Assemble the Facts in Sequence PhL 3.6.14

[5.11.14] A version of all this stuff is now installed in Section 3.7. I could not show φ = constant for the entire range of ω, just ω = 0 and high ω. I would have thought low ω proof would be easy, but I don't know how to do it. Perhaps it is not true, but I don't need this information. Final review 5/15/14.

For the time being, I am going to be thinking always of the two cylinder problem rather than the general case. Later I can hopefully generalize.

Fact 1: $\mathbf{E}_t = 0$ around a conductor cross section perimeter at constant z.

[for a while I was confused by using $\mathbf{E}_t$ for both tangential and transverse E, but this is cleared up now in lines doc and E_t is the transverse while E_θ is the tangential]

Here $\mathbf{E}_t$ refers to E tangential to the conductor surface in the cross section picture. For the two-cylinder problem, this is E_θ with θ defined as in bipolar doc. In Section D.8 (a) we argue that this "quasi static" approximation is valid up to 10^{14} Hz which is 100,000 GHz, well beyond transmission lines of interest to us.

Fact 2: In the King gauge, for a transmission line operating in the transmission line limit, the transverse vector potential is very small: $|\mathbf{A}_t| < 10^{-4} |A_z|$ for f = DC to 1000 GHz.

This is demonstrated in Appendix M, see (M.22). [added this as a separate item]

Fact 3: φ = constant on the conductor cross section perimeter at all ω of interest.

At ω = 0 ("electrostatics") we know that $\mathbf{E}_t = 0$ implies that scalar potential φ = constant on a conductor surface, since $\mathbf{E}_t = -\nabla_t\varphi$ and $\mathbf{E}_t = 0$. For ω > 0, we have instead from (1.3.1) that $\mathbf{E}_t = -\nabla_t\varphi - j\omega\mathbf{A}_t$. In order to continue to claim that φ = constant on a conductor cross section, we must show that $-j\omega\mathbf{A}_t$ can be neglected relative to $-\nabla_t\varphi$. Then we will have $\mathbf{E}_t = -\nabla_t\varphi$ and by Fact 1 we know $\mathbf{E}_t = 0$ for ω > 0 and then once again φ must to constant on each conductor cross section surface.

We shall now demonstrate this fact in a series of 6 steps:

1. By either of two methods shown below, we argue first that

$$A_z \approx (\beta_d/\omega)\,\varphi \ . \qquad \text{// note that } (\beta_d/\omega) = 1/v_d \qquad (1)$$

[Note: Both methods are OK, and both methods implicitly assume the strong skin effect regime! The two methods appear in my recent Review of Pardox 1 and 2.doc]

Method A: Make the ansatz that $(\partial_x A_x + \partial_y A_y) << (\partial_z A_z)$ based on the Appendix M claim that $|A_{x,y}| < 10^{-4} |A_z|$ from DC to 1000 GHz. [certainly a fudge!] Then the King gauge says (in the dielectric),

$\partial_z A_z \approx$ div $\mathbf{A}$ $= -j\,(\beta_d^2/\omega)\varphi$. *(1.5.5)*

As with all transmission line quantities, we assume $A_z(x,y,z) = e^{-j\beta_d z}\, A_z(x,y)$ so then

$-j\beta_d A_z(x,y) \approx -j\,(\beta_d^2/\omega)\varphi(x,y)$ => $A_z \approx (\beta_d/\omega)\varphi$

Method A Revisited. Imagine as done below that $\partial_x A_x + \partial_y A_y \approx (A_x + A_y)/D$ where D is some transverse dimension. Just simplify and write this as $\partial_x A_x + \partial_y A_y \approx 2(A_x/D)$. Now to achieve Method A's goal, we have to show that

$(\partial_x A_x + \partial_y A_y) << (\partial_z A_z)$

or

$(2/D)\, A_x << -j\beta_d A_z$

or

$(2/D)\, A_x << -j(2\pi/\lambda) A_z$

or

$(\lambda/D) << (A_z/A_x)$ // all should be absolute values

or

$(D/\lambda) >> (A_x/A_z)$

or

$(A_x/A_z) << (D/\lambda)$ (*)

Now we supposedly know from Appendix M that

$(A_x/A_z) < 10^{-4}$

but this fact does not tell us anything about whether $(A_x/A_z) << (D/\lambda)$. For example, we might have for some transmission line that $(A_x/A_z) = 10^{-4}$ but $(D/\lambda) = 10^{-6}$ and then the << thing is false. So I guess I am arguing here that Method A is based on the claim that $(\partial_x A_x + \partial_y A_y) << (\partial_z A_z)$ but I know that this claim really can only be justified in the strong skin effect limit, and so Method A as stated above is an invalid argument. That is a good thing, since it does not require the skin limit.

Method B: From (1.3.1) we have $E_z = -\partial_z\varphi - \partial_t A_z$. We know that E_z is extremely small inside and just outside a conductor surface (and vanishes for a perfect conductor), so

$\partial_z\varphi \approx -\partial_t A_z$ => $-j\,\beta_d\varphi \approx -j\omega A_z$ => $\beta_d\varphi \approx \omega A_z$ => $A_z \approx (\beta_d/\omega)\varphi$

This is a valid argument, since E_z extremely small requires strong skin limit!

2. We then argue that for φ evaluated near one of the conductors in the active region between the conductors,

$$|\partial_x\varphi| \approx \frac{\Delta\varphi}{\Delta x} \approx \frac{V}{D} \approx (1/D)\,|\varphi| \qquad D = \text{general transverse dimension} \qquad (2)$$

3. We argue next, based on Appendix M and Fact 2 above :

$$|A_x| << |A_z|$$

and then setting $|A_z| \approx (\beta_d/\omega)\,|\varphi|$ according to item (1) above, we get

$$|A_x| << (\beta_d/\omega)\,|\varphi|$$

which we rewrite slightly,

$$\omega|A_x| << \beta_d\,|\varphi|$$

or

$$|\omega A_x| << (2\pi/\lambda)\,|\varphi| = 2\pi\,|\varphi|\,(1/\lambda) \qquad (3)$$

4. Now consider the transmission lime limit inequality,

$$\lambda >> D$$

$$(1/\lambda) << (1/D)$$

$$2\pi\,|\varphi|\,(1/\lambda) << 2\pi\,|\varphi|\,(1/D)\ . \qquad (4)$$

Thus from (3) and (4) and then finally (2)

$$|\omega A_x| << 2\pi\,|\varphi|\,(1/\lambda) << 2\pi\,|\varphi|\,(1/D) \approx 2\pi\,|\partial_x\varphi|\ .$$

From this we conclude, ignoring the 2π factor,

$$|\omega A_x| << |\partial_x\varphi| \qquad (5)$$

5. Finally, consider

$$E_x = -\partial_x\varphi - j\omega A_x\ .$$

Based on (5) , we can ignore the last term and we get

$$E_x \approx -\partial_x\varphi \qquad (6)$$

6. Making this same argument for $\partial_y\varphi$, we end up with

$$\mathbf{E}_t \approx -\nabla_t\,\varphi$$

which says that the transverse electric field $\mathbf{E}_t$ for a transmission line operating at frequency $\omega > 0$, but operating in the transmission line limit of $\lambda >> D$, is the same as the electrostatic electric field $\mathbf{E}_t$ obtained from the $\omega = 0$ DC capacitor problem. This is the basis of our "electro-quasi-static" approach to the

transmission line analysis. Thus for any $\omega > 0$ up to 1000 GHz the conductor cross section surfaces each have constant φ, just as they do at $\omega = 0$.

[On 5/8/14 I think the above 6 step proof is OK. The argument requires (1) the transmission line limit and (2) the strong skin effect regime and (3) Appendix M to get the conclusion.]

Comment: If we are at low ω, then the strong skin is not true, and the above proof fails, though the conclusion may still be true.

Commentary:

So above I have supposedly nailed down the idea that φ = constant on a conductor accurate to 10^{-4} at any ω including low and high frequencies. This would seem to validate the V(z) portion of Ch 4. Above I argue by Method A and Method B that $A_z = (\beta_d/\omega)\varphi + 10^{-4}$*stuff. Question: if φ = constant on conductor, doesn't this mean A_z= constant as well to this same 10^{-4} accurate at all ω ? So this would say A_z = constant at DC, which I KNOW is not the case. Something is confused here, as usual.

I think I have resolved this last issue.