1/\omega)j\omega\varphi = -\mu_0\,\xi_1 j\omega\varphi$

I would like now to do the round wire computation *in this new gauge* and see what happens!

Plan E3. Do the DC round wire in this new gauge. Use Appendix B as a guide.

[This section is completely wrong because I am including only the surface part of J_m in a calculation that requires all of J_m!]

Start off with these modified equations:

$$\nabla^2_{2D}\mathbf{A}(\mathbf{x}) = -\mu_0\mathbf{J}_m(\mathbf{x}) - \mu_0\mathbf{J}_c(\mathbf{x})\,. \qquad \text{(B.6.1a)}$$

$$A_z = -(1/4\pi)\left\{\mu_0\int dS'\,[K_z^m(\mathbf{x}')]\ln(R^2) + \mu_0\int dV'[J_{cz}(\mathbf{x}')]\ln(R^2)\right\} + H. \qquad \text{(B.6.1b)}$$

We now insert the uniform prescribed current J_{cz} *and* our previously computed surface current K_z,

$$J_{cz}(\mathbf{x}') = J_{cz} = I/(\pi a^2) \qquad \text{// uniform in round wire} \qquad \text{(B.6.2)}$$

$$K_z^m(\mathbf{x}') = K_z^m = -[(\mu_2-\mu_1)/\mu_0]\,I/(2\pi a) \qquad \text{// from (B.4.4)} \qquad \text{(B.6.3)}$$

to get

$$A_z = -[I/(4\pi^2a^2)]\left\{(\mu_1-\mu_2)(a/2)\int dS'\,\ln(R^2) + \mu_0\int dV'\ln(R^2)\right\} + H. \qquad \text{(B.6.4)}$$

It seems that everything stays the same except for this $\mu_2 \rightarrow \mu_0$ in the volume term, so I should be able to jump far ahead in the calculation:

$$\int dV' \ln(R^2) = \begin{cases} 2\pi a^2 \ln(r) & r > a \\ 2\pi a^2 \ln a + \pi(r^2 - a^2) & r < a \end{cases} \tag{B.6.14}$$

$$\int dS' \ln(R^2) = 4\pi a \begin{cases} \ln(r) & r > a \\ \ln(a) & r < a \end{cases} . \tag{B.6.15}$$

The above integrals are "just geometry". We then get

$$A_z = - [I/(4\pi^2 a^2)] \{(\mu_1 - \mu_2)(a/2) \int dS' \ln(R^2) + \mu_0 \int dV' \ln(R^2) \} + H. \qquad (B.6.4)$$

$$= - [I/(4\pi^2 a^2)] \{(\mu_1 - \mu_2)(a/2) 4\pi a \begin{cases} \ln(r) & r > a \\ \ln(a) & r < a \end{cases} + \mu_0 \begin{cases} 2\pi a^2 \ln(r) & r > a \\ 2\pi a^2 \ln a + \pi(r^2 - a^2) & r < a \end{cases} \} + H \tag{B.6.16}$$

Then write these out for the two separate regions :

$$A_z(r>a) = -[I/(4\pi^2 a^2)] \{ \mu_1 - \mu_2)(a/2) \ 4\pi a \ln(r) + \mu_0 \ 2\pi a^2 \ \ln(r) \}$$

$$= -[I/(4\pi^2 a^2)] \{ \mu_1 - \mu_2) \ 2\pi a^2 \ln(r) + \mu_0 \ 2\pi a^2 \ \ln(r) \}$$

$$= -[I/(2\pi)] \ (\mu_1 - \mu_2 + \mu_0) \ln(r) \ + H(r>a) \qquad \text{region 1, } r>a \tag{B.6.17}$$

$$A_z(r<a) = -[I/(4\pi^2 a^2)] \{ (\mu_1 - \mu_2)(a/2) \ 4\pi a \ln(a) + \mu_0(\ 2\pi a^2 \ln(a) + \pi(r^2 - a^2) \ ) \ \}$$

$$= -[I/(4\pi a^2)] \{ (\mu_1 - \mu_2) \ 2a^2 \ln(a) + \mu_0 2a^2 \ln(a) + \mu_0(r^2 - a^2) \ ) \ \}$$

$$= -[I/(4\pi a^2)] \{ (\mu_1 - \mu_2 + \mu_0) \ 2a^2 \ln(a) + \mu_0(r^2 - a^2) \ ) \ \}$$

$$= -[I/(4\pi a^2)] \{ (\mu_1 - \mu_2 + \mu_0) \ 2a^2 \ln(a) + \ 2a^2 \mu_0 (r^2 - a^2)/(2a^2) \ ) \ \}$$

$$= -[I/(2\pi)] \{ (\mu_1 - \mu_2 + \mu_0) \ln(a) + \mu_0 (r^2 - a^2)/(2a^2) \ ) \ \} \quad + H(r<a) \quad \text{region 2, } r < a \tag{B.6.18}$$

There results are now different because we are in a different gauge!

The next step is to compute the magnetic field **B** = curl **A** given our function $A_z(r)$. We find from the usual cylindrical coordinates curl formula (B.5.2),

$$\mathbf{B} = \text{curl } \mathbf{A} = - \ \partial_r A_z(r) \ \hat{\boldsymbol{\theta}} \ . \tag{B.6.20}$$

At the boundary r = 0 both expressions give

$$A_z(r=a) = \ = -[I/(2\pi)] \{ (\mu_1 - \mu_2 + \mu_0) \ln(a) \tag{B.6.19}$$

First, for r > a one finds,

$$-\partial_r A_z(r) = \partial_r[[I/(2\pi)]\ (\mu_1-\mu_2+\mu_0)\ \ln(r)\ = [I/(2\pi)]\ (\mu_1-\mu_2+\mu_0)\ (1/r) \qquad r > a \qquad \text{(B.6.21)}$$

and then for r < a,

$$-\partial_r A_z(r) = I/(2\pi)\ \partial_r\ \{(\mu_1-\mu_2+\mu_0)\ \ln(a) + \mu_0(r^2-a^2)/(2a^2)\ )\ \} = (I\ \mu_0/2\pi)\ r/a^2 \qquad r < a \qquad \text{(B.6.22)}$$

Thus

$$\mathbf{B} = [I/(2\pi)]\ (\mu_1-\mu_2+\mu_0)\ (1/r)\ \hat{\boldsymbol{\theta}} \qquad \text{for } r > a \text{ which is in the dielectric medium with } \mu_1$$
$$\mathbf{B} = (I\ \mu_0/2\pi)\ r/a^2\hat{\boldsymbol{\theta}} \qquad \text{for } r < a \text{ which is in the conductor medium with } \mu_2\ . \qquad \text{(B.6.23)}$$

Using $\mathbf{B} = \mu\mathbf{H}$ in each region we then get

$$\mathbf{H} = [I/(2\pi)]\ (\mu_1-\mu_2+\mu_0)/\mu_1\ (1/r)\ \hat{\boldsymbol{\theta}} \qquad \text{for } r > a \qquad \Rightarrow \qquad H_\theta = (I/2\pi r)$$
$$\mathbf{H} = (I\ \mu_0/2\pi)/\mu_2\ r/a^2\ \hat{\boldsymbol{\theta}} \qquad \text{for } r < a \qquad \Rightarrow \qquad H_\theta = (Ir/2\pi a^2) \qquad \text{(B.6.24)}$$

These are NOT my standard results. Wow. This new "honest gauge" has given a different result! I KNOW that the standard results are right for arbitrary μ_1 and μ_2 so there is a LOOSE BOLT somewhere in the works that is messing up everything I do.

[As now expected, if you omit the remaining bulk pieces of J_m, you get the wrong answer!]

Let's now look back at the "simple derivation" of these results in Section B.4 [I keep wanting to verify even the simplest things. That is what you do when you don't know where to go!]

For a round wire of radius a with uniform J_z (as would be the DC case $\omega = 0$) , geometric symmetry makes the calculation of **H** very easy. One need only apply Ampere's Law separately for a point r outside the wire, and for another point r inside the wire. For the outside case one finds

$$2\pi r\ H_\theta(r) = I \qquad \Rightarrow \qquad H_\theta(r) = I/(2\pi r) \qquad r \geq a\ . \qquad \text{(B.4.1)}$$

And then for the inside case the "current enclosed" is determined by a simple area fraction.

$$2\pi r\ H_\theta(r) = I\ (\pi r^2/\pi a^2) \qquad \Rightarrow \qquad H_\theta(r) = I\ r/(2\pi a^2) \qquad r \leq a\ . \qquad \text{(B.4.2)}$$

Ampere's Law is curl $\mathbf{H} = \mathbf{J}_c$ and is not supposed to "see" the mag charge on the surface. Then we have this integral form,

$$\text{curl}\ \mathbf{H} = j\omega\varepsilon\mathbf{E} + \mathbf{J} \qquad \Leftrightarrow \qquad \oint_C \mathbf{H} \bullet d\mathbf{s}\ = \int_S\ [j\omega\varepsilon\mathbf{E} + \mathbf{J}] \bullet d\mathbf{S} \qquad \text{(1.1.36)}$$

Hmmm. But we are at DC so $\omega = 0$ and then

$$\text{curl } \mathbf{H} = \mathbf{J} \quad \Leftrightarrow \quad \oint_C \mathbf{H} \bullet d\mathbf{s} = \int_S \mathbf{J} \bullet d\mathbf{S} \qquad (1.1.36)$$

and then we do what is done above and reach the usual conclusions.

New Paradox: I have come up with a new gauge for **A** in which $\mathbf{J}_m$ appears in the **A** wave equation in an "honest way". But in this new gauge, when I compute **H** or **B** for a round wire, I get the wrong answer! There is indeed a loose bolt somewhere. My model flunks the simplest n = 2 test you can think of.

[The loose bolt in this case is the omission of the extra J_m stuff in the bulk regions. I never fully did the problem with that extra J_m stuff with success, but I am pretty sure it would succeed. But knowledge a priori of J_m requires knowing H or B, and that requires the problem to already be solved! Catch 22.]

The problem originates right here:

$$\nabla^2_{2D}\mathbf{A}(\mathbf{x}) = -\mu_0\mathbf{J}_m(\mathbf{x}) - \mu_0\mathbf{J}_c(\mathbf{x}) \,. \qquad (B.6.1a)$$

If we replace $\mu_0 \rightarrow \mu_2$ for the conductor of region 2, then everything comes out right. But the above derivation shows that it really is μ_0 that appears in this equation. [And I know at least in BC terms why you get the right answer with $\mu_0 \rightarrow \mu_2$ for the round wire, but I have not yet been able to prove this in any general sense.]

Are we not allowed to merge the regions into a single region R? Is that the fallacy? [no it is not]

Plan E4. Adding homo solutions to new gauge solution

[The plan here was this: still not knowing about the bulk J_m contributions, I use the μ_0 gauge to try and compute A. I accept that I might have to add homo solutions to make the BC's work. I add in such homo solutions, and I tune them to make BC's match. But after doing this, I get the wrong answers for the round wire H field. I now think this can never work because you have to include the bulk J_m terms in the calculation, and if you omit them, homo solutions probably won't help.]

Start off with these modified equations:

$$\nabla^2_{2D}\mathbf{A}(\mathbf{x}) = -\mu_0\mathbf{J}_m(\mathbf{x}) - \mu_0\mathbf{J}_c(\mathbf{x}) \,. \qquad (B.6.1a)$$

$$A_z = -(1/4\pi) \{ \mu_0 \int dS' \, [K_z^m(\mathbf{x}')] \ln(R^2) + \mu_0 \int dV' [J_{cz}(\mathbf{x}')] \ln(R^2) \} \,. \qquad (B.6.1b)$$

Question: I have just blindly assumed that the particular solution is THE solution. But how do I know that some homo solution might not be needed to be added in to get the correct full solution? Is this my loose bolt in this case? OK, let's go back to the derivation and just add H everywhere. If I do this, I end up with this final result for A_z

$$A_z(r>a) = -[I/(2\pi)] \, (\mu_1-\mu_2+\mu_0) \ln(r) + H_1(r>a) \qquad \text{region 1, } r>a \qquad (B.6.17)$$

$$A_z(r<a) = -[I/(2\pi)]\ \{\ (\mu_1-\mu_2+\mu_0)\ \ln(a) + \mu_0(r^2-a^2)/(2a^2)\ )\ \}\ \ + H_2(r<a) \quad \text{region 2, } r<a \qquad \text{(B.6.18)}$$

Now maybe I have to select the homo function H such that some known BC's are met. Here are the BC's I know have to be met

$A_z(a)^1 = A_z(a)^2$ the vector potential is continuous at the surface

$(1/\mu_1)\ \partial_r A_z(a)^1 = (1/\mu_2)\ \partial_r A_z(a)^2$ there is no K_z^{free} at the boundary

This second condition says that the slope of A_z still jumps, and the reason is that there is as surface mag charge at the boundary which makes it jump. This fact is embedded in the fact that $\mu_1 \neq \mu_2$.

Now recall from above:

First, for r > a one finds,

$$-\partial_r A_z(r) = \partial_r[[I/(2\pi)]\ \ (\mu_1-\mu_2+\mu_0)\ \ln(r) - \partial_r\ H_1(r)$$

$$= [I/(2\pi)]\ \ (\mu_1-\mu_2+\mu_0)\ (1/r) - \partial_r\ H_1(r) \qquad r > a \qquad \text{(B.6.21)}$$

and then for r < a,

$$-\partial_r A_z(r) = I/(2\pi)\ \partial_r\ \{(\mu_1-\mu_2+\mu_0)\ \ln(a) + \mu_0(r^2-a^2)/(2a^2)\ )\ \}- \partial_r\ H_2(r)$$

$$= (I\ \mu_0/2\pi)\ r/a^2 - \partial_r\ H_2(r) \qquad r < a \qquad \text{(B.6.22)}$$

So now write down the two BC's:

$A_z(a)^1 = A_z(a)^2$

$$-[I/(2\pi)]\ \ (\mu_1-\mu_2+\mu_0)\ \ln(r) + H_1(r>a) = -[I/(2\pi)]\ \{\ (\mu_1-\mu_2+\mu_0)\ \ln(a) + \mu_0(r^2-a^2)/(2a^2)\ )\ \}\ + H_2(r<a)$$

or

$$-[I/(2\pi)]\ \ (\mu_1-\mu_2+\mu_0)\ \ln(a) + H_1(a) = -[I/(2\pi)]\ \{\ (\mu_1-\mu_2+\mu_0)\ \ln(a)\ \}\ + H_2(a)$$

To make this work, we just have

$H_1(a) = H_2(a)$

Next, consider

$-(1/\mu_1)\ \partial_r A_z(a)^1 = -(1/\mu_2)\ \partial_r A_z(a)^2$

which says

$$(1/\mu_1)\{\ [I/(2\pi)]\ \ (\mu_1-\mu_2+\mu_0)\ (1/r) - \partial_r\ H_1(r)\ \} = (1/\mu_2)\ \{(I\ \mu_0/2\pi)\ r/a^2 - \partial_r\ H_2(r)\}$$

or

$$(1/\mu_1)\{\ [I/(2\pi)]\ \ (\mu_1-\mu_2+\mu_0)\ (1/a) - \partial_r\ H_1(a)\ \} = (1/\mu_2)\ \{(I\ \mu_0/2\pi)1/a - \partial_r\ H_2(a)\}$$

Let's rescale the homo function so that

$$H = [I/(2\pi)]\,(1/a)\,h$$

Then our condition is

$$(1/\mu_1)\{\,[I/(2\pi)]\;(\mu_1-\mu_2+\mu_0)\,(1/a) - \;[I/(2\pi)]\,(1/a)\partial_r\, h_1(a)\,\}$$

$$= (1/\mu_2)\;\{(I\,\mu_0/2\pi)1/a - \;[I/(2\pi)]\,(1/a)\partial_r\, h_2(a)\}$$

Then we can cancel lots of stuff to get

$$(1/\mu_1)\{\;(\mu_1-\mu_2+\mu_0)\;-\;\partial_r\, h_1(a)\,\}$$

$$= (1/\mu_2)\;\{(\,\mu_0) - \;\partial_r\, h_2(a)\}$$

or

$$(1/\mu_1)\{\;(\mu_1-\mu_2+\mu_0)\;-\;\partial_r\, h_1(a)\,\} = \;(1/\mu_2)\;\{(\,\mu_0) - \;\partial_r\, h_2(a)\}$$

or

$$(1/\mu_2)\,\partial_r\, h_2(a)\; - (1/\mu_1)\,\partial_r\, h_1(a) \;=\; (1/\mu_2)\,(\,\mu_0) - (1/\mu_1)(\mu_1-\mu_2+\mu_0)$$

$$= [\mu_1\mu_0\; - (\mu_1-\mu_2+\mu_0)\mu_2\;]/(\mu_1\mu_2)$$

$$= [\mu_1\mu_0\; - (\mu_1\,\mu_2-\mu_2^2+\mu_0\,\mu_2)\;]/(\mu_1\mu_2)$$

$$= [\;\mu_1\mu_0\; - \mu_1\,\mu_2+\mu_2^2-\mu_0\,\mu_2)\;]/(\mu_1\mu_2)$$

$$= [\;(\mu_2-\,\mu_1)(\mu_2-\,\mu_0)\;]/(\mu_1\mu_2)$$

Now multiply both sides by $\mu_1\mu_2$ to get

$$\mu_1\partial_r\, h_2(a)\; - \mu_2\,\partial_r\, h_1(a) \;=\; (\mu_2-\,\mu_1)(\mu_2-\,\mu_0)$$

So here then are our two conditions on the h functions

$$h_1(a) = h_2(a)$$

$$\mu_1\partial_r h_2(a)\; - \mu_2\,\partial_r h_1(a) \;=\; (\mu_2-\,\mu_1)(\mu_2-\,\mu_0)$$

So we have to find functions that make this work! Wow. the atoms are the 2D Laplace atoms which include of course ln(r) and constant and linear r and 1/r. We need to pick Smythian forms which are reasonable for the range of r. Thus we might pick

$h_1(r) = C\ln r$ // since this does the right thing as $r \to \infty$

$h_2(r) = B$ // which is OK at $r = 0$

Then

$\partial_r h_1 = C/r$

$\partial_r h_2 = 0$

and then the conditions are

$Clna = B$

$-\mu_2(C/a) = (\mu_2-\mu_1)(\mu_2-\mu_0)$

Then solve the second to get

$C = (\mu_1-\mu_2)(\mu_2-\mu_0)(a/\mu_2)$

$B = C \ln a = (\mu_1-\mu_2)(\mu_2-\mu_0)(a/\mu_2) \ln a$

OK, then we have

$h_1(r) = Clnr$
$h_2(r) = Clna$

and then

$H_1 = [I/(2\pi)]\ (1/a)\ Clnr$
$H_2 = [I/(2\pi)]\ (1/a)\ Clna$

Then go back to

$A_z(r>a) = -[I/(2\pi)]\ (\mu_1-\mu_2+\mu_0)\ln(r) + H_1(r>a)$

$A_z(r<a) = -[I/(2\pi)]\ \{\ (\mu_1-\mu_2+\mu_0)\ln(a) + \mu_0(r^2-a^2)/(2a^2)\ )\ \} + H_2(r<a)$

and add in the homo components to get

$A_z(r>a) = -[I/(2\pi)]\ (\mu_1-\mu_2+\mu_0)\ln(r) + [I/(2\pi)]\ (1/a)\ C\ lnr$

$A_z(r<a) = -[I/(2\pi)]\ \{\ (\mu_1-\mu_2+\mu_0)\ln(a) + \mu_0(r^2-a^2)/(2a^2)\ )\ \} + [I/(2\pi)]\ (1/a)\ Clna$

where

$C = (\mu_1-\mu_2)(\mu_2-\mu_0)(a/\mu_2)$

STOP. As $\mu_2 \rightarrow 0$, $C \rightarrow \infty$ and so $A_z(r>a) \rightarrow \infty$, but in the correct answer $A_z(r>a) = -[I/(2\pi)]\ \{\ \mu_1 \ln(r)\ \}$ is independent of μ_2, so something is wrong above this point.

Try to get simpler forms:

$A_z(r>a) = -[I/(2\pi)]\ (\mu_1-\mu_2+\mu_0)\ln(r) + [I/(2\pi)]\ (1/a)\ (\mu_1-\mu_2)(\mu_2-\mu_0)(a/\mu_2)\ lnr$

$= -[I/(2\pi)] \ln(r) \ \{ \ (\mu_1-\mu_2+\mu_0) - \ (\mu_1- \mu_2)(\mu_2- \mu_0)(1/\mu_2) \ \}$

$= -[I/(2\pi)] \ln(r) \ \{ \ \mu_2(\mu_1-\mu_2+\mu_0) + \ (\mu_2- \mu_1)(\mu_2- \mu_0) \ \}/\mu_2$

$= -[I/(2\pi)] \ln(r) \ \{ \ \mu_2\mu_1- \mu_2\mu_2+ \mu_2\mu_0 + \mu_1\mu_0 \ - \mu_1 \mu_2+\mu_2^2-\mu_0 \mu_2\}/\mu_2$

$= -[I/(2\pi)] \mu_0 (\mu_1/\mu_2) \ln(r)$ STOP here, this is the wrong result!

Then

$A_z(r<a) = -[I/(2\pi)] \{ (\mu_1-\mu_2+\mu_0) \ln(a) + \mu_0(r^2-a^2)/(2a^2)) \ \} \ + [I/(2\pi)] (1/a) \text{Cln}a$

$= -[I/(2\pi)] \{ (\mu_1-\mu_2+\mu_0) \ln(a) + \mu_0(r^2-a^2)/(2a^2)) \ \}$

$+ [I/(2\pi)] (1/a) (\mu_1- \mu_2)(\mu_2- \mu_0)(a/\mu_2) \text{ln}a$

$= -[I/(2\pi)]\{ (\mu_1-\mu_2+\mu_0) \ln(a) + \mu_0(r^2-a^2)/(2a^2) - \ (\mu_1- \mu_2)(\mu_2- \mu_0)(1/\mu_2) \text{ln}a \ \}$

$= -[I/(2\pi)]\{ + \mu_0(r^2-a^2)/(2a^2) + \ (\mu_1-\mu_2+\mu_0) \ln(a) \ - \ (\mu_1- \mu_2)(\mu_2- \mu_0)(1/\mu_2) \text{ln}a \ \}$

$= -[I/(2\pi)]\{ + \mu_0(r^2-a^2)/(2a^2) + \ [\ (\mu_1-\mu_2+\mu_0) \ - \ (\mu_1- \mu_2)(\mu_2- \mu_0)(1/\mu_2)] \ \text{ln}a \ \}$

$= -[I/(2\pi)]\{ + \mu_0(r^2-a^2)/(2a^2) + \ [\ (\mu_1-\mu_2+\mu_0)\mu_2 \ + \ (\mu_2- \mu_1)(\mu_2- \mu_0)] \ \text{ln}a/\mu_2 \ \}$

$= -[I/(2\pi)]\{ + \mu_0(r^2-a^2)/(2a^2) + \ \mu_0(\mu_1/\mu_2) \text{ln}a \ \}$

So when the homo pieces are added, our new solution A_z is this

$A_z(r>a) = \ -[I/(2\pi)] \mu_0 (\mu_1/\mu_2) \ln r$ // these meet BC at r = a

$A_z(r<a) = -[I/(2\pi)]\{\mu_0(r^2-a^2)/(2a^2) + \ \mu_0(\mu_1/\mu_2) \text{ln}a \ \}$

From these we find

$\partial_r A_z(r>a)^1 = -[I/(2\pi)] \mu_0 (\mu_1/\mu_2)(1/r)$

$\partial_r A_z(r<a)^2 = \ -[I/(2\pi)] \mu_0 r /(a^2)$

Then

$\partial_r A_z(a) = -[I/(2\pi)] \mu_0 (\mu_1/\mu_2)(1/a)$ region 1

$\partial_r A_z(a) = \ -[I/(2\pi)] \mu_0 1/a$

And then

$(1/\mu_1) \partial_r A_z(a)^1 = -[I/(2\pi)] \mu_0 (1/\mu_2)(1/a)$

$(1/\mu_2)\,\partial_r A_z(a)^2 = -[I/(2\pi)]\,\mu_0\,1/a(1/\mu_2)$ // 2nd BC is also met at r = a

which then verifies our matching boundary conditions.

Now what about the fields?

$$\mathbf{B} = \text{curl } \mathbf{A} = -\,\partial_r A_z(r)\,\hat{\boldsymbol{\theta}}\ . \qquad \text{(B.6.20)}$$

$B_1 = -\,\partial_r A_z(r)^1 = [I/(2\pi)]\,\mu_0\,(\mu_1/\mu_2)(1/r)$

$B_2 = -\,\partial_r A_z(r)^2 = [I/(2\pi)]\,\mu_0\, r\,/(a^2)$

and then

$H_1 = B_1/\mu_1 = [I/(2\pi)]\,\mu_0\,(1/\mu_2)(1/r)$ compare $H_\theta(r) = I/(2\pi r)$

$H_2 = B_2/\mu_2 = [I/(2\pi)]\,\mu_0(1/\mu_2)\, r\,/(a^2)$ compare $H_\theta(r) = I\, r/(2\pi a^2)$

The result is CLOSE, but we have an extra (μ_0/μ_2) factor in both equations!

So if I did not make an algebraic error, this says that adding the homo solutions did NOT fix up my paradox.

[Again, the above computation ignored bulk J_m contributions so is doomed to fail.]

Plan E4. Show directly that the correct A_z satisfies a certain wave equation

Collect More Evidence. I did the following exercise. Start with this equation showing a μ_2 :

$$\nabla^2_{2D} A_z(\mathbf{x}) = -\mu_0 J_{mz}(\mathbf{x}) - \mu_2 J_{cz}(\mathbf{x}) = -[\,(\mu_1-\mu_2)(I/2\pi a)]\,\delta(a-r) - \mu_2(I/\pi a^2)\,\theta(r < a)$$

Then if you take the Appendix B solutions

$A_z(r>a) == -[I/(2\pi)]\,\{\,\mu_1 \ln(r)\,\}$

$A_z(r<a) = = -[I/(2\pi)]\,\{\,\mu_1 \ln(a) + \mu_2(r^2-a^2)/(2a^2)\,)\ \}$

and if you write

$\nabla^2_{2D} A_z = (1/r)\partial_r(rA_z)$

then you find that the A_z solution does in fact satisfy the equation for r > a and for r < a. [yes, this is true.] I did this on the whiteboard and won't repeat it here, it is pretty simple. Now write the equation as

$$\nabla \bullet \nabla A_z(\mathbf{x}) = -[\,(\mu_1-\mu_2)(I/2\pi a)]\,\delta(a-r) - \mu_2(I/\pi a^2)\,\theta(r < a) \qquad (*)$$

[This is a useful general idea, to use the divergence theorem to deal with a delta function.]

Then integrate both sides over all 2D space

$$\int dV\ \nabla \bullet \nabla A_z(\mathbf{x}) = \int dV\ \{ -[(\mu_1-\mu_2)(I/2\pi a)]\ \delta(a-r) - \mu_2(I/\pi a^2)\ \theta(r < a) \}$$

$$= \int r dr \int d\theta\ \{ -[(\mu_1-\mu_2)(I/2\pi a)]\ \delta(a-r) - \mu_2(I/\pi a^2)\ \theta(r < a) \}$$

$$= 2\pi \int_0^\infty r dr\ \{ -[(\mu_1-\mu_2)(I/2\pi a)]\ \delta(a-r) - \mu_2(I/\pi a^2)\ \theta(r < a) \}$$

$$= 2\pi \int_0^\infty r dr\ \{ [(\mu_2-\mu_1)(I/2\pi a)]\ \delta(a-r) - \mu_2(I/\pi a^2)\ \theta(r < a) \}$$

$$= 2\pi\ [(\mu_2-\mu_1)(I/2\pi a)] \int_0^\infty r dr\ \delta(a-r) - 2\pi\ \mu_2(I/\pi a^2) \int_0^\infty r dr\ \theta(r < a)$$

$$= 2\pi\ [(\mu_2-\mu_1)(I/2\pi a)]\ a - 2\pi\ \mu_2(I/\pi a^2) \int_0^a r dr$$

$$= 2\pi\ [(\mu_2-\mu_1)(I/2\pi a)]\ a - 2\pi\ \mu_2(I/\pi a^2)\ (a^2/2)$$

$$= [(\mu_2-\mu_1)I] - \mu_2(I\} = -\mu_1 I$$ // the $\delta(a-r)$ made a contribution here

So we find then that, by evaluating the *right* side above, we have

$$\int dV\ div_{2D} \nabla A_z(\mathbf{x}) = -\mu_1 I$$

Now we want to evaluate the *left* side to see what it comes out being.

Use the divergence theorem over all space (maybe a large cylinder) which says

$$\int dV\ div_{2D} \nabla A_z(\mathbf{x}) = \int d\mathbf{s} \bullet \nabla A_z(\mathbf{x}) = \oint_C ds\ (\partial_r A_z)$$

where the C curve is a great circle. On this great circle, the r>a solution for A_z should apply so we get

$$\oint_C ds\ (\partial_r A_z) = \oint_C ds\ \partial_r[-[I/(2\pi)]\ \{ \mu_1 \ln(r) \}] = -[I/(2\pi)]\mu_1 \oint_C ds\ \partial_r\ \ln(r)$$

$$= -[I/(2\pi)]\mu_1 \oint_C ds\ (1/r) = -[I/(2\pi)]\mu_1 \int r d\theta\ (1/r) = -[I/(2\pi)]\mu_1\ 2\pi = -[I]\mu_1$$

Thus, our solution is verified not only for r > a and for r < a, but the delta function term δ(r-a) is also verified as we show by integrating both sides of our equation (*) over all space.

[Well, I only used the exterior solution to get the $-[I]\mu_1$, so maybe I only have verified the r>a solution by this method. I could maybe redo the above for a volume inside the wire and then verify the r<a .]

Conclusion: By direct calculation using *just the PDE* (with μ_2) and no Helmholtz integrals or any of that stuff -- by direct calculation I have shown that the Appendix B solution for A_z does in fact solve

$$\nabla^2_{2D} A_z(\mathbf{x}) = -\mu_0 J_{mz}(\mathbf{x}) - \mu_2 J_{cz}(\mathbf{x})$$

Therefore, that second term must have a μ_2 and cannot have a μ_0 despite earlier work which shows μ_0 to be the constant! I further verify this by doing the Helmholtz integral and getting the right answer with no homo terms added, and I show that with no homo terms, the BC's both are satisfied.

[everything in this Conclusion is correct! I did show directly that my known-good solution satisfies the PDE with μ_2 and that "extra" J_m surface-only thing present.]

So, the Grand Master Debugger is slowly collecting evidence. Many fingers are now pointing to the requirement for μ_2 going with the current J_{c2}. [and that is how it ends up!]

Plan E5: Gauge Violation?

[An interesting thing to examine.]

In my most recent derivation of a potential wave equation, I had this gauge condition

$$\text{div}\mathbf{A} = -\mu_0\varepsilon_1 j\omega\varphi - \mu_0\sigma_1\varphi$$

so at $\omega = 0$ that seems to say

$$\text{div}\mathbf{A} = -\mu_0\sigma_1\varphi \ .$$

But if I assume that $\mathbf{A} = A_z(x,y)\hat{\mathbf{z}}$, then div $\mathbf{A} = \partial_z A_z = 0$, so I am not respecting the gauge! So just making that assumption violates the gauge unless it happens that $\varphi = 0$, which might be the case in a purely magnetic problem. Suddenly my little world has an internal inconsistency. [I think that is correct. If you assume $\mathbf{A} = A_z(x,y)\hat{\mathbf{z}}$, you always get div $\mathbf{A}$ =0. If you are in some Lorenz or King gauge, you have to assume $\varphi = 0$ to be consistent. The other solution is either $A_z(x,y,z)$ or A_x and/or $A_y \neq 0$.]

Let's go back then to our full derived 3D μ_0-gauge equations which were

$$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0 \mathbf{J}_{m1} \qquad \text{region 1}$$
$$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0 \mathbf{J}_{c2} - \mu_0 \mathbf{J}_{m2} \qquad \text{region 2}$$

$$(\nabla^2 + \beta_{01}^2)\varphi = -\rho_1/\varepsilon_1 \qquad \text{region 1}$$
$$(\nabla^2 + \beta_{01}^2)\varphi = -\rho_2/\varepsilon_2 \qquad \text{region 2}$$

$$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0 \mathbf{J}_{c2} - \mu_0 \mathbf{J}_{m} \qquad \text{region R}$$
$$(\nabla^2 + \beta_{01}^2)\varphi = -\rho_1/\varepsilon_1 - \rho_2/\varepsilon_2 \qquad \text{region R}$$

$$\text{div}\mathbf{A} = -\mu_0\varepsilon_1 j\omega\varphi - \mu_0\sigma_1\varphi$$

[Reminder: I think this is all correct if you include all J_m in J_m!]

Now I just showed that A_x and A_y have to exist or we have to have $A_z(x,y,z)$ with change in z, otherwise the gauge condition is not satisfied. If we have a simple infinite wire with some ρ on the outside surface and with some DC current, then A_z is not going to depend on z, so A_x and A_y must then exist. I think I can continue to insist the J_{c2} is entirely in the z direction as is J_{m1}, so we then have

$$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_m \qquad \text{region R}$$

meaning

$$(\nabla^2 + \beta_{01}^2)A_z = -\mu_0 J_{c2z} - \mu_0 J_{mz} \qquad \text{region R}$$
$$(\nabla^2 + \beta_{01}^2)A_x = 0$$
$$(\nabla^2 + \beta_{01}^2)A_y = 0$$

and this then allows homo solutions for A_x and A_y which in turn will allow us to satisfy the gauge condition. But the z equation is unaltered, I still claim that A_z does not depend on z, so we then have

$$(\nabla^2_{2D} + \beta_{01}^2)A_z(r) = -\mu_0 J_{c2z}(r) - \mu_0 J_{mz}(r) \qquad \text{region R}$$

So what is new here is that there are some other components A_x and A_y floating around which are non-zero! But since **B** = curl **A** these extra components must somehow not alter **B** from what the "standard solution" gives for **B**. [and assuming φ = 0 is always another way out.]

General Dialog

I am surprised that in my μ_0 gauge, I could not get a viable solution for A_z by doing the particular integral and then adding homo terms to meet the BC's. How can this be explained?

(1) the wave equation extended to region R is wrong

$$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1 \partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2} \qquad \text{region 2}$$

(2) I did something wrong adding the homo terms. However, after I added them, my solution did at least meet the BC's as I verified from the solution.

(3) But maybe the BC's themselves are not correct

(4) my K_z expression is wrong.

(5) there is something else wrong that I am just not aware of, am too blind to see at this time.

[Number (5) wins! I was blind to the fact that J_m was not just a surface current!]

Redo the μ_0 gauge solution as a two-region solution with BC's.

Perhaps once again that unifying R is to blame [wrong] . So let's look again at our separated solutions which I will take to be these:

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{m1}$ region 1
$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1 \partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{c2} - \mu_0\mathbf{J}_{m2}$ region 2

I will decide to throw all the surface current into region 1, so we then have [but when bulk J_m is present you cannot do this!]

$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1\partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{m}$ region 1
$(\nabla^2 - \mu_0\varepsilon_1 \partial_t^2 - \mu_0\sigma_1 \partial_t)\mathbf{A} = -\mu_0\mathbf{J}_{c}$ region 2

and then the Helmholtz is this

$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0\mathbf{J}_{m}$ region 1
$(\nabla^2 + \beta_{01}^2)\mathbf{A} = -\mu_0\mathbf{J}_{c2}$ region 2
$\beta_{01}^2(\omega) = \mu_0\varepsilon_1\omega^2 - \mu_0\sigma_1 j\omega = \omega(\mu_0\varepsilon_1\omega - j\,\mu_0\sigma_1)$ // = 0 at DC

So at DC, here are our equations

$\nabla^2 A_z = -\mu_0 J_m$ region 1
$\nabla^2 A_z = -\mu_0 J_{c2}$ region 2

We assume that $A_z = A_z(r)$ so no z dependence, and then

$\nabla^2_{2D} A_z = -\mu_0 J_m$ region 1
$\nabla^2_{2D} A_z = -\mu_0 J_{c2}$ region 2

[By ignoring bulk J_m in the μ_0 gauge you doom the problem to fail and solution to be wrong!]

Region 1 Solution

The particular integral here is this (we are only doing r > a)

$$A_z = -(1/4\pi)\ \{\ \mu_0 \int dS'\ [K_z^m(\mathbf{x}')]\ \ln(R^2) = +(1/4\pi)(\mu_2-\mu_1)\ I/(2\pi a) \int dS'\ \ln(R^2)$$

$$= (1/4\pi)(\mu_2-\mu_1)\ I/(2\pi a) * 4\pi a \ln r \qquad \text{// from B..6.15}$$

$$= (\mu_2-\mu_1)\ I/(2\pi)\ \ln r \qquad \text{// which is a homo solution for r > a, as expected}$$

I will add more homo solution of the form $A_z^{h1} = C \ln r$ since this seems right for large r. Then

$A_z(r>a) = [(\mu_2-\mu_1)\, I/(2\pi) + C]\, \ln(r)$.

Region 2 Solution

The particular integral here is this (we are only doing r < a)

$$A_z(r) = -(1/4\pi)\, \mu_0 \int dV'[J_{cz}(\mathbf{x}')]\, \ln(R^2) = -(1/4\pi)\, \mu_0\, I/(\pi a^2) \int dV'\, \ln(R^2)$$

$$= -(1/4\pi)\, \mu_0\, I/(\pi a^2)\, [\, 2\pi a^2 \ln(a) + \pi(r^2-a^2)\,] \qquad \text{// from B..6.14}$$

$$= -(I/2\pi)\, \mu_0\, [\, \ln(a) + (r^2-a^2)/2a^2\,]$$

STOP: (then resume) Isn't this a homo solution? No, the only atoms for axial symmetry are A + B lnr, so r^2 is not an atom, so this is not a homo solution, no problem. In fact, $\nabla^2_{2D}(r^2) = 4$ so $\nabla^2_{2D}(A_z(r)) = -(I/2\pi)\, \mu_0\, (1/2a^2)\, 4 = -(I/\pi)\, \mu_0\, (1/a^2) = -\mu_0\, I/(\pi a^2) = -\mu_0 J_c$, all is fine.

[I was a little late realizing that only A and Blnr are legal atoms for this azisym situation.]

I will add a homo solution of the form $A_z^{h2} = B$ since this seems right for small r. Then

$$A_z(r<a) = -(I/2\pi)\, \mu_0\, [\, \ln(a) + (r^2-a^2)/2a^2\,] + B.$$

So far then we have these solutions for the two regions

$$A_z(r>a) = [(\mu_2-\mu_1)\, I/(2\pi) + C]\, \ln(r).$$
$$A_z(r<a) = -(I/2\pi)\, \mu_0\, [\, \ln(a) + (r^2-a^2)/2a^2\,] + B.$$

$$\partial_r A_z(r>a) = [(\mu_2-\mu_1)\, I/(2\pi) + C]/r \qquad \text{// made error here earlier!}$$
$$\partial_r\, A_z(r<a) = -(I/2\pi)\, \mu_0\, r/a^2$$

We now write these four objects at r = a:

$$A_z(a)^1 = [(\mu_2-\mu_1)\, I/(2\pi) + C]\, \ln(a).$$
$$A_z(a)^2 = -(I/2\pi)\, \mu_0\, [\, \ln(a)\,] + B.$$

$$\partial_r A_z(a)^1 = [(\mu_2-\mu_1)\, I/(2\pi) + C]/a \qquad \text{// ditto}$$
$$\partial_r A_z(a)^2 = -(I/2\pi)\, \mu_0\, (1/a)$$

Our boundary conditions are supposedly

$$A_z(a)^1 = A_z(a)^2$$
$$\partial_r A_z(a)^1/\mu_1 = \partial_r A_z(a)^2/\mu_2$$

which we then write out as

$[(\mu_2-\mu_1)\, I/(2\pi) + C]\, \ln(a) = -(I/2\pi)\, \mu_0 \ln(a) + B$

$(1/\mu_1)[(\mu_2-\mu_1)\, I/(2\pi) + C]/a = -(I/2\pi)\, \mu_0\, (1/a)(1/\mu_2)$ // ditto

Solve the second equation for C in steps

$(1/\mu_1)[(\mu_2-\mu_1)\, I/(2\pi) + C] = -(I/2\pi)\, \mu_0\, (1/\mu_2)$

$[(\mu_2-\mu_1)\, I/(2\pi) + C] = -(I/2\pi)\, \mu_0\, (\mu_1/\mu_2)$

$C = -(I/2\pi)\, \mu_0\, (\mu_1/\mu_2) - (1/\mu_1)[(\mu_2-\mu_1)\, I/(2\pi)$

$= -(I/2\pi)\, [\mu_0\, (\mu_1/\mu_2) + (1/\mu_1)(\mu_2-\mu_1)\,]$

$= -(I/2\pi)\, [\mu_0\, \mu_1^2 + (\mu_2^2-\mu_1\mu_2)\,]/(\mu_1\mu_2)$

$= -(I/2\pi)\, [\mu_0\, \mu_1^2 + \mu_2(\mu_2-\mu_1)\,]/(\mu_1\mu_2)$ // new and ugly, never needed

Then the first equation may be solved for B

$B = [(\mu_2-\mu_1)\, I/(2\pi) + C]\, \ln(a) + (I/2\pi)\, \mu_0\, \ln(a)$

Well, what we really want here is this fact, a form of our solution for C,

$[(\mu_2-\mu_1)\, I/(2\pi) + C] = -(I/2\pi)\, \mu_0\, (\mu_1/\mu_2)$

and then we find

$B = -(I/2\pi)\, \mu_0\, (\mu_1/\mu_2)\ln(a) + (I/2\pi)\, \mu_0\, \ln(a)$

$= (I/2\pi)\, \ln(a)\, [-\mu_0\, (\mu_1/\mu_2) + \mu_0]$

$= (I/2\pi)\, \ln(a)\mu_0\, [-(\mu_1/\mu_2) + 1\,]$ // ditto

Then with these constants determined, we end up with

$\partial_r A_z(r>a) = [(\mu_2-\mu_1)\, I/(2\pi) + C]/r = -(I/2\pi)\, \mu_0\, (\mu_1/\mu_2)/r$

$\partial_r A_z(r<a) = -(I/2\pi)\, \mu_0\, r/a^2$

and then the magnetic B fields are

$\mathbf{B} = \text{curl}\ \mathbf{A} = -\partial_r A_z(r)\, \hat{\boldsymbol{\theta}}$. (B.6.20)

$B_1 = -\partial_r A_z(r)^1 = (I/2\pi)\, \mu_0\, (\mu_1/\mu_2)/r$ // did not change

$B_2 = -\partial_r A_z(r)^2 = (I/2\pi)\, \mu_0\, r/a^2$

These results are incorrect, so there is some malfunction in this method. I thought maybe maintaining the separate regions would fix things up rather than a unified R, but that is not the case.

[Again: the malfunction is omitting bulk J_m contributions.]

Comment: The first time through I got the C expression wrong, but the end result did not change and it is still wrong.

By the way, in cylindricals we could redo our slope BC this way

Assume $\mathbf{A} = A_z(r)\,\hat{\mathbf{z}}$

curl $\mathbf{A} = \hat{\mathbf{r}}\,[\,r^{-1}\partial_\theta A_z - \partial_z A_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z A_r - \partial_r A_z] + \hat{\mathbf{z}}\,[\,r^{-1}\partial_r(rA_\theta) - r^{-1}\partial_\theta A_r\,]$

$= \hat{\boldsymbol{\theta}}\,[-\partial_r A_z] = \hat{\boldsymbol{\theta}}\,B_\theta$ where $B_\theta = -\partial_r A_z$

Then lines (1.1.44) says

$$(1/\mu_2)B_\theta^{\,2} - (1/\mu_1)B_\theta^{\,1} = K_z^{\,free}$$

or

$$-(1/\mu_2)\,\partial_r A_z^{\,2} + (1/\mu_1)\,\partial_r A_z^{\,1} = K_z^{\,free}$$

and that is where our slope BC comes from. I just assume A_z itself is continuous (gulp).

[I think this is pretty well nailed down, except maybe sign of K_z , and installed in Section 1.1]

So this is a **Major Problem**. I claim to be teaching how to solve problems in this area, but my solution of the simplest possible problem is failing!! I have spent perhaps 4 whole days trying to find out why with no success. And before that maybe 90 days on Lines, and before that maybe 1 year on Stak!

I need to develop some debugger probing tools. [yes you do!] The bug is deep in the Machine here [yes it was] . This is going to go well into January 2014, I can see that now [wrong]. If four days does not nail it, it is something very serious and hard to find. I'm glad no one is having to pay me for this flailing work. [but the fifth day was golden. An argument to support "persistence".]

There is the little annoying fact about the integral and Maple

[an interesting side line. Maple does it weird, but I know GR7 is correct as is my sum method.]

[Again, *every single item* is on the suspect list, including this integral, since I am thrashing!]

In Appendix B I quote this result,

This integral may be evaluated using GR7 p 531 4.224,

$$9.^8 \qquad \int_0^{\pi} \ln(a + b\cos x)\, dx = \pi \ln \frac{a + \sqrt{a^2 - b^2}}{2} \qquad [a \geq |b| > 0] \qquad \text{GW (322)(15)}$$

but when I tell Maple to do this, here is what I get

```
> restart;
> assume (a>b,b>0);
> Int(log(a+b*cos(x)),x=0..Pi);
```

$$\int_0^{\pi} \ln(a + b\cos(x))\, dx$$

```
> int(log(a+b*cos(x)),x=0..Pi);
```

$$-I\ln\left(-\frac{b-a+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)\ln(b)+\pi\ln(a-b)+\frac{1}{2}I\pi^2+I\ln\left(-\frac{-b-a+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)\ln(b)+I\,\mathrm{dilog}\left(\frac{b}{-a+\sqrt{a^2-b^2}}\right)$$
$$-\pi\ln(-b+a+\sqrt{(a-b)(a+b)})+I\ln(a+\sqrt{(a-b)(a+b)})\ln(-b+a+\sqrt{(a-b)(a+b)})-I\ln(b)\ln(-b+a+\sqrt{(a-b)(a+b)})$$
$$+I\ln\left(-\frac{b-a+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)\ln(-a+\sqrt{(a-b)(a+b)})+I\ln\left(\frac{b-a+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)\ln(b)+I\ln\left(\frac{b-a+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)\ln\left(-\frac{1}{-a+\sqrt{a^2-b^2}}\right)$$
$$-I\,\mathrm{dilog}\left(\frac{b}{a+\sqrt{a^2-b^2}}\right)+I\,\mathrm{dilog}\left(\frac{b-a+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)-I\,\mathrm{dilog}\left(\frac{b+a+\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right)+\ln\left(\frac{b+a+\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right)\pi-I\ln\left(\frac{b+a+\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right)\ln(b)$$
$$+\pi\ln(b+a+\sqrt{(a-b)(a+b)})-I\ln(a+\sqrt{(a-b)(a+b)})\ln(b+a+\sqrt{(a-b)(a+b)})+I\ln(b)\ln(b+a+\sqrt{(a-b)(a+b)})$$
$$-I\ln\left(-\frac{-b-a+\sqrt{a^2-b^2}}{-a+\sqrt{a^2-b^2}}\right)\ln(-a+\sqrt{(a-b)(a+b)})+I\ln\left(\frac{b+a+\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right)\ln(a+\sqrt{(a-b)(a+b)})$$

```
a := 3: b := 2:
int(log(a+b*cos(x)),x=0..Pi);
```

$$\mathrm{dilog}\left(2\frac{1}{3+\sqrt{5}}\right)+I\ln(-3-\sqrt{5})\ln(\sqrt{5}+1)+\pi\ln(5-\sqrt{5})+I\ln(-3+\sqrt{5})\ln(\sqrt{5}-1)+I\ln(2)\ln(5+\sqrt{5})-I\ln(-3-\sqrt{5})\ln(5+\sqrt{5})$$
$$-I\ln(2)\ln(\sqrt{5}+1)-I\ln(2)\ln(\sqrt{5}-1)+I\ln(2)\ln(-1-\sqrt{5})-I\ln(-3+\sqrt{5})\ln(5-\sqrt{5})-I\,\mathrm{dilog}\left(1+2\frac{1}{3+\sqrt{5}}\right)+\ln(-3+\sqrt{5})\pi$$
$$-2I\ln(2)\ln(3+\sqrt{5})+I\ln(2)\ln(4)-2\pi\ln(2)+\frac{1}{2}I\pi^2+I\ln(-1-\sqrt{5})\ln(3+\sqrt{5})-I\ln(-1-\sqrt{5})\ln(4)+I\ln(3+\sqrt{5})\ln(5-\sqrt{5})$$
$$+I\,\mathrm{dilog}\left(1+2\frac{1}{-3+\sqrt{5}}\right)+I\,\mathrm{dilog}\left(2\frac{1}{-3+\sqrt{5}}\right)$$

```
evalf(%);
```

$$7.088853573 + 19.73920881\,I$$

The simple integral says that the integral should be

$$\pi \ln((1/2) (3 + \sqrt{5}))$$

```
evalf(Pi*log((1/2)*(3+sqrt(5))));
```

$$3.023543070$$

Wolfram Alpha can't do it unless you pay them. Hmmm. I think Stak has a series for this log thing that I used somewhere. Page 104 (6.22) says this

$$\ln(1 + \alpha^2 - 2\alpha \cos x) = -2 \sum_{n=1}^{\infty} \alpha^n \cos(nx)/n \qquad \text{valid for } |\alpha| < 1.$$

Then write, using $\alpha = r'/r < 1$

$$\ln(r^2 + r'^2 - 2rr' \cos x) = \ln[(r^2)(1 + \alpha^2 - 2\alpha \cos x)] = \ln(r^2) + \ln(1 + \alpha^2 - 2\alpha \cos x)$$

$$= 2\ln(r) - 2 \sum_{n=1}^{\infty} \alpha^n \cos(nx)/n$$

And then we have

$$Q(r',r) = \int_0^{\pi} dx \ln [r'^2 + r^2 - 2rr' \cos x)] = \int_0^{\pi} dx \{ 2\ln(r) - 2 \sum_{n=1}^{\infty} \alpha^n \cos(nx)/n \}$$

$$= 2\pi \ln(r) - 2 \sum_{n=1}^{\infty} \alpha^n/n \left(\int_0^{\pi} dx \cos(nx) \right) = 2\pi \ln(r) - 2 \sum_{n=1}^{\infty} \alpha^n/n \sin(\pi n)/n$$

$$= 2\pi \ln(r) \qquad \text{valid for } r' < r$$

You could swap $r \leftrightarrow r'$ in the above few lines to obtain

$$Q(r',r) = \int_0^{\pi} dx \ln [r'^2 + r^2 - 2rr' \cos x)] = 2\pi \ln(r') \qquad \text{valid for } r' > r$$

So we end up then with

$$Q(r',r) = 2\pi \begin{cases} \ln(r') & r' > r \\ \ln(r) & r' < r \end{cases}$$

and this gives the same result as the GR7 quoted integral. So I don't know what Maple is thinking.

So I guess I will cross off worrying about the GR7 integral and Maple's problem with it.

Here is another Maple attempt which shows conclusively that Maple gets this integral wrong:

```
restart;
a := Int(log(3 + 2*cos(x)), x=0..Pi);
```

$$a := \int_0^{\pi} \ln(3 + 2\cos(x))\, dx$$

```
value(a);
```

$$\ln(2)\ln(4) + I\ln(-1-\sqrt{5})\ln(3+\sqrt{5}) - 2\pi\ln(2) + I\,\mathrm{dilog}\left(1 + 2\frac{1}{-3+\sqrt{5}}\right) + \frac{1}{2}I\pi^2$$
$$- I\,\mathrm{dilog}\left(2\frac{1}{3+\sqrt{5}}\right) - 2I\ln(2)\ln(3+\sqrt{5}) - I\ln(2)\ln(\sqrt{5}+1) + I\ln(2)\ln(5+\sqrt{5})$$
$$+ I\ln(-3+\sqrt{5})\ln(\sqrt{5}-1) + I\ln(-3-\sqrt{5})\ln(\sqrt{5}+1) + I\,\mathrm{dilog}\left(2\frac{1}{-3+\sqrt{5}}\right)$$
$$- I\,\mathrm{dilog}\left(1 + 2\frac{1}{3+\sqrt{5}}\right) - I\ln(-3+\sqrt{5})\ln(5-\sqrt{5}) + \ln(5-\sqrt{5})\pi + I\ln(3+\sqrt{5})\ln(5-\sqrt{5})$$
$$+ \ln(-3+\sqrt{5})\pi - I\ln(-3-\sqrt{5})\ln(5+\sqrt{5}) - I\ln(-1-\sqrt{5})\ln(4) + I\ln(2)\ln(-1-\sqrt{5})$$
$$- I\ln(2)\ln(\sqrt{5}-1)$$

```
evalf(%);
```

$$7.088853569 + 19.73920880\,I$$

```
evalf(Int(log(3 + 2*cos(x)), x=0..Pi));   # numerical !!!
```

$$3.023543069$$

```
evalf(Pi*log((1/2)*(3 + sqrt(3^2-2^2))));
```

$$3.023543070$$

In the above explicit example, Maple's numerical integration agrees with the GR7 value. I think Maple has gone off on some strange branch of the analytic function ln(3 +2cos(z)) which is not the obvious branch. I will not pursue this further!

Resume 11.21.13.

Signal tracing day. [Amen] I will start this time at the end of the chain and trace backwards to look for the malfunction.

(a) What are the "right" A_z expressions for the round wire?

We know the correct **B** fields for the round wire

$$\mathbf{B} = (I\,\mu_1/2\pi)\,(1/r)\;\hat{\boldsymbol{\theta}} \qquad \text{for } r > a \text{ which is in the dielectric medium with } \mu_1$$
$$\mathbf{B} = (I\,\mu_2/2\pi)\,(r/a^2)\;\hat{\boldsymbol{\theta}} \qquad \text{for } r < a \text{ which is in the conductor medium with } \mu_2 \quad . \qquad \text{(B.6.23)}$$

With our assumption of $\mathbf{A} = A_z(r)\,\hat{\mathbf{z}}$ we also know that

$$\mathbf{B} = \text{curl}\,\mathbf{A} = -\,\partial_r A_z(r)\,\hat{\boldsymbol{\theta}} \quad . \qquad \text{(B.6.20)}$$

So we then have

$$- \partial_r A_z(r)^1 = (I \mu_1/2\pi)\,(1/r)$$
$$- \partial_r A_z(r)^2 = (I \mu_2/2\pi)\,(r/a^2)$$

Now integrate both these equations to get

$$A_z(r)^1 = -(I \mu_1/2\pi)\ln(r) + C_1$$
$$A_z(r)^2 = -(I \mu_2/4\pi)\,(r^2/a^2) + C_2$$

So these are the forms that the potential must have. The potentials already meet the slope condition by inspection. And I think $C_1 = 0$ for correct large r behavior. Then value match says

$$-(I \mu_2/4\pi)\,(r^2/a^2) + C_2 = -(I \mu_1/2\pi)\ln(r)$$
$$-(I \mu_2/4\pi) + C_2 = -(I \mu_1/2\pi)\ln(a)$$
$$C_2 = -(I \mu_1/2\pi)\ln(a) + (I \mu_2/4\pi) = -(I/2\pi)[\,\mu_1 \ln a - \mu_2/2]$$

The potential must then be this

$$A_z(r)^1 = -(I \mu_1/2\pi)\ln(r)$$
$$A_z(r)^2 = -(I \mu_2/4\pi)\,(r^2/a^2) - (I/4\pi)[\,2\mu_1 \ln a - \mu_2] = -(I/4\pi)\,[\,\mu_2(r^2/a^2) - \mu_2 + 2\mu_1 \ln a\,]$$
$$= -(I/4\pi)\,[\,\mu_2\{(r^2/a^2) - 1\} + 2\mu_1 \ln a\,]$$

Once again

$$A_z(r)^1 = -(I \mu_1/2\pi)\ln(r)$$
$$A_z(r)^2 = -(I/4\pi)\,[\,\mu_2\{(r^2/a^2) - 1\} + 2\mu_1 \ln a\,]$$

Let's now verify by tracing forward again

$$- \partial_r A_z(r)^1 = (I \mu_1/2\pi)\,1/r \quad = (I \mu_1/2\pi)\,1/r \quad = \text{correct}$$
$$- \partial_r A_z(r)^2 = (I/4\pi)\,[\,\mu_2\{(2r/a^2)\}\,] \quad = (I \mu_2/2\pi)\,(r/a^2) \quad = \text{correct}$$

Now the gauge condition says div $A = \partial_z A_z = 0$ which trivially true, so not violating this at $\omega = 0$.

Conclusion: given the requirement $A \rightarrow$ constant * lnr for large r, and given the *known* B fields, and given the A_z assumption, the potentials MUST have this form for the round wire [correct]

$$A_z(r)^1 = -(I \mu_1/2\pi)\ln(r)$$
$$A_z(r)^2 = -(I/4\pi)\,[\,\mu_2\{(r^2/a^2) - 1\} + 2\mu_1 \ln a\,] \quad \text{// both agree with original App B calc}$$

and these do meet both our boundary conditions.

(b) Do these correct A_z solutions solve the μ_0 gauge wave equation [as I perceived it]

Now that we know the required A_z fields, let's check to see if they satisfy our wave equations:

$$\nabla^2_{2D} A_z = -\mu_0 J_m \quad \text{region 1} \quad [\text{ I imagined } J_m \text{ only on boundary}]$$
$$\nabla^2_{2D} A_z = -\mu_0 J_{c2} \quad \text{region 2}$$

We can write these as

$$r^{-1}\partial_r(r\partial_r A_z) = -\mu_0 \{- [(\mu_2-\mu_1)/\mu_0] [I/(2\pi a)\}\delta(r-a) \qquad \text{region 1}$$
$$r^{-1}\partial_r(r\partial_r A_z) = -\mu_0 \{I/2\pi a^2\} \qquad \text{region 2}$$

Now compute left sides: First quote from a few lines above

$$- \partial_r A_z(r)^1 = (I \mu_1/2\pi)\, 1/r \qquad = \text{correct}$$
$$- \partial_r A_z(r)^2 = (I \mu_2/2\pi)\, (r/a^2) \qquad = \text{correct}$$

Then

$$r\partial_r A_z(r)^1 = -(I \mu_1/2\pi)$$
$$r\partial_r A_z(r)^2 = -(I \mu_2/2\pi)\, (r^2/a^2)$$

$$\partial_r(r\partial_r A_z(r)^1) = 0$$
$$\partial_r(r\partial_r A_z(r)^2) = -(I \mu_2/\pi)\, (r/a^2)$$

$$r^{-1}\partial_r(r\partial_r A_z(r)^1) = 0$$
$$r^{-1}\partial_r(r\partial_r A_z(r)^2) = - (I \mu_2/\pi)\, (1/a^2) = -\mu_2\, (I/\pi a^2)$$

Looking at region 2, we have a gaping contradiction.

Fact: The known-correct A_z potential in region 2 does NOT satisfy this wave equation:

$$\nabla^2_{2D} A_z = - \mu_0 J_{c2} \qquad \text{[a major signal tracing discovery]}$$

Instead, it satisfies THIS wave equation

$$\nabla^2_{2D} A_z = - \mu_2 J_{c2} \qquad \text{// NOTE WELL !!!}$$

We have a forward path to the wave equation which is like this

$$\text{curl } \mathbf{B} = \mu_0(\partial_t \mathbf{D} + \mathbf{J}_c + \mathbf{J}_m) \qquad \text{first step}$$
$$\nabla^2_{2D} A_z = - \mu_0 J_{c2} \qquad \text{last step region 2}$$

Somewhere along this path, something is going wrong! [yup!] This is our signal tracer debug method.

Let's check the very first step! Start with the known correct fields

$$\mathbf{B} = (I \mu_1/2\pi)\, (1/r)\ \hat{\boldsymbol{\theta}} \qquad \text{for } r > a \text{ which is in the dielectric medium with } \mu_1$$
$$\mathbf{B} = (I \mu_2/2\pi)\, (r/a^2)\ \hat{\boldsymbol{\theta}} \qquad \text{for } r < a \text{ which is in the conductor medium with } \mu_2 \ . \qquad \text{(B.6.23)}$$

Then use

$$\text{curl } \mathbf{B} = \hat{\mathbf{r}} [r^{-1}\partial_\theta B_z - \partial_z B_\theta] + \hat{\boldsymbol{\theta}} [\partial_z B_r - \partial_r B_z] + \hat{\mathbf{z}} [r^{-1}\partial_r(rB_\theta) - r^{-1}\partial_\theta B_r]$$

$$= \hat{\mathbf{r}} [- \partial_z B_\theta] + \hat{\mathbf{z}} [r^{-1}\partial_r(rB_\theta)]$$

$= \hat{\mathbf{z}}\ [\ r^{-1}\partial_r\{rB_\theta\}\ ]$

Then

$\text{curl}\mathbf{B}^1 = \hat{\mathbf{z}}\ [\ r^{-1}\partial_r\{r\ (I\ \mu_1/2\pi)\ (1/r)\}\ ] = 0$

$\text{curl}\mathbf{B}^2 = \hat{\mathbf{z}}\ [\ r^{-1}\partial_r\{r\ (I\ \mu_2/2\pi)\ (r/a^2)\}\ ] = (I\ \mu_2/2\pi a^2)\ \hat{\mathbf{z}}\ [\ r^{-1}\partial_r\{r^2\}\ ]$

$= (I\ \mu_2/2\pi a^2)\ \hat{\mathbf{z}}\ [\ r^{-1}2r\ ] = (I\ \mu_2/\pi a^2)\ \hat{\mathbf{z}}$

So our very first step in region 2 says

$\text{curl}\ \mathbf{B} = \mu_0(\partial_t\mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$

$[\text{curl}\ \mathbf{B}]_z = \mu_0 J_{cz} = \mu_0 I/(\pi a^2)$

Conclusion: The very first step is wrong, signal tracing procedure is concluded. [a breakthrough!]

Statement of Conclusion: The known B fields for the round wire in region 2 (inside wire) do NOT satisfy this Maxwell equation

$\text{curl}\ \mathbf{B} = \mu_0(\partial_t\mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$ [with J_m as I was imagining it, just a surface thing]

(c) So let's continue the reverse trace debug with a narrowed range

We start with

$\text{curl}\ \mathbf{H} = \partial_t\mathbf{D} + \mathbf{J_c}$ // a true Maxwell equation (1.1.1), very first of lines doc!

We then claim that

$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mu_0(\mathbf{H}+ \chi_{m2}\ \mathbf{H}) = \mu_0(1+\chi_{m2})\mathbf{H} = \mu_2\mathbf{H}$

which looks OK. Then we can say, since μ_0 is surely a constant,

$\text{curl}\ \mathbf{B} = \mu_0(\text{curl}\ \mathbf{H}+\text{curl}\ \mathbf{M})$

We then insert $\text{curl}\ \mathbf{H} = \partial_t\mathbf{D} + \mathbf{J_c}$ along with $\mathbf{J_m} = \text{curl}\ \mathbf{M}$ to get

$\text{curl}\ \mathbf{B} = \mu_0([\partial_t\mathbf{D} + \mathbf{J_c}]+[\mathbf{J_m}]) = \mu_0(\partial_t\mathbf{D} + \mathbf{J_c} + \mathbf{J_m})$

So our problem lies somewhere right in these last few lines.

Let's simplify this step sequence even more. Just set $\partial_t D = 0$ for $\omega = 0$, Then
(green mean step verified, purple means step fails to verify. I colored them after doing each below.)

Step 1: curl $\mathbf{H} = \mathbf{J_c}$ in all of region R = 1 + 2

Step 2: $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$ in all of region R = 1 + 2

We note in passing that the entire wire interior has $\mathbf{M} \neq 0$ as indicated by my App B picture. The curl of M is 0 in the bulk region and only non-zero at the boundary.

Step 3: curl $\mathbf{B} = \mu_0$(curl $\mathbf{H}$+curl $\mathbf{M}$) // no question about this step

Step 4: curl $\mathbf{M} = \mathbf{J_m}$ = isolated to boundary region // this is where the error was!

We know insert step 1 and step 4 into step 3 to get

Step 5: curl $\mathbf{B} = \mu_0(\mathbf{J_c} + \mathbf{J_m})$ we showed above this is violated.

In region 2, we have shown that Step 5 is violated by the known B fields of a round wire.

Verify Step 1. Let's verify that Step 1 is at least valid for our round wire. The known fields are

$$\mathbf{H^1} = (I/2\pi)(1/r)\,\hat{\boldsymbol{\theta}} \quad \text{for } r > a \quad \Rightarrow \quad H_\theta = (I/2\pi r)$$
$$\mathbf{H^2} = (I/2\pi)(r/a^2)\,\hat{\boldsymbol{\theta}} \quad \text{for } r < a \quad \Rightarrow \quad H_\theta = (Ir/2\pi a^2) \qquad \text{(B.6.24)}$$

$$\text{curl } \mathbf{H} = \hat{\mathbf{r}}\,[\, r^{-1}\partial_\theta H_z - \partial_z H_\theta] + \hat{\boldsymbol{\theta}}\,[\partial_z H_r - \partial_r H_z] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rH_\theta) - r^{-1}\partial_\theta H_r\,]$$

$$= \hat{\mathbf{r}}\,[\, -\partial_z H_\theta] + \hat{\mathbf{z}}\,[\, r^{-1}\partial_r(rH_\theta)\,]$$

$$= \hat{\mathbf{z}}\,[\, r^{-1}\partial_r\{rH_\theta\}\,]$$

curl $\mathbf{H^1} = 0$

curl $\mathbf{H^2} = \hat{\mathbf{z}}\,[\, r^{-1}\partial_r\{r(I/2\pi)(r/a^2)\}\,] = \hat{\mathbf{z}}\,(I/2\pi a^2)\, r^{-1}\partial_r\{r^2\} = \hat{\mathbf{z}}\,(I/2\pi a^2)\, r^{-1}2r = \hat{\mathbf{z}}\,(I/\pi a^2)$

$\mathbf{J_c^2} = \hat{\mathbf{z}}\,(I/\pi a^2)$

Thus I have just verified that in region 2 we have curl $\mathbf{H^2} = \mathbf{J_c^2}$, so Step 1 is OK.

Verify Step 2. Next, let's check Step 2 which claims that

$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$

I think I know **B** and **H**, but what is **M**? Well, we think that

$\mathbf{M} = \chi_m \mathbf{H} = [\, \mu_2/\mu_0 - 1\,]\,\mathbf{H}$

$$\mathbf{M^2} = [\,\mu_2/\mu_0 - 1]\,\mathbf{H^2} = [\,\mu_2/\mu_0 - 1]\,(I/2\pi)\,(r/a^2)\,\hat{\boldsymbol{\theta}}$$

$$= [\,\mu_2/\mu_0 - 1]\,(I/2\pi a^2)\,r\,\hat{\boldsymbol{\theta}}$$

Note that this magnetization is circumferential, not constant, and not longitudinal.

We then want to verify this equation

$$\mathbf{B^2} = \mu_0(\mathbf{H^2}+\mathbf{M^2})$$

$$\text{LHS} = \mu_2\mathbf{H^2} = \mu_2\,(Ir/2\pi a^2)\,\hat{\boldsymbol{\theta}}$$

$$\text{RHS} = \mu_0\,\{\,(Ir/2\pi a^2)\hat{\boldsymbol{\theta}} + [\,\mu_2/\mu_0 - 1]\,(I/2\pi a^2)\,r\,\hat{\boldsymbol{\theta}}\,\}$$

$$= \mu_0\hat{\boldsymbol{\theta}}\,(I/2\pi a^2)\{\,(r + [\,\mu_2/\mu_0 - 1]\,r\,\}$$

$$= \mu_0\hat{\boldsymbol{\theta}}\,(I/2\pi a^2)\,r\{\,1 + [\,\mu_2/\mu_0 - 1]\,\} = \mu_0\hat{\boldsymbol{\theta}}\,(I/2\pi a^2)\,r\{\,\mu_2/\mu_0\,\} = \hat{\boldsymbol{\theta}}\,(I/2\pi a^2)\,\mu_2 r$$

Thus we have LHS = RHS and so this Step 2 is verified in region 2, it goes green.

Verify Step 3. We have shown in Step 2 that $\mathbf{B^2} = \mu_0(\mathbf{H^2}+\mathbf{M^2})$ in the bulk region 2. It seems a no brainer that we would then have

$$\text{curl}\,\mathbf{B^2} = \mu_0(\text{curl}\,\mathbf{H^2}+\text{curl}\,\mathbf{M^2})$$

Since everything is a function only of r and nothing depends on z, we write this as

$$\hat{\mathbf{z}}\,[\,r^{-1}\partial_r\{rB_\theta^2\}\,] = \hat{\mathbf{z}}\,\mu_0\,(\,r^{-1}\partial_r\{rH_\theta^2\} + r^{-1}\partial_r\{rM_\theta^2\}\,) \qquad (*)$$

where from above

$$H_\theta^2 = (Ir/2\pi a^2)$$

$$B_\theta^2 = \mu_2\,H_\theta^2 = \mu_2\,(Ir/2\pi a^2)$$

$$M_\theta^2 = [\,\mu_2/\mu_0 - 1]\,H_\theta^2 = [\,\mu_2/\mu_0 - 1]\,(Ir/2\pi a^2)$$

Looking now at (*) we have

$$\text{LHS} = r^{-1}\partial_r(r\,\mu_2\,(Ir/2\pi a^2)) = \mu_2(I/2\pi a^2)\,r^{-1}\partial_r(r^2) = \mu_2(I/\pi a^2)$$

$$\text{RHS} = \mu_0\,\{\,r^{-1}\partial_r(r\,(Ir/2\pi a^2)) + r^{-1}\partial_r\{r\,[\,\mu_2/\mu_0 - 1]\,(Ir/2\pi a^2)\,\}$$

$= \mu_0(I/2\pi a^2)\, r^{-1}\partial_r \{ r\, r + [\mu_2/\mu_0 - 1] r\, r\} = \mu_0(I/2\pi a^2)\, r^{-1}\partial_r \{r^2 + [\mu_2/\mu_0 - 1] r^2\}$

$= \mu_0(I/2\pi a^2)\, r^{-1}\partial_r \{ \mu_2/\mu_0] r^2\} = \mu_0(I/2\pi a^2)\, r^{-1}\{ \mu_2/\mu_0] 2r\} = (I/\pi a^2) \{ \mu_2\}$

and we have then verified Step 3 with our known fields, so it goes green.

Verify Step 4. Let's compute

$\mathbf{J_m}^2 = \text{curl } \mathbf{M}^2 = \hat{\mathbf{z}} [\, r^{-1}\partial_r \{ r M_\theta^2\} \,] = \hat{\mathbf{z}} [\, r^{-1}\partial_r \{ r [\, \mu_2/\mu_0 - 1] (Ir/2\pi a^2)\} \,]$

$= [\, \mu_2/\mu_0 - 1] (I/2\pi a^2)\, \hat{\mathbf{z}} [\, r^{-1}\partial_r \{ rr \} \,] = [\, \mu_2/\mu_0 - 1] (I/2\pi a^2)\, \hat{\mathbf{z}} [\, 2 \,]$

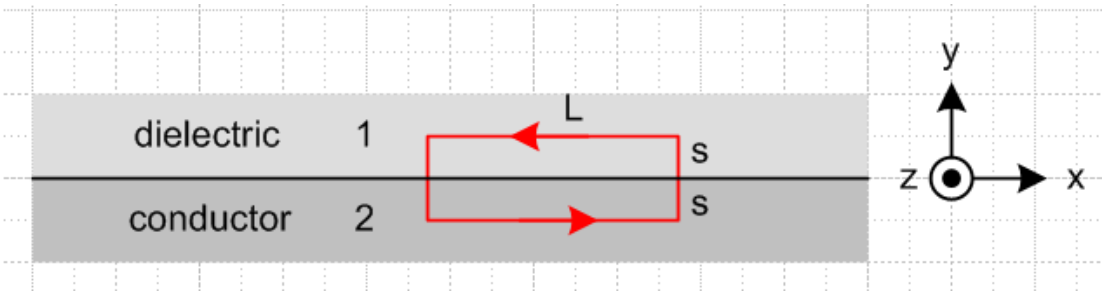
$= [\, \mu_2/\mu_0 - 1] (I/\pi a^2)\, \hat{\mathbf{z}}$

Bug Found !!! **// there was indeed a Loose Bolt**

DING !!!!!!! [maybe 11.21.13 at 2 PM circa] I was thinking that J_m existed only on the boundary of the round wire!!! That is the way things work at a planar boundary between μ_1 and μ_2 materials in scenarios #2 and #3 below. But here you see that $\mathbf{J_m}^2$ is non-vanishing *everywhere inside* the round wire!! (outside as well) That was a big mistake in my interpretation of things. $\mathbf{J_m}^2$ is in fact a constant everywhere inside the wire, it is not 0 as I thought. This is a big game changer. [yes it is!]

Planar Scenarios

Look again at the **planar** boundary situation: We imagine current in the z direction in region 2.



We have here

$\mathbf{M} = \chi_m \mathbf{H} = [\, \mu_2/\mu_0 - 1] \mathbf{H}$

$M_x^1 = \chi_m^1 H_x = [\, \mu_1/\mu_0 - 1] H_x^1$
$M_x^2 = \chi_m^2 H_x = [\, \mu_2/\mu_0 - 1] H_x^2 \qquad H_x^2 = H_x^1$

Scenario #1

The physical situation here is supposed to be uniform J_z in the bottom infinite half space. If we put our little loop entirely in the lower half space, we would find that

$$\text{curl } H = J \qquad \Rightarrow \qquad [H_x(y+2s) - H_x(y)]\, L = J_z L 2s$$

so that

$$[H_x(y+2s) - H_x(y)]/2s = J_z = d\,H_x/dy \qquad =? \qquad H_x(y) = J_z y + H_x(y=0)$$

So the H field in the conductor is not constant, but is linear in y. Then

$$M_x^2 = [\,\mu_2/\mu_0 - 1]\, H_x^2 = [\,\mu_2/\mu_0 - 1]\,[\,H_x(0) + J_z y\,]$$

So the magnetization in this scenario is NOT constant (since of course $\mathbf{M} = \chi_m \mathbf{H}$). Then the magnetization current is given by

$$\mathbf{J_m} = \text{curl } \mathbf{M} = \hat{\mathbf{x}}\,(\partial_y M_z - \partial_z M_y) + \hat{\mathbf{y}}\,(\partial_z M_x - \partial_x M_z) + \hat{\mathbf{z}}\,(\partial_x M_y - \partial_y M_x)$$

$$= \hat{\mathbf{z}}\,(-\partial_y M_x) = \hat{\mathbf{z}}\,(-\partial_y\{\,[\,\mu_2/\mu_0 - 1]\,[\,H_x(0) + J_z y\,]\}) = -[\,\mu_2/\mu_0 - 1] J_z \hat{\mathbf{z}}$$

So even in this planar boundary case, we find that J_m is NOT a surface current, it is a constant throughout region 2, just as in the round wire. So where did this "surface current" fixation come from?

Scenario #2

Well, suppose in this same scenario instead of having a uniform J_z, suppose we have J_z existing only as a surface current at some $\mathbf{y} = -y_0\hat{\mathbf{y}}$. That is a sheet down somewhere in the conductor. Then for loops outside that sheet, you find that H_x is constant in y. For a loop around the sheet we would get

$$H_x(-y_0 + s) - H_x(-y_0 - s) = K_z \qquad \text{where } K_z \text{ is our sheet current.}$$

So what we have in THIS scenario is

$$H_x(y) = H_x(0) \qquad \text{for y in the range y = 0 down to y = } -y_0$$

$$H_x(y) = H_x(0) - K_z \qquad \text{for } y < -y_0 \qquad \text{ignoring sign of } K_z$$

Then in this situation we have region 2 divided into two constant H regions, which means that we have then two constant M regions down there. So with a surface current as our source, we STILL end up with bulk magnetizations in this planar boundary case.

Now lets go up into region 1. We know that H_x continues across the boundary, and if we assume there is no current in region 1, it stays constant as you go up, using our loop rule. Then we have

$$H_x(y) = H_x(0) \qquad \text{for y in the range } y > 0$$

$H_x(y) = H_x(0)$ for y in the range y = 0 down to y = $-y_0$

$H_x(y) = H_x(0) - K_z$ for $y < -y_0$ ignoring sign of K_z

Now we then have, since $M_x = [\mu_i/\mu_0 - 1] H_x$ we have

$M_x(y) = [\mu_1/\mu_0 - 1] H_x(0)$ for y in the range y > 0

$M_x(y) = [\mu_2/\mu_0 - 1] H_x(0)$ for y in the range y = 0 down to y = $-y_0$

$M_x(y) = [\mu_2/\mu_0 - 1] (H_x(0) - K_z)$ for $y < -y_0$ ignoring sign of K_z

So now we have three distinct regions of constant magnetization. Then the mag current is this:

$$M_x(y) = [\mu_1/\mu_0 - 1] H_x(0)\, \theta(y>0) + [\mu_2/\mu_0 - 1] H_x(0)\, \theta(y<0)\, \theta(y>-y_0)$$

$$+ [\mu_2/\mu_0 - 1] (H_x(0) - K_z)\, \theta(y<-y_0)$$

$$\mathbf{J}_m = \hat{\mathbf{z}} (- \partial_y M_x)$$

$$- J_{mz} = \partial_y M_x =$$

Now

$$\partial_y\, \theta(y>0) = \partial_y \theta(y) = \delta(y)$$

$$\partial_y\, [\theta(y<0)\, \theta(y>-y_0)] = \delta(-y_0) - \delta(y)$$

$$\partial_y\, [\theta(y<-y_0)] = - \delta(-y_0)$$

So we find that

$$- J_{mz} = [\mu_1/\mu_0 - 1] H_x(0)\, \delta(y) + [\mu_2/\mu_0 - 1] H_x(0) \{ \delta(-y_0) - \delta(y)\}$$

$$+ [\mu_2/\mu_0 - 1] (H_x(0) - K_z)[- \delta(-y_0)]$$

$$= [\mu_1/\mu_0 - 1] H_x(0)\, \delta(y) - [\mu_2/\mu_0 - 1] H_x(0)\, \delta(y)$$

$$+ [\mu_2/\mu_0 - 1] H_x(0) \{ \delta(-y_0) - [\mu_2/\mu_0 - 1] (H_x(0) - K_z)\, \delta(-y_0) \}$$

$$= [\mu_1/\mu_0 - \mu_2/\mu_0] H_x(0)\, \delta(y) + K_z\, \delta(-y_0)$$

$$= (\mu_1-\mu_2)/\mu_0\, H_x(0)\, \delta(y) + K_z\, \delta(-y_0)$$

so then

$$J_{mz} = (\mu_2-\mu_1)/\mu_0\, H_x(0)\, \delta(y) - K_z\, \delta(-y_0)$$

Thus, in this case we have TWO sheets of magnetization current! One sheet is at the boundary between μ_1 and μ_2, while the other is at that sheet source current. So at $y = -y_0$ we have both a real current sheet with K_z and we have a magnetization current sheet with $-K_z$.

So to some extent, THIS is the situation perhaps analogous to the "Jackson problem" with a point charge and ε_1 and ε_2 where we get a surface polarization charge on the boundary.

Scenario #3

I suppose we could take a "point current" $J_z d^3x$ at say $x = 0$ and $y = -y_0$. That is really a wire in the z direction down there which seems an OK thing to have. This might be the true analog to the Jackson problem. Then B&B p 142 suggest $d\mathbf{H} = (1/4\pi r^3)\,\mathbf{J}\, d^3x \times \mathbf{r}$ and we have some H field which is trying to go around the conductor. Since there will be a sudden change in M all along the boundary (H_x continuous through the boundary), we will have a magnetization current all along the boundary, but it won't be constant. it will be something like the Jackson polarization charge density. So this would I guess make a very good "exercise" to perhaps include in an appendix with the Jackson problem. It would have the same image type solution. I suspect up in region 1 we would have perfectly circular magnetic field lines. But save all this for the future, right now I want to de-thread from my Disaster! It is 8:30 AM on 11.21.13,

Dethreading the Nov 16 Disaster after Bug Found

OK, back to the round wire. What does J_m really look like here? We have for this problem:

$$M_\theta^1 = [\,\mu_1/\mu_0 - 1]\, H_\theta^1 = [\,\mu_1/\mu_0 - 1]\,(I/2\pi r)$$
$$M_\theta^2 = [\,\mu_2/\mu_0 - 1]\, H_\theta^2 = [\,\mu_2/\mu_0 - 1]\,(Ir/2\pi a^2)$$

You can see that at r = a, there will be a sudden jump in this thing, so there will be a surface current component. We can write

$$M_\theta = [\,\mu_1/\mu_0 - 1]\,(I/2\pi r)\,\theta(r>a) + [\,\mu_2/\mu_0 - 1]\,(Ir/2\pi a^2)\theta(r<a)$$

$$\partial_r M_\theta = [\,\mu_1/\mu_0 - 1]\,(I/2\pi r)\,\delta(r-a) + [\,\mu_2/\mu_0 - 1]\,(Ir/2\pi a^2)[\,-\delta(r-a)]$$

$$= [\,\mu_1/\mu_0 - \mu_2/\mu_0]\,(I/2\pi r)\,\delta(r-a)$$

[OUCH! On 2.7.14 I realized I did the $\partial_r M$ wrong above because I have omitted two terms!!!]
[The error is now corrected in Appendix G, we end up with only one bulk term, same δ term]

And then

$$\mathbf{J}_m = \text{curl}\,\mathbf{M} = \hat{\mathbf{z}}\,[\,r^{-1}\partial_r\{rM_\theta\}\,]$$

so that [I think this little calculation is done correctly including all signs]

$$J_{mz} = r^{-1}\partial_r\{rM_\theta\} = r^{-1}[\,M_\theta + r\partial_r M_\theta] = r^{-1}M_\theta + \partial_r M_\theta$$

$$= r^{-1}[\,\mu_1/\mu_0 - 1]\,(I/2\pi r)\,\theta(r>a) + [\,\mu_2/\mu_0 - 1]\,(Ir/2\pi a^2)\theta(r<a)] + [\,\mu_1 - \mu_2]\,(I/2\pi r\mu_0)\,\delta(r-a)$$

$$= [\,\mu_1/\mu_0 - 1]\,(I/2\pi r^2)\,\theta(r>a) + [\,\mu_2/\mu_0 - 1]\,(I/2\pi a^2)\theta(r<a) + [\,\mu_1 - \mu_2]\,(I/2\pi a\mu_0)\,\delta(r-a)$$

So in fact there are three distinct regions for J_m : [WRONG as of 2.7.14]

$J_{mz} = [\,\mu_2/\mu_0 - 1]\,(I/2\pi a^2)$ inside the wire, it is a constant in this region

$J_{mz} = [\,\mu_1/\mu_0 - 1]\,(I/2\pi r^2)$ outside the wire, it decays with $1/r^2$

$J_{mz} = [\,\mu_1 - \mu_2]\,(I/2\pi a\mu_0)\,\delta(a)$ on the surface of the wire

Now, the third term I used to think was the only place J_{mz} existed, and I wrote in App B

$K_z^m = -\,[(\mu_2-\mu_1)/\mu_0]\,[\,I/(2\pi a)]$ round wire of radius a and μ_2, dielectric μ_1 (B.1.11)

and this agrees exactly with my third component above [including sign]. So I had that part right!

But I did not have the other two parts right!

Attempt to solve the μ_0 gauge wave equation using the correct J_{mz}

[I raced through this long calculation and probably made many errors. I suspect the divergence stuff is really there though and has to be dealt with. In the end I did not get the correct result, but I think if you did it carefully you would get the correct result. But I have no interest now in this μ_0 gauge!]

So let's now go back to our putative wave equation (from far above)

$$\nabla^2 A_z = -\,\mu_0 J_{c2z} - \mu_0 J_{mz} \qquad \text{region R}$$

$$J_{c2z}(r) = (I/\pi a^2)\theta(r<a)$$

$$J_{mz}(r) = [\,\mu_1/\mu_0 - 1]\,(I/2\pi r^2)\,\theta(r>a) + [\,\mu_2/\mu_0 - 1]\,(I/2\pi a^2)\theta(r<a) + [\,\mu_1 - \mu_2]\,(I/2\pi a\mu_0)\,\delta(a)$$

This is a whole new world now! Let's compute the Helmholtz integral particular in this strange gauge which exposes the magnetization currents:

$$A_z = +\mu_0 \int dV' \{\,[\,\mu_1/\mu_0 - 1]\,(I/2\pi r'^2)\,\theta(r'>a) + [\,\mu_2/\mu_0 - 1]\,(I/2\pi a^2)\theta(r'<a)\,\}\,[\,\ln(1/R)/2\pi]$$

$$+ \mu_0 \int dS' \,[\,\mu_1 - \mu_2]\,(I/2\pi a\mu_0)\,[\,\ln(1/R)/2\pi] + \int dV'\,(I/\pi a^2)\theta(r'<a)\,[\,\ln(1/R)/2\pi]$$

We might as well replace $\ln(1/R)$ by $-(1/2\,\ln(R^2)$ right at the start to get

$$A_z = -(\mu_0/2) \int dV' \{ [\mu_1/\mu_0 - 1] (I/2\pi r'^2) \theta(r'>a) + [\mu_2/\mu_0 - 1] (I/2\pi a^2)\theta(r'<a) \} [\ln(R^2)/2\pi]$$

$$-(\mu_0/2) \int dS' [\mu_1 - \mu_2] (I/2\pi a\mu_0) [\ln(R^2)/2\pi] - (1/2) \int dV' (I/\pi a^2)\theta(r'<a) [\ln(R^2)/2\pi]$$

Now pull out the 2π from the propagator

$$A_z = -(\mu_0/4\pi) \int dV' \{ [\mu_1/\mu_0 - 1] (I/2\pi r'^2) \theta(r'>a) + [\mu_2/\mu_0 - 1] (I/2\pi a^2)\theta(r'<a) \} [\ln(R^2)]$$

$$-(\mu_0/4\pi) \int dS' [\mu_1 - \mu_2] (I/2\pi a\mu_0) [\ln(R^2)] - (\mu_0/4\pi)\mu_0 \int dV' (I/\pi a^2)\theta(r'<a) [\ln(R^2)]$$

Now move the obvious factor to the left side to get (include I in this)

$$(-4\pi/\mu_0 I) A_z = \int dV' \{ [\mu_1/\mu_0 - 1] (1/2\pi r'^2) \theta(r'>a) + [\mu_2/\mu_0 - 1] (1/2\pi a^2)\theta(r'<a) \} [\ln(R^2)]$$

$$+ \int dS' [\mu_1 - \mu_2] (1/2\pi a\mu_0) [\ln(R^2)] + \int dV' (1/\pi a^2)\theta(r'<a) [\ln(R^2)]$$

Now break into four separate terms

$$(-4\pi/\mu_0 I) A_z = \int dV' [\mu_1/\mu_0 - 1] (1/2\pi r'^2) \theta(r'>a) [\ln(R^2)]$$
$$+ \int dV' [\mu_2/\mu_0 - 1] (1/2\pi a^2) \theta(r'<a) [\ln(R^2)]$$
$$+ \int dS' [\mu_1 - \mu_2] (1/2\pi a\mu_0) [\ln(R^2)] + \int dV' (1/\pi a^2)\theta(r'<a) [\ln(R^2)]$$

Now take out constants in each term

$$(-4\pi/\mu_0 I) A_z = [\mu_1/\mu_0 - 1] (1/2\pi) \int dV' (1/r'^2) \theta(r'>a) [\ln(R^2)]$$
$$+ [\mu_2/\mu_0 - 1] (1/2\pi a^2) \int dV' \theta(r'<a) [\ln(R^2)]$$
$$+ [\mu_1 - \mu_2] (1/2\pi a\mu_0) \int dS' [\ln(R^2)]$$
$$+ (2/2\pi a^2) \int dV' \theta(r'<a) [\ln(R^2)]$$

Notice that we can combine the 2nd and 4th terms to get a single term with $[\mu_2/\mu_0 + 1]$. So

$$(-4\pi/\mu_0 I) A_z = [\mu_1/\mu_0 - 1] (1/2\pi) \int dV' (1/r'^2) \theta(r'>a) [\ln(R^2)]$$
$$+ [\mu_2/\mu_0 + 1] (1/2\pi a^2) \int dV' \theta(r'<a) [\ln(R^2)]$$
$$+ [\mu_1 - \mu_2] (1/2\pi a\mu_0) \int dS' [\ln(R^2)]$$

We now have three integrals to calculate. Let's write things out in more detail using

$$\int dV' = \int_0^{\infty} r'dr' \int_{-\pi}^{\pi} d\theta'$$

$$\int dS' = a \int_{-\pi}^{\pi} d\theta'$$

Then we have

$$(-4\pi/\mu_0 I)\, A_z = [\,\mu_1/\mu_0 - 1]\,(1/2\pi) \int_0^{\infty} r'dr' \int_{-\pi}^{\pi} d\theta'(1/r'^2)\,\theta(r'>a)\,[\,\ln(R^2)]$$
$$+ [\,\mu_2/\mu_0 + 1]\,(1/2\pi a^2) \int_0^{\infty} r'dr' \int_{-\pi}^{\pi} d\theta'\theta(r'<a)\,[\,\ln(R^2)]$$
$$+ [\,\mu_1 - \mu_2]\,(1/2\pi a\mu_0)\, a \int_{-\pi}^{\pi} d\theta'\,[\,\ln(R^2)]$$

Now remove θ's

$$(-4\pi/\mu_0 I)\, A_z = [\,\mu_1/\mu_0 - 1]\,(1/2\pi) \int_a^{\infty} r'dr' \int_{-\pi}^{\pi} d\theta'(1/r'^2)\,[\,\ln(R^2)]$$
$$+ [\,\mu_2/\mu_0 + 1]\,(1/2\pi a^2) \int_0^{a} r'dr' \int_{-\pi}^{\pi} d\theta'\,[\,\ln(R^2)]$$
$$+ [\,\mu_1 - \mu_2]\,(1/2\pi a\mu_0)\, a \int_{-\pi}^{\pi} d\theta'\,[\,\ln(R^2)] \qquad \text{// but here } r' = a \text{ inside } R^2$$

and reorder things

$$(-4\pi/\mu_0 I)\, A_z = [\,\mu_1/\mu_0 - 1]\,(1/2\pi) \int_a^{\infty} r'^{-1}dr' \int_{-\pi}^{\pi} d\theta'\,[\,\ln(R^2)]$$
$$+ [\,\mu_2/\mu_0 + 1]\,(1/2\pi a^2) \int_0^{a} r'dr' \int_{-\pi}^{\pi} d\theta'\,[\,\ln(R^2)]$$
$$+ [\,\mu_1 - \mu_2]\,(1/2\pi a\mu_0)\, a \int_{-\pi}^{\pi} d\theta'\,[\,\ln(R^2)] \qquad \text{// } r' = a$$

Now write, with advanced knowledge,

$$\int_{-\pi}^{\pi} d\theta'\,[\,\ln(R^2)] = 2 \int_0^{\pi} d\theta'\,[\,\ln(R^2)] = 2\,Q(r',r)$$

$$Q(r',r) \equiv \int_0^{\pi} dx\, \ln [r'^2 + r^2 - 2rr' \cos x]$$

Thus we have

$$(-4\pi/\mu_0 I)\, A_z = [\,\mu_1/\mu_0 - 1]\,(1/\pi) \int_a^{\infty} r'^{-1}dr'\, Q(r',r)$$
$$+ [\,\mu_2/\mu_0 + 1]\,(1/\pi a^2) \int_0^{a} r'dr'\, Q(r',r)$$
$$+ [\,\mu_1 - \mu_2]\,(1/\pi a\mu_0)\, a\, Q(a,r)$$

Now from App B we know that

$$Q(r',r) = 2\pi \begin{cases} \ln(r') & r' > r \\ \ln(r) & r' < r \end{cases} = 2\pi \ln(r')\,\theta(r'>r) + 2\pi \ln(r)\theta(r'<r)$$

So we have two integrals to compute. First

$$\int_a^\infty r'^{-1}dr'\, Q(r',r) = \int_a^\infty r'^{-1}dr' \,[2\pi \ln(r')\,\theta(r'>r) + 2\pi \ln(r)\theta(r'<r)]$$

$$= 2\pi \{ \int_a^\infty r'^{-1}dr' \,[\ln(r')\,\theta(r'>r) + \ln(r)\theta(r'<r)] \}$$

$$= 2\pi \{ \int_a^\infty r'^{-1}dr' \ln(r')\,\theta(r'>r) + \int_a^\infty r'^{-1}dr' \ln(r)\theta(r'<r) \}$$

$$= 2\pi \{ \int_a^\infty r'^{-1}dr' \ln(r')\,\theta(r'>r) + \ln(r) \int_a^\infty r'^{-1}dr'\theta(r'<r) \}$$

$$= 2\pi \{ \int_{\max(a,r)}^\infty r'^{-1}dr'\ln(r') + \ln(r) \int_a^r r'^{-1}dr' \}$$

Now separate the two cases

r > a	region 1	outside
r < a	region 2	inside

Then

$$[\int_a^\infty r'^{-1}dr'\, Q(r',r)]^1 = 2\pi \{ \int_r^\infty r'^{-1}dr' \ln(r') + \ln(r) \int_a^r r'^{-1}dr' \}$$

We have Maple do this integration to avoid errors.

```
int1 := 2*Pi*(Int(log(x)/x,x=r..W) + log(r)*Int(1/x,x=a..r));
```

$$int1 := 2\,\pi \left(\int_r^W \frac{\ln(x)}{x}\,dx + \ln(r) \int_a^r \frac{1}{x}\,dx \right)$$

```
value(int1);
```

$$2\,\pi \left(\frac{1}{2}\ln(W)^2 - \frac{1}{2}\ln(r)^2 + \ln(r)\,(\ln(r) - \ln(a)) \right)$$

So my surprise, we get a divergence on the first integral so I put in a cutoff W. Then we have

$$[\int_a^\infty r'^{-1}dr'\, Q(r',r)]^1 = 2\pi \{(1/2)\ln(W^2) - \ln(r) + [\ln(r)]^2 - \ln a \ln r \}$$

This was not what I was expecting. Meanwhile for r < a

$$[\int_a^\infty r'^{-1}dr'\, Q(r',r)\,]^2 = 2\pi\,\{\ \int_{\max(a,r)}^\infty r'^{-1}dr'\,\ln(r') + \ln(r)\int_a^r r'^{-1}dr'\ \}$$

$$= 2\pi\,\{\ \int_a^\infty r'^{-1}dr'\ln(r')\ + 0\ \}$$

```
int2 := 2*Pi*(Int(log(x)/x,x=a..W));
```

$$int2 := 2\,\pi\int_a^W \frac{\ln(x)}{x}\,dx$$

```
value(int2);
```

$$2\,\pi\left(\frac{1}{2}\ln(W)^2 - \frac{1}{2}\ln(a)^2\right)$$

We then have

$$[\int_a^\infty r'^{-1}dr'\, Q(r',r)\,]^2 = 2\pi\,\{\ (1/2)\ln(W^2) - \ln(a)\ \}$$

So we summarize our results for this painful integral:

$$[\int_a^\infty r'^{-1}dr'\, Q(r',r)\,]^1 = 2\pi\,\{(1/2)\ln(W^2) - \ln(r) + [\ \ln(r)]^2 - \ln a\ \ln r\ \} \qquad r > a$$

$$[\int_a^\infty r'^{-1}dr'\, Q(r',r)\,]^2 = 2\pi\,\{\ (1/2)\ln(W^2) - \ln(a)\ \} \qquad r < a$$

Now we have our second integral to deal with

$$\int_0^a r'dr'\, Q(r',r) = 2\pi\ \int_0^a r'dr'\,[\ \ln(r')\,\theta(r'>r) + \ln(r)\theta(r'<r)]$$

$$= 2\pi\ \int_0^a r'dr'\ \ln(r')\,\theta(r'>r)\ +\ 2\pi\ \int_0^a r'dr'\,\ln(r)\theta(r'<r)$$

$$= 2\pi\{\ \int_0^a r'dr'\ \ln(r')\,\theta(r'>r)\ + \ln(r)\ \int_0^a r'dr'\theta(r'<r)\ \}$$

$$= 2\pi\{\ \int_r^a r'dr'\ \ln(r')\ + \ln(r)\ \int_0^{\min(a,r)} r'dr'\ \}$$

So again we do our two regions

$$[\int_0^a r'dr'\, Q(r',r)]^1 = 2\pi\{\ 0\ + \ln(r)\ \int_0^a r'dr'\ \} = 2\pi\,\ln(r)\,(1/2)a^2 = \pi a^2\,\ln r \qquad r>a$$

$$[\int_0^a r'dr'\, Q(r',r)]^2 = 2\pi\{ \int_r^a r'dr'\ \ln(r') + \ln(r) \int_0^r r'dr' \} \qquad r<a$$

```
int3 := 2*Pi* (Int(x*log(x),x=r..a) + log(r)*Int(x,x=0..r));
```

$$int3 := 2\pi \left(\int_r^a x \ln(x)\, dx + \ln(r) \int_0^r x\, dx \right)$$

```
value(int3);
```

$$2\pi \left(\frac{1}{2} a^2 \ln(a) - \frac{1}{4} a^2 + \frac{1}{4} r^2 \right)$$

So we summarize these as

$$[\int_0^a r'dr'\, Q(r',r)]^1 = \pi a^2 \ln r \qquad r>a$$

$$[\int_0^a r'dr'\, Q(r',r)]^2 = 2\pi\, [\, (a^2/2)\ln a + (1/4)(r^2-a^2)] \qquad r<a$$

Finally, we have to consider

$$Q(a,r) = 2\pi \ln(a)\, \theta(a>r) + 2\pi \ln(r)\theta(a<r)$$

$$[Q(a,r)]^1 = 2\pi \ln(r)$$
$$[Q(a,r)]^2 = 2\pi \ln(a)$$

OK, we now put the pieces together

$$(-4\pi/\mu_0 I)\, A_z^1 = [\, \mu_1/\mu_0 - 1]\, (1/\pi)\, [\int_a^\infty r'^{-1} dr'\, Q(r',r)]^1$$
$$+ [\, \mu_2/\mu_0 + 1]\, (1/\pi a^2)\ [\int_0^a r'dr'\, Q(r',r)\,]^1$$
$$+ [\, \mu_1 - \mu_2]\, (1/\pi a \mu_0)\, a\, [Q(r',a)]^1 \qquad r > a$$

$$(-4\pi/\mu_0 I)\, A_z^1 = [\, \mu_1/\mu_0 - 1]\, (1/\pi)\, 2\pi\, \{(1/2)\ln(W^2) - \ln(r) + [\, \ln(r)]^2 - \ln a \ln r\,\}$$
$$+ [\, \mu_2/\mu_0 + 1]\, (1/\pi a^2)\, \pi a^2 \ln r$$
$$+ [\, \mu_1 - \mu_2]\, (1/\pi a \mu_0)\, a 2\pi \ln(r) \qquad r > a$$

$$(-4\pi/\mu_0 I)\, A_z^1 = [\, \mu_1/\mu_0 - 1]\, \{\ln(W^2) - 2\ln(r) + 2[\, \ln(r)]^2 - 2\ln a \ln r\,\}$$
$$+ [\, \mu_2/\mu_0 + 1]\ \ln r$$
$$+ [\, \mu_1 - \mu_2]\, (1/\mu_0)\, 2 \ln(r) \qquad r > a$$

$$= [\, \mu_1/\mu_0 - 1]\, \{\ln(W^2) + 2[\, \ln(r)]^2\,\}$$

$$+ \ln(r)\, \{\ -2\, [\, \mu_1/\mu_0 - 1] - 2 \ln a\, [\, \mu_1/\mu_0 - 1] + [\, \mu_2/\mu_0 + 1] + 2\, [\, \mu_1 - \mu_2]\, (1/\mu_0)\,\}$$

$$= [\, \mu_1/\mu_0 - 1]\, \{\ln(W^2) + 2[\, \ln(r)]^2\,\}$$

$$+ \ln(r)\, \{\ -2\mu_1/\mu_0 + 2 - 2 \ln a\, [\, \mu_1/\mu_0 - 1] + \mu_2/\mu_0 + 1 + 2\, [\, \mu_1/\mu_0 - \mu_2/\mu_0]\,\}$$

$$= [\,\mu_1/\mu_0 - 1]\,\{\ln(W^2) + 2[\,\ln(r)]^2\,\}$$

$$+ \ln(r)\,\{\; - 2\mu_1/\mu_0 + 3 - 2\,\ln a\,[\,\mu_1/\mu_0 - 1] + \mu_2/\mu_0 + 2\;\mu_1/\mu_0 - 2\mu_2/\mu_0]\,\}$$

$$= [\,\mu_1/\mu_0 - 1]\,\{\ln(W^2) + 2[\,\ln(r)]^2\,\}$$

$$+ \ln(r)\,\{ +3 - 2\,\ln a\,[\,\mu_1/\mu_0 - 1] - \mu_2/\mu_0]\,\}$$

It's ugly and certainly wrong, but leave for the moment. Next

$$(-4\pi/\mu_0 I)\,A_z^2 = [\,\mu_1/\mu_0 - 1]\,(1/\pi)\,[\int_a^\infty r'^{-1}dr'\,Q(r',r)]^2$$

$$+ \;[\,\mu_2/\mu_0 + 1]\,(1/\pi a^2)\;[\int_0^a r'dr'\,Q(r',r)\,]^2$$

$$+ [\,\mu_1 - \mu_2]\,(1/\pi a\mu_0)\,a\,[Q(r',a)]^2 \qquad r < a$$

$$(-4\pi/\mu_0 I)\,A_z^2 = [\,\mu_1/\mu_0 - 1]\,(1/\pi)\,2\pi\,\{\,(1/2)\ln(W^2) - \ln(a)\,\}$$
$$+ \;[\,\mu_2/\mu_0 + 1]\,(1/\pi a^2)\,2\pi\,[\,(a^2/2)\ln a + (1/4)(r^2 - a^2)]$$
$$+ [\,\mu_1 - \mu_2]\,(1/\pi a\mu_0)\,a\,2\pi\,\ln(a) \qquad r < a$$

$$(-4\pi/\mu_0 I)\,A_z^2 = [\,\mu_1/\mu_0 - 1]\;\;\{\,\ln(W^2) - 2\ln(a)\,\}$$
$$+ \;[\,\mu_2/\mu_0 + 1]\;\;[\,\ln a + (1/2a^2)(r^2 - a^2)]$$
$$+ [\,\mu_1 - \mu_2]\,(1/\mu_0)\;\,2\,\ln(a) \qquad r < a$$

$$= [\,\mu_1/\mu_0 - 1]\;\;\ln(W^2)$$

$$- 2\ln(a)\,[\,\mu_1/\mu_0 - 1] + [\,\mu_2/\mu_0 + 1]\;\;[\,\ln a + (1/2a^2)(r^2 - a^2)] + [\,\mu_1 - \mu_2]\,(1/\mu_0)\;\,2\,\ln(a)$$

$$= [\,\mu_1/\mu_0 - 1]\;\;\ln(W^2) + [\,\mu_2/\mu_0 + 1]\,(1/2a^2)(r^2 - a^2) +$$

$$+ \ln a\,\{\,-2\;\,[\,\mu_1/\mu_0 - 1] + [\,\mu_2/\mu_0 + 1] + [\,\mu_1 - \mu_2]\,(1/\mu_0)\;\,\}$$

$$= [\,\mu_1/\mu_0 - 1]\;\;\ln(W^2) + [\,\mu_2/\mu_0 + 1]\,(1/2a^2)(r^2 - a^2) +$$

$$+ \ln a\,\{\,-2\,\mu_1/\mu_0 + 2 + \mu_2/\mu_0 + 1 + \mu_1/\mu_0 - \mu_2/\mu_0\;\,\}$$

$$= [\,\mu_1/\mu_0 - 1]\;\;\ln(W^2) + [\,\mu_2/\mu_0 + 1]\,(1/2a^2)(r^2 - a^2) +$$

$$+ \ln a\,\{\,-\mu_1/\mu_0 + 3\,\}$$

So including various no doubt algebra mistakes, here is the result:

$$(-4\pi/\mu_0 I)\,A_z^1 = \{\,+3 - 2\,\ln a\,[\,\mu_1/\mu_0 - 1] - \mu_2/\mu_0]\,\}\,\ln(r) \qquad r > a$$
$$(-4\pi/\mu_0 I)\,A_z^2 = [\,\mu_1/\mu_0 - 1]\;\;\ln(W^2) + [\,\mu_2/\mu_0 + 1]\,(1/2a^2)(r^2 - a^2) + \ln a\,\{\,-\mu_1/\mu_0 + 3\,\} \qquad r<a$$

These are of course wrong, but they have some semblance of truth in them. Recall the correct results,

$$A_z(r)^1 = -(I\,\mu_1/2\pi)\,\ln(r) + C_1$$
$$A_z(r)^2 = -(I\,\mu_2/4\pi)\,(r^2/a^2) + C_2$$

I think errors resulted in failure to get cancellations, as suggested by the "3" for example, and the fact that μ_1 and μ_2 both appear in each equation (cancellation failures).

I will assume that with enough corrective hours I could make things work out here. But this is more important:

Death Knell for the μ_0 Gauge

I only considered the μ_0 gauge because I wanted to "expose" the magnetization current $\mathbf{J_m}$. I thought at that time $\mathbf{J_m}$ was something very simple, just the surface K_z thing, and then I was getting what looked like a simple problem to solve :

$$\nabla^2_{2D} A_z = -\mu_0 J_{c2z} - \mu_0 J_{mz} \qquad \text{region R}$$

where $\mathbf{J_m}$ was localized to the boundary K_z thing. Since that interpretation was wrong about $\mathbf{J_m}$, naturally this equation is not satisfied by the correct solution for A_z. In retrospect, since $\mathbf{J_m}$ is much more complicated than I thought, and also *unknown* at the start, this is a terrible gauge in which to work!! In the messy calculation just finished above, I had to *assume* the solution in order to know H and thus to know J_m so I could put it on the right side! So certainly this is the end of the μ_0 gauge for me, though it was instructive and led me to my misunderstanding of J_m. [Sayonara to the μ_0 gauge]

Reminder of the Just-Fine two region Case Study solution to the round wire problem.

This μ_0 gauge was a huge distraction used as a probe into things. If I then go back to the μ_1 gauge, I still have this situation,

$$\nabla^2_{2D}\,\mathbf{A_z} = -\mu_2\,J_{cz} \qquad \text{x in region R} \qquad \text{the King } \mu_1 \text{ gauge}$$

I now *appreciate* this equation much more because all that messy mag current J_m is completely hidden from sight, we do not include it in J_{cz}. Remember that J_m exists in both bulk regions 1 and 2, *and* on the boundary, so it is *very ugly*. Also notice that μ_2 goes with J_{c2z} which I know is very helpful. In the μ_0 gauge we have $-\mu_0\,J_{cz}$ which caused headaches galore.

Now remember that I successfully solved the above wave equation in "Case Study" and got the exact right answer. Here is how that solution went

1. Treat things as two regions, so have this

$$\nabla^2_{2D}\,\mathbf{A_z} = 0 \qquad \text{region 1}$$
$$\nabla^2_{2D}\,\mathbf{A_z} = -\mu_2\,J_{cz} \qquad \text{region 2}$$

2. Compute the particular integral for region 2 and add a homo function to this solution in the form of a simple constant d based on the idea that the other atom lnr is not allowed in region 2.

$$A_z^2 = (1/2\pi) \int dV' \, [\mu_2 J_{cz}(\mathbf{x}')] \ln(1/R) \quad + \quad d \; = \; W + d$$

3. In region 1, assume a homo solution of this form

$$A_z^1 = A \ln r$$

We could have added a second atom so perhaps $A_z^1 = A \ln r + A'$, but we would have found A' = 0 when we did the BC match.

4. Compute the W integral to find

$$W = -(I/2\pi)\,\mu_2 \begin{cases} \ln(r) & r > a \\ \ln a + (r^2/a^2 - 1)/2 & r < a \end{cases}$$

but we only make use of this result in region 2 (r<a) where it showed up above.

5. The solutions at this point are (where A and d are to be determined)

$$A^1 = A \ln r$$
$$A^2 = - \; (I/2\pi)\,\mu_2 \, [\, \ln a + (1/2)(r^2/a^2 - 1) \,] \; + d$$

6. Match both function and derivative BC's at r = a to find that

$$A = -\mu_1 \, (I/2\pi)$$
$$d = (I/2\pi)(\mu_2 - \mu_1)\ln a$$

7. Install these to get the final results which we know are the correct results!

$$A^1 = -(I/2\pi) \, [\; \mu_1 \ln a \; + \; (1/2)\mu_2(r^2/a^2 - 1)$$
$$A^2 = -\mu_1 \, (I/2\pi)\ln r$$

So basically "all is well" with the round wire and the King Helmholtz method, BUT you have to play with homo atoms in addition to the particular integral, something King did not mention but he would say was an obvious fact of life.

The one-region Case Study solution to the round wire problem.

What happens if we reconsider the above wave equation in the **single-region R** view

$$\nabla^2_{2D} \mathbf{A}_z = - \mu_2 J_{cz} \qquad \text{x in region R} \qquad \text{the King } \mu_1 \text{ gauge}$$

You could compute the particular integral just as done above for all of R to get

$$A_z^2 = (1/2\pi) \int dV' \, [\mu_2 J_{cz}(x')] \ln(1/R) + A_z^h = W + A_z^h \qquad \text{// for all R}$$

$$W = -(I/2\pi)\,\mu_2 \begin{cases} \ln(r) & r > a \\ \ln a + (r^2/a^2 - 1)/2 & r < a \end{cases}$$

Your task is then to come up with a homo adder function A_z to make the BC's work at the boundary. You might conjecture that

$$\begin{aligned} A_z^1 &= -(I/2\pi)\,\mu_2 \ln(r) + C \ln(r) && r > a \\ A_z^2 &= -(I/2\pi)\,\mu_2 [\, \ln(a) + (r^2/a^2 - 1)/2 \,] + d && r < a \end{aligned}$$

If you were then to match the boundary conditions and thereby compute d and C, you would then arrive at the correct solution. No matter how you cut it, the BC's are really what determine the solution! So let's do this explicitly. We have

$$\begin{aligned} A_z^1 &= [-(I/2\pi)\,\mu_2 + C \,] \ln(r) && r > a \\ A_z^2 &= -(I/2\pi)\,\mu_2 [\, \ln(a) + (r^2/a^2 - 1)/2 \,] + d && r < a \end{aligned}$$

$$\begin{aligned} -\partial_r A_z^1 &= [(I/2\pi)\,\mu_2 - C \,]1/r && r > a \\ -\partial_r A_z^2 &= (I/2\pi)\,\mu_2 [\, (r/a^2) \,] && r < a \end{aligned}$$

Then evaluate both at the boundary

$$\begin{aligned} A_z^1 &= [-(I/2\pi)\,\mu_2 + C \,] \ln(a) && r > a \\ A_z^2 &= -(I/2\pi)\,\mu_2 [\, \ln(a) \,] + d && r < a \end{aligned}$$

$$\begin{aligned} -\partial_r A_z^1 &= [(I/2\pi)\,\mu_2 - C \,]1/a && r > a \\ -\partial_r A_z^2 &= (I/2\pi)\,\mu_2 [\, (1/a) \,] && r < a \end{aligned}$$

then we find

$$[-(I/2\pi)\,\mu_2 + C \,] \ln(a) = -(I/2\pi)\,\mu_2 [\, \ln(a) \,] + d$$
$$(1/\mu_1)[(I/2\pi)\,\mu_2 - C \,]1/a = (1/\mu_2)\,(I/2\pi)\,\mu_2 [\, (1/a) \,]$$

$$[(I/2\pi)\,\mu_2 - C \,] \ln(a) = (I/2\pi)\,\mu_2 [\, \ln(a) \,] - d$$
$$[(I/2\pi)\,\mu_2 - C \,] = (I/2\pi)\mu_1$$

$$C = (I/2\pi)\,\mu_2 - (I/2\pi)\mu_1 = (I/2\pi)(\mu_2 - \mu_1)$$

$$d = (I/2\pi)\,\mu_2 [\, \ln(a) \,] - [(I/2\pi)\,\mu_2 - C \,] \ln(a)$$

$= (I/2\pi)\,\mu_2\,[\,\ln(a)\,] - (I/2\pi)\mu_1 \ln(a)$

$= (I/2\pi)\ln(a)(\mu_2 - \mu_1)$

Thus our values are

$C = (I/2\pi)(\mu_2-\mu_1)$
$d = (I/2\pi)\,(\mu_2 - \mu_1)\ln(a)$

and then our solution is

$\mathbf{A_z}^1 = [-(I/2\pi)\,\mu_2 + C\,]\ln(r)$ r > a
$\mathbf{A_z}^2 = -(I/2\pi)\,\mu_2\,[\,\ln(a) + (r^2/a^2-1)/2\,] + d$ r < a

$\mathbf{A_z}^1 = [-(I/2\pi)\,\mu_2 + (I/2\pi)(\mu_2-\mu_1)\,]\ln(r)$ r > a
$\mathbf{A_z}^2 = -(I/2\pi)\,\mu_2\,[\,\ln(a) + (r^2/a^2-1)/2\,] + (I/2\pi)\,(\mu_2 - \mu_1)\ln(a)$ r < a

$\mathbf{A_z}^1 = [-(I/2\pi)\,\mu_1\,]\ln(r)$ r > a
$\mathbf{A_z}^2 = -(I/2\pi)\,\mu_2\,\ln(a) - (I/2\pi)\,\mu_2\,(r^2/a^2-1)/2 + (I/2\pi)\,(\mu_2 - \mu_1)\ln(a)$
$= -(I/2\pi)\,\mu_1\,\ln(a) - (I/2\pi)\,\mu_2\,(r^2/a^2-1)/2$ r < a
$= -(I/2\pi)[\,\mu_1\,\ln(a) + \mu_2\,(r^2/a^2-1)/2\,]$ r < a

and this is the "correct" solution as found in Appendix B. [reviewed to here]

There does remain this old mystery

Which I am not sure I am worried about. It was presented as Method II in my Case Study doc. I found that IF you assumed a wave equation of this form

$$\nabla^2_{2D}\,\mathbf{A} = -\mu_0\mathbf{K_m}\,\delta(r-a) - \mu_2\mathbf{J_{c2}}$$

THEN the particular integral *happens* to give the exact right solution with no homo terms needed. This was precisely the subject of Appendix B Version 2. I fretted for a long time trying to find why this is true. Notice that you are only using "some" of the full mag current J_m in this equation, and that with no justification whatsoever. Somehow it happens that adding $-\mu_0\mathbf{K_m}\,\delta(r-a)$ must generate exactly the precise homo adder solution which appears in the one-region case study above. In other words, it must happen that this is true:

$Z = (1/2\pi)\int dV'\,[\mu_0\,K_z\,\delta(r-a)]\ln(1/R) = d = (I/2\pi)(\mu_2-\mu_1)\ln a$ r < a

$Z = (1/2\pi)\int dV'\,[\mu_0 K_z\,\delta(r-a)]\ln(1/R) = C\ln r = (I/2\pi)(\mu_2-\mu_1)\ln r$ r > a

I can quickly check on this using App D stuff.

$$Z = (1/2\pi) \int dV' \, [\mu_0 K_z \, \delta(r-a)] \ln(1/R)$$

$$= - (1/2\pi)(1/2) \int dV' \, [\mu_0 K_z \, \delta(r-a)] \ln(R^2)$$

$$= - (1/2\pi)(1/2) \, [(\mu_1-\mu_2) \, I/(2\pi a) \int dV' \, \delta(r-a) \ln(R^2)$$

$$= - (1/2\pi)(1/2) \, [(\mu_1-\mu_2) \, I/(2\pi a) \int dr' \, r' \, d\theta \, \delta(r-a) \ln(R^2)$$

$$= - (1/2\pi)(1/2) \, [(\mu_1-\mu_2) \, I/(2\pi a) \, a \int d\theta \ln(R^2)$$

$$= - (1/2\pi)(1/2) \, [(\mu_1-\mu_2) \, I/(2\pi a) \, a \, 2 \, Q(a,r)$$

$$= - (1/2\pi)(1/2) \, [(\mu_1-\mu_2) \, I/(2\pi a) \, a \, 2 * 2\pi \begin{cases} \ln(a) & a > r \\ \ln(r) & a < r \end{cases}$$

$$= (I/2\pi) \, [(\mu_2-\mu_a) \begin{cases} \ln(a) & r<a \\ \ln(r) & r>a \end{cases}$$

and this then all checks out right.

So the cloying question remains: WHY does adding source $-\mu_0 K_z \, \delta(r-a)$ to the right side of the wave equation generate these exactly correct homo adder solutions? Here is the problem with just this term

$$\nabla^2_{2D} A_z = -\mu_0 K_z \, \delta(r-a) \qquad \text{all R}$$
$$\nabla^2_{2D} A_z = 0 \qquad r>a$$
$$\nabla^2_{2D} A_z = 0 \qquad r<a$$

Whatever the solution is, it *will* be a homo solution in the two separate regions, so we know THAT much. And we expect continuity of A_z at the boundary, and the solution shows that. Maybe this added current somehow cancels the mag current, causing B and hence A to "see nothing there" and then you get the regular jump on $\partial_r A_z$ with no μ_i factors. I do not think it is worth much to come up with a precise interpretation here, and I will let it lay. It certainly is a special case I would say!

I did show earlier that

$$J_{mz} = [\mu_1/\mu_0 - 1] \, (I/2\pi r^2) \, \theta(r>a) + [\mu_2/\mu_0 - 1] \, (I/2\pi a^2) \theta(r<a) + [\mu_1 - \mu_2] \, (I/2\pi a \mu_0) \, \delta(a)$$

I think this Disaster of Nov 16 has come to an end, I am now going to back thread through all related docs with red text to make sure I have not missed something.

[I went through this entire 67 page doc in much detail and still may pursue the "cloying question" a bit more.]