Computation of Az for two cylinders case PhL 3.3.14

[5/14/14 final review done. No need to study this doc other than to read the summary. There are no field plots in this doc!]

[Lots of Chapter 6.5 data in this doc. Proximity effect! A LOT happened in this doc.]

Summary: In **Sections 1-8** I do the two-cylinder analysis which is now all written up in lines doc in Section 6.5, so no need to ponder these sections any more. Here is where I developed it.

Section 9 just shows you can write $K_z(\varphi) = \delta J_z(a,\varphi)$ if you want the extreme skin current as a surface current K_z.

Section 10 tries to compute A_z from the Helmholtz integral and leads to a messy $\sin\theta \cos\theta$ integral that is so ugly that I wrote a whole separate doc about it. This section is then a failure.

Section 11 relates $n(\theta)$ to J_z (or K_z) and this is in lines doc now in Section 6.5. I have this here in a simpler form $J_z(\theta,z) = (v_d/\delta)\, n(\theta,z)$ which now appears fancier in (6.5.21), but basically same idea. I refer to this equation as a "new continuity fact".

In **Section 12** I use $-j\omega A_z(\mathbf{x}) = E_z(\mathbf{x}) - j\beta_d\varphi(\mathbf{x})$ with my newly found $E_z(\mathbf{x}) = J_z/\sigma$ to produce an expression for A_z at the conductor surface, using also φ from the two-cylinder problem. I then get some expression for $A_z(a,\theta)$ which is a constant plus a θ dependent term.

Section 13 digresses on notational problems in bipolar.doc which I go fix

Section 14 continues to study the new A_z expression. I show that the θ-dependent 2nd term is negligible in the extreme skin regime, so A_z= constant on perimeter and King is justified! This was a big deal for me when I first did it. I then ponder paradoxes 1,2 and 3 which now are all resolved.

Section 15 on "hierarchy" is about different levels of a "good conductor" and this is now embodied in my low, strong and extreme δ limits of (3.7.3).

which is non-constant on the surface, though I seem to ignore that fact. Recall the title of this doc "finding A_z for two cylinders". In retrospect

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Reviewed this earlier, and then again on 5/9/14. There is a lot of good stuff here. Things happened here for the first time.

1. Formula to use

Statement of the relationship between $A_z(\mathbf{x})$ and $E_z(\mathbf{x})$ and $\varphi(\mathbf{x})$ for wave solution.

I will try to use this formula:

$$\mathbf{E} = -\text{ grad } \varphi - \partial_t \mathbf{A} \qquad (1.3.1)$$

$$E_z = -\partial_z \varphi - \partial_t A_z$$

$$E_z = j\beta_d \varphi - j\omega A_z$$

$$-j\omega A_z(\mathbf{x}) = E_z(\mathbf{x}) - j\beta_d \varphi(\mathbf{x})$$

2. Bipolar doc solution for φ

Statement $\varphi(\mathbf{x})$ for two-cylinders in bipolar coordinates.

My bipolar doc has these results:

$$\varphi(\xi) = -\frac{\xi}{\xi_2-\xi_1} V \qquad (10.8)$$

$$V_2 = -\frac{\xi_2}{\xi_2-\xi_1} V = -\frac{\xi_2}{\xi_2+|\xi_1|} V < 0$$

$$V_1 = -\frac{\xi_1}{\xi_2-\xi_1} V = +\frac{|\xi_1|}{\xi_2+|\xi_1|} V > 0 \ . \qquad (10.9)$$

$$\varphi(\xi) = -\frac{\xi}{\xi_2-\xi_1} V = -\frac{V}{2(\xi_2-\xi_1)} \ln\left[\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}\right] = -\frac{V}{2(\xi_2-\xi_1)} \ln\left[\frac{s_1^2}{s_2^2}\right] \qquad \textit{(10.8)} \qquad (10.18)$$

So I have the solution potential φ in lots of different forms.

3. Review of the moments η_m

Statement of the charge moments for the two-cylinder problem. I need to put this somewhere!

In "details of Chap 6" I compute from the above φ first E, then surface charge n, and then the moments and I find that

$$\eta_m \equiv N_m / N_0 = (-1)^m \exp(-|m|\xi_1)$$

$$n(\varphi)/N_0 = 1 + 2\,\Sigma_{m=1}^{\infty}\eta_m \cos(m\varphi) = 1 + 2\,\Sigma_{m=1}^{\infty}(-1)^m \exp(-|m|\xi_1)\cos(m\varphi)$$

and I plotted this thing for $\xi_1 = 0.25$ and it looked good.

4. Use moments to compute $E_z(r,\varphi)$

Compute E_z from η_m for two-cylinder problem.

Using these moments, I am able then to compute E_z

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\, (a\beta')\left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] \equiv K_{m,a}\,\eta_m$$

$$E_z(r,\varphi) = \sum_{m=-\infty}^{\infty} E_z(r,m)\, e^{jm\varphi} = \Sigma_m\, K_{m,a}\,\eta_m\, e^{jm\varphi}$$

So I can insert my explicit moments to get

$$E_z(r,m) = (1/4)\, I\, R_{dc}\, (a\beta')\left\{ (-1)^m \exp(-|m|\xi_1)\left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]\right\}$$

$$E_z(r,\varphi) = \sum_{m=-\infty}^{\infty} E_z(r,m)\, e^{jm\varphi}$$

$$= \sum_{m=-\infty}^{\infty} e^{jm\varphi} \quad (1/4)\, I\, R_{dc}\, (a\beta')\left\{ (-1)^m \exp(-|m|\xi_1)\left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]\right\}$$

5. Symmetry of f_m and value of f_0.

Show that $f_{-m} = f_m$.

Question: Define:

$$f_m \equiv \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]$$

Then

$$f_{-m} = \left[\frac{J_{-m}(x)}{J_{-m+1}(x_a)} - \frac{J_{-m}(x)}{J_{-m-1}(x_a)}\right]$$

But Spiegel p 136 tells me that

$$J_{-m}(x) = (-1)^m J_m(x) \qquad m = \text{integer}$$

which I recall from dim past. Then we get

$$J_{-m}(x) = (-1)^m J_m(x)$$

$$J_{-m+1} = (-1)^{m+1} J_{m-1}$$

$$J_{-m-1} = (-1)^{m+1} J_{m+1}$$

Then

$$f_{-m} = \left[\frac{J_{-m}(x)}{J_{-m+1}(x_a)} - \frac{J_{-m}(x)}{J_{-m-1}(x_a)}\right]$$

$$= -\left[\frac{J_m(x)}{J_{m-1}(x_a)} - \frac{J_m(x)}{J_{m+1}(x_a)}\right] = f_m$$

which is a good result

$$f_{-m} = f_m$$

Also, while we are at it

$$f_0 = \left[\frac{J_0(x)}{J_1(x_a)} - \frac{J_0(x)}{J_{-1}(x_a)}\right] = \left[\frac{J_0(x)}{J_1(x_a)} + \frac{J_0(x)}{J_1(x_a)}\right] = 2\frac{J_0(x)}{J_1(x_a)}$$

6. Final formula for $E_z(r,\varphi)$.

State simplified sum for E_z in terms of the moments.

Now go back to

$$E_z(r,\varphi) = \sum_{m=-\infty}^{\infty} E_z(r,m)\, e^{jm\varphi}$$

$$= \sum_{m=-\infty}^{\infty} e^{jm\varphi} \quad (1/4)\, I\, R_{dc}\ (a\beta')\ \{\ (-1)^m \exp(-|m|\xi_1)\left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right]\ \}$$

$$= (1/4)\, I\, R_{dc}\ (a\beta') \sum_{m=-\infty}^{\infty} e^{jm\varphi}\ \{\ (-1)^m \exp(-|m|\xi_1)\, f_m\ \}$$

To reflect the negative part of the sum, write

$$\Sigma_{m=-\infty}^{\infty} e^{jm\varphi} (-1)^m \exp(-|m|\xi_1) f_m$$

$$= f_0 + \Sigma_{m=-\infty}^{-1} e^{jm\varphi} (-1)^m \exp(-|m|\xi_1) f_m + \Sigma_{m=1}^{\infty} e^{jm\varphi} (-1)^m \exp(-|m|\xi_1) f_m$$

$$= f_0 + \Sigma_{m=1}^{\infty} e^{jm\varphi} (-1)^m \exp(-|m|\xi_1) f_m + \Sigma_{m=1}^{\infty} e^{-jm\varphi} (-1)^m \exp(-|m|\xi_1) f_{-m}$$

$$= f_0 + \Sigma_{m=1}^{\infty} (-1)^m \exp(-|m|\xi_1) f_m [e^{jm\varphi} + e^{-jm\varphi}]$$

$$= f_0 + 2 \Sigma_{m=1}^{\infty} (-1)^m \exp(-|m|\xi_1) f_m \cos(m\varphi) .$$

Then we have

$$E_z(r,\varphi) = (1/4) I R_{dc} (a\beta') [f_0 + 2 \Sigma_{m=1}^{\infty} (-1)^m \exp(-|m|\xi_1) f_m \cos(m\varphi)]$$

$$= (1/4) I R_{dc} (a\beta') \{ [2 \frac{J_0(x)}{J_1(x_a)}] + 2 \Sigma_{m=1}^{\infty} (-1)^m \exp(-|m|\xi_1) [\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}] \cos(m\varphi) \}$$

$$= (1/4) I R_{dc} (a\beta') \{ [2 \frac{J_0(x)}{J_1(x_a)}] + 2 \Sigma_{m=1}^{\infty} \eta_m [\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}] \cos(m\varphi) \}$$

$$= I R_{dc} \{ (x_a/4) [2 \frac{J_0(x)}{J_1(x_a)}] + (1/2) x_a \Sigma_{m=1}^{\infty} \eta_m [\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}] \cos(m\varphi) \}$$

$$= I R_{dc} \{ (x_a/4) f_0 + (1/2) x_a \Sigma_{m=1}^{\infty} \eta_m f_m \cos(m\varphi) \}$$

This is something I could plot! The r dependence is in x = β'r.

7. What value of β' should be used?

Claim can set $\beta' = \beta = (j-1)\, 10\, (1/a)$ for $1/10^{th}$ skin depth.

From "current asymmetry issue" I have

$$\beta = e^{j3\pi/4} (\sqrt{2}/\delta)$$

$$\beta_d = \beta_{d0} [1 - j (1/2)\tan_L] \qquad \beta_{d0} = (\omega/v_d)$$

The main observation is that β_d is mainly real with a small negative imaginary part. Also,

$$|\beta_d| \approx |\beta_{d0}| = (\omega/v_d)$$

I shows that if we are in the skin effect regime of $\delta = a/10$ or better, then β_d is about 1/22.5 of β in abs value. I could put in the exact stuff, but for the moment, I will assume $\beta' = \beta$. So then

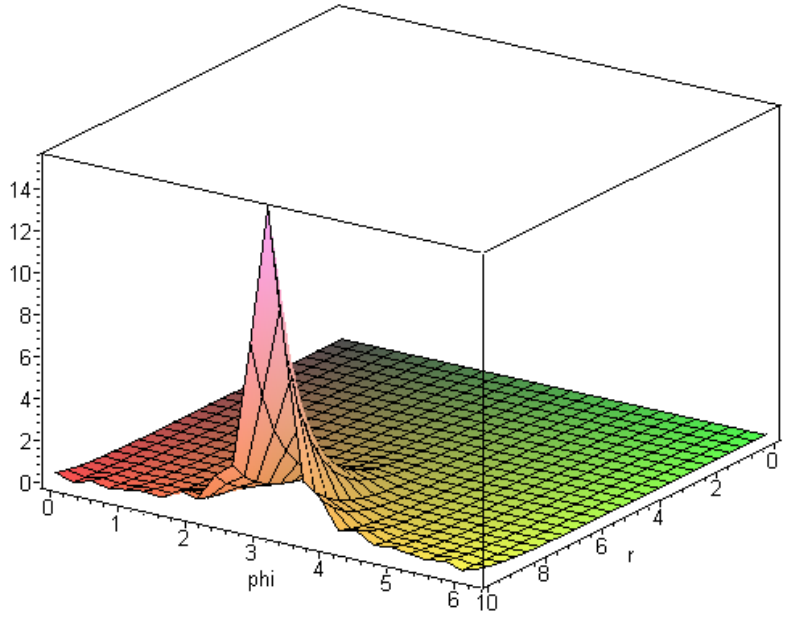
$$\beta' = \beta = e^{j3\pi/4}\, 10\, (\sqrt{2}\,/a) \quad = (j\text{-}1)\, 10\, (1/a)$$

where I will work directly with $\delta = a/10$ as a specific example. Perhaps make this a/N with variable N.

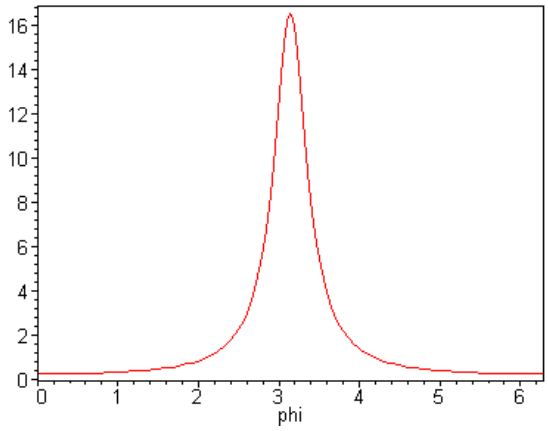
8. Maple program to compute $E_z(r,\theta)$.

Maple plots of the sum for E_z derived above. See that J_z is indeed "asymmetric".

The program was quick to write, I keep 10 terms, set $\delta = a/10$ and here is a plot of the result,

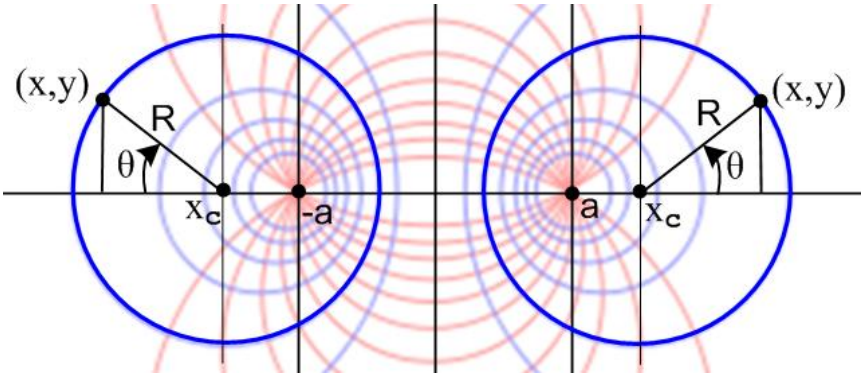


Here I keep 20 terms and plot $E_z(a,\theta)$,



It sure looks like n(φ) !!!! Is there some more direct way to get from n(φ) to $E_z(a,\varphi)$? See Section 11 below.

You see that most current ($J_z = \sigma E_z$) is near the surface which is r = 10 as expected, and you see that the current is strongly peaked at φ = π, as I have always thought !! Recall how φ = θ is defined:



$$J_z(r,\varphi) = \sigma\,(1/4)\,I\,R_{dc}\,(a\beta')\,[f_0 + 2\,\Sigma_{m=1}^{\infty}\,(-1)^m \exp(-|m|\xi_1)\,f_m \cos(m\varphi)\,]$$

So angle θ = φ, and I am dealing with the left conductor where φ is CCW which I don't think is a problem.

Now I should be able to use this J_z expression to compute A_z. I have to assume this in each of the two conductors. This is likely to be a heavy duty numeric integration. Maybe I should first go extreme skin effect and try to make it a 1D integration. The above says

$$J_z(r,\varphi) = \sigma\,(1/4)\,I\,R_{dc}\,(a\beta')\,[f_0 + 2\,\Sigma_{m=1}^{\infty}\,(-1)^m \exp(-|m|\xi_1)\,f_m \cos(m\varphi)\,]$$

$$f_m \equiv \left[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\right] \qquad f_0 = 2\,\frac{J_0(x)}{J_1(x_a)}$$

How do I turn the expo decay surface layer into a true surface current?

[Note added 5/9/14. I think I dropped the ball right here. I had just computed E_z above to be

$$E_z(r,\varphi) == (1/4)\,I\,R_{dc}\,(a\beta')\;\{\,[2\,\frac{J_0(x)}{J_1(x_a)}\,] + 2\,\Sigma_{m=1}^{\infty}\,\eta_m\,[\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}\,]\cos(m\varphi)\,\}$$

where [] = f_0. Perhaps nicer to write this as in my review of paradox a and 2,

$$E_z(r,\theta) = I\,R_{dc}\,(1/4)\,x_a\,[\,f_0 + 2\sum_{m=1}^{\infty}\eta_m\,f_m(x,x_a)\cos(m\theta)\,] \qquad x_a = \beta' a$$

So I laboriously computed this $E_z(r,\theta)$ and I failed to continue on to get A_z from:

$$E_z = j\beta_d\varphi - j\omega A_z \qquad \Rightarrow \qquad j\omega A_z = j\beta_d\varphi - E_z$$

where E_z as above and φ as

$$\varphi(\xi) = -\frac{\xi}{\xi_2-\xi_1} V = -\frac{V}{2(\xi_2-\xi_1)} \ln\left[\frac{(x+a)^2 + y^2}{(x-a)^2 + y^2}\right] = -\frac{V}{2(\xi_2-\xi_1)} \ln\left[\frac{s_1^2}{s_2^2}\right] \qquad (10.8) \qquad (10.18)$$

which I could of course write n (r,θ) coordinates for the left wire, say, to match E_z's coordinates. But then we have I appearing in E_z and we have V appearing in φ and since $V/I = Z_0$ this brings in Z_0 which has a hazy evaluation although a clear definition here as shown. But I think what I would find if I carried through here is that E_z, though highly variable (as the plot above shows), is still always quite small (in the strong skin limit) and so A_z would then basically follow φ according to $j\beta_d\varphi - j\omega A_z = 0$ or $A_z = (1/v_d)\varphi$. This subject is explored more in Review of Paradox 1 and 2.doc.

Instead of computing A_z as just described, below I start with J_z and try to get A_z using the Helm integral!]

9. How to get surface current K_z from J_z

How to represent skin depth current J_z as a purely surface current K_z.

Suppose $J_z(r) = e^{-r/\delta}$. Since δ is very small, we just treat this as a 1D problem and say

$$K = \int_0^\infty e^{-r/\delta}\, dr = \delta\, J_z(0)$$

So in our case this translates to

$$K_z(\varphi) = \delta\, J_z(a,\varphi)$$

and there is our surface current which is a function of φ.

10. Attempt computation of A_z

Here I try to compute A_z for the two-cylinder problem *using* the King Helmholtz integral method as developed in lines Ch 4, making use of a pure surface current <u>in the extreme skin depth limit</u>, and I end up with this rather intractable result:

$$-A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi} i(z) \left\{ \int_0^{2\pi} d\theta_1\, a\, K_z(\pi-\theta_1) \ln[x^2 + y^2 + a_1^2 - 2xa_1\cos\theta_1 - 2ya_1\sin\theta_1] \right.$$
$$\left. - \int_0^{2\pi} d\theta_2\, a\, K_z(\theta_2) \ln[(x-b)^2 + y^2 + a_1^2 - 2(x-b)\, a_1\cos\theta_1 - 2ya_1\sin\theta_1] \right\}$$

$$K_z(\theta) = \delta\, J_z(a,\theta)$$

$$J_z(a,\theta) = \sigma\,(1/4)\,I\,R_{dc}\,(a\beta')\,[f_0 + 2\,\Sigma_{m=1}^{\infty}\,(-1)^m \exp(-|m|\xi_1)\,f_m \cos(m\theta)\,] \qquad \text{with } x \to x_a$$

The required integral here is then

$$\int_0^{2\pi} d\theta \cos(m\theta) \ln [A + B\sin\theta + C\cos\theta]$$

I think I can do this by converting all to expo notation and then letting $z = e^{j\theta}$ and doing it as a contour integral, after first doing parts. I am used to the King method giving horrible integrals. I show in a separate doc how to do this integral as a contour integral in Maple and for each m we get a simple analytic function for this integral, so in theory I can actually compute $A_z(x,y)$ at any point for the two cylinder problem, but I have not carried out that task.

Analytically this requires some fancy dθ integrals

Now consider from lines doc,

$$A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi}\, i(z) \int_{-\infty}^{\infty} dz' \{ \int_{C_1} dx_1'\, dy_1'\, b_1(x_1',y_1') \frac{1}{R_1} - \int_{C_2} dx_2'\, dy_2'\, b_2(x_2',y_2') \frac{1}{R_2} \} \qquad (4.9.1)$$

$$R_1^2 = (x-x_1')^2 + (y-y_1')^2 + (z-z')^2 = s_1^2 + (z-z')^2 \qquad s_1^2 = (x-x_1')^2 + (y-y_1')^2 \qquad (4.8.1)$$
$$R_2^2 = (x-x_2')^2 + (y-y_2')^2 + (z-z')^2 = s_2^2 + (z-z')^2 \qquad s_2^2 = (x-x_2')^2 + (y-y_2') \ .$$

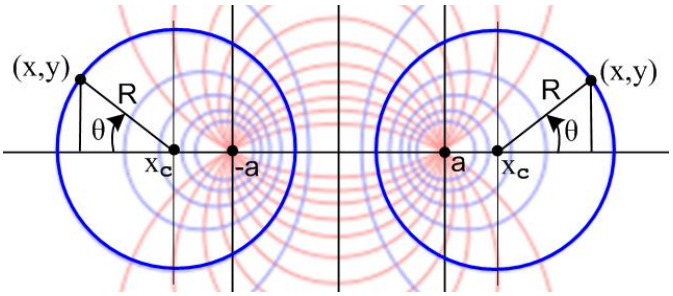
Let's replace

$$\int_{C_1} dx_1'\, dy_1'\, b_1(x_1',y_1') = \int_0^{2\pi} d\theta_1'\, a\, K_z(\theta'_1)$$

Then we have

$$A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi} \int_{-\infty}^{\infty} dz'\, i(z') \{ \int_0^{2\pi} d\theta_1'\, a\, K_z(\theta'_1) \frac{1}{R_1} - \int_0^{2\pi} d\theta_2'\, a\, K_z(\theta'_2) \frac{1}{R_2} \}$$

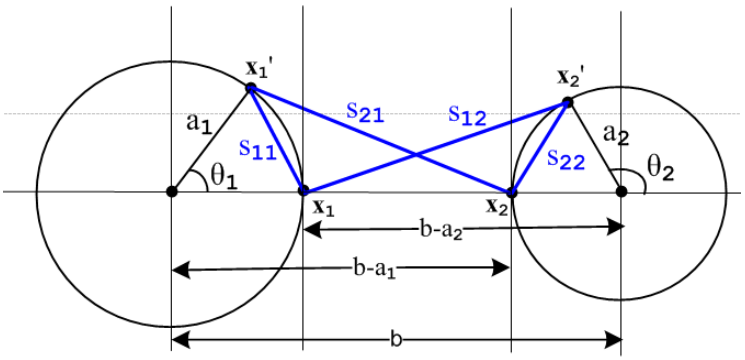
where we have to be a little careful now. Recall again,



The angle θ'_1 is the θ shown on the left, and θ'_2 is the θ shown on the right. Let's rewrite things this way

$$A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi} \int_{-\infty}^{\infty} dz'\, i(z') \left\{ \int_0^{2\pi} d\theta_1\, a\, K_z(\pi - \theta_1) \frac{1}{R_1} - \int_0^{2\pi} d\theta_2\, a\, K_z(\theta_2) \frac{1}{R_2} \right\}$$

where I am now using this *only* to show the two angles θ_1 and θ_2



But I need now a different picture in order to see R_1 and R_2 . But luckily I have this one

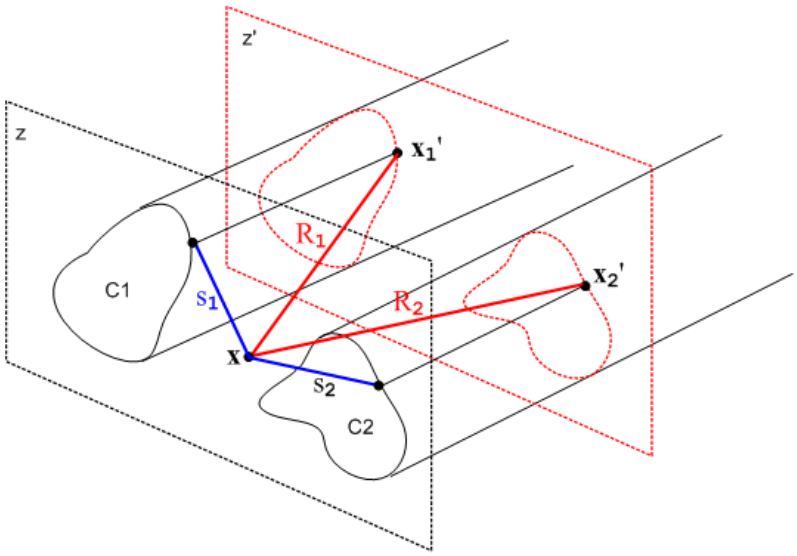


Fig 4.1

where **x** is at some arbitrary location. I know that

$$\int_{-\infty}^{\infty} dz' \frac{1}{R_1} = -2 \ln(s_1/\Lambda) \qquad \text{(J.10)}$$

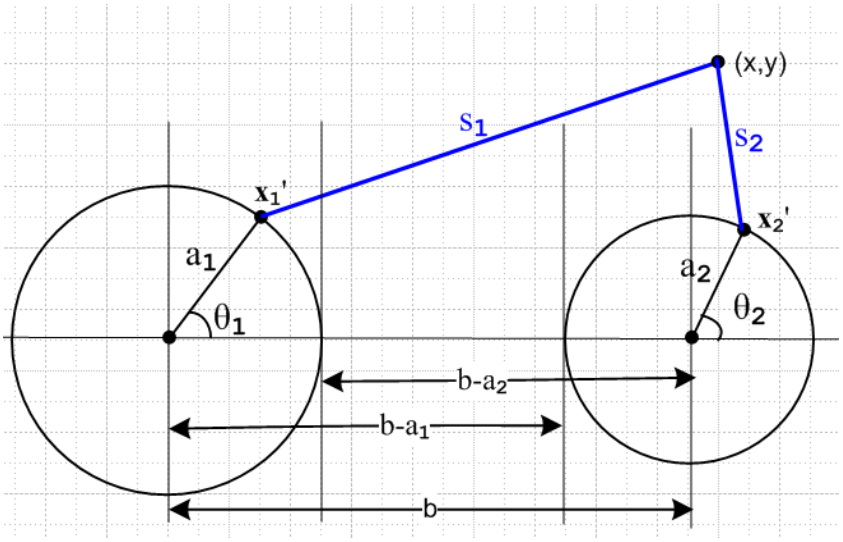
So also taking out i(z) in the usual way, we get

$$A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi} i(z) \left\{ \int_0^{2\pi} d\theta_1\, a\, K_z(\pi - \theta_1) [-2 \ln(s_1/\Lambda)] - \int_0^{2\pi} d\theta_2\, a\, K_z(\theta_2) [-2 \ln(s_2/\Lambda)] \right\}$$

where

$$s_1^2 = (x-x_1')^2 + (y-y_1')^2$$
$$s_2^2 = (x-x_2')^2 + (y-y_2')^2$$

But now I have to draw a NEW transverse picture in order to understand these distances:



Now

$$x'_1 = a_1 \cos\theta_1$$
$$y'_1 = a_1 \sin\theta_1$$

$$x'_2 = a_2 \cos\theta_2 + b$$
$$y'_2 = a_2 \sin\theta_2$$

So our distances are quite messy

$$s_1^2 = (x-x_1')^2 + (y-y_1')^2 = (x- a_1 \cos\theta_1)^2 + (y- a_1 \sin\theta_1)^2$$

$$s_2^2 = (x-x_2')^2 + (y-y_2')^2 = (x- a_2 \cos\theta_2 - b)^2 + (y- a_2 \sin\theta_2)^2$$

So we then have

$$- A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi} i(z) \left\{ \int_0^{2\pi} d\theta_1 \, a \, K_z(\pi - \theta_1) \ln(s_1/\Lambda)^2 - \int_0^{2\pi} d\theta_2 \, a \, K_z(\theta_2) \ln(s_2/\Lambda)^2 \right\}$$

I know I can just set $\Lambda = 1$, so we still have this huge mess

$$-A_{z12}(\mathbf{x}) = \frac{\mu}{4\pi} i(z) \left\{ \int_0^{2\pi} d\theta_1\, a\, K_z(\pi - \theta_1) \ln[(x - a_1\cos\theta_1)^2 + (y - a_1\sin\theta_1)^2] \right.$$
$$\left. - \int_0^{2\pi} d\theta_2\, a\, K_z(\theta_2) \ln[(x - a_2\cos\theta_2 - b)^2 + (y - a_2\sin\theta_2)^2] \right\}$$

$$K_z(\varphi) = \delta\, J_z(a,\varphi)$$

$$J_z(a,\varphi) = \sigma\, (1/4)\, I\, R_{dc}\, (a\beta') \left[f_0 + 2 \sum_{m=1}^{\infty} (-1)^m \exp(-|m|\xi_1)\, f_m \cos(m\varphi)\right] \qquad \text{with } x \to x_a$$

Now maybe we can simplify a bit,

$$s_1^2 = (x-x_1')^2 + (y-y_1')^2 = (x - a_1\cos\theta_1)^2 + (y - a_1\sin\theta_1)^2$$

$$= x^2 + a_1^2\cos\theta_1^2 - 2xa_1\cos\theta_1 + y^2 + a_1^2\sin\theta_1^2 - 2ya_1\sin\theta_1$$

$$= x^2 + a_1^2 - 2xa_1\cos\theta_1 + y^2 - 2ya_1\sin\theta_1$$

$$= x^2 + y^2 + a_1^2 - 2xa_1\cos\theta_1 - 2ya_1\sin\theta_1$$

$$s_2^2 = (x-x_2')^2 + (y-y_2')^2 = (x - a_2\cos\theta_2 - b)^2 + (y - a_2\sin\theta_2)^2$$

just replace x by x-b everywhere and 1→2 so

$$s_2^2 = (x-b)^2 + y^2 + a_1^2 - 2(x-b)\, a_1\cos\theta_1 - 2ya_1\sin\theta_1$$

The general form is

$$s^2 = A + B\sin\theta + C\cos\theta$$

so to do this stuff analytically we would need this integral

$$\int_0^{2\pi} d\theta \cos(m\theta) \ln[A + B\sin\theta + C\cos\theta]$$

I don't offhand see this in GR7, but Maple can do the integral for m = 1,2,3 and results are of a poly nature with increasing complexity of the result as m increases.

I think a numerical approach would be better.

Another approach would be to try this all directly in bipolar coordinates! Actually, I think that is going to work pretty well.

11. Getting J_z directly from n(φ) ?

[I added the (j+1) factors below on 5.20.14]

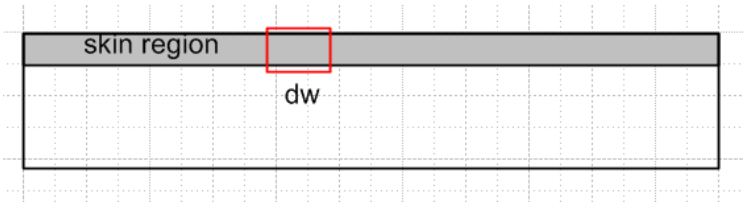
This is the big new realization for working in the skin depth limit:

$J_z(\varphi,z) = (\omega/\beta_d)\, n(\varphi,z)/\delta = (v_d/\delta)\, n(\varphi,z)$ dim OK

$K_z(\varphi,z) = v_d\, n(\varphi,z)$ dim OK

This shows that indeed J_z is asymmetrical on the two-cylinder cross section since n is asymmetrical.

Consider a Gaussian box where z is perpendicular to paper



Imagine some $J_z(r,\varphi)$ which characterize by

$$K_z(\varphi,z) = \delta\, J_z(a,\varphi,z)$$

where I now show a z argument. [K_z is not the Debye surface current, it is the total surf current]

The **front and back sides** (other sides added in later) of the above box give

$$\int J \bullet dS = [\, J_z(\varphi,z+dz) - J_z(\varphi,z)]\, dw\, \delta = -j\omega\, Q_{enclosed} = -j\omega\, n(\varphi,z)\, dw\, dz$$

[perhaps I am using the average values of J_z on front and back; well, as shown in "continuity at conductor surfaces", the integrals of J_z over the front and back sides creates 1/(j+1) due to the assumed skin depth model of $J_x(y) = J_x(0)e^{-(j+1)y/\delta}$ so then

$$[\, J_z(\varphi,z+dz) - J_z(\varphi,z)]\, dw\, [\delta/(j+1)] = -j\omega\, n(\varphi,z)\, dw\, dz$$

$$\partial_z J_z(\varphi,z)\, [\delta/(j+1)] = -j\omega\, n(\varphi,z)$$

$$-j\beta_d J_z(\varphi,z)\, [\delta/(j+1)] = -j\omega\, n(\varphi,z)$$

$$\beta_d J_z(\varphi,z)\, [\delta/(j+1)] = \omega\, n(\varphi,z)$$

$J_z(\varphi,z) = (\omega/\beta_d)\, n(\varphi,z)(j+1)/\delta = (j+1)(v_d/\delta)\, n(\varphi,z)$ dim OK

$K_z(\varphi,z) = (j+1)v_d\, n(\varphi,z)$ dim OK

Something seems fishy here. We know that n "really is on the surface itself" whereas J_z has some thickness to it, but you see the derivation here! This then explains why $J_z(\varphi,z)$ looks like $n(\varphi,z)$ when I plot it above. This simple result could be a major breakthrough for The Dim Wit.

But what about the two other pairs of box faces? The top and bottom faces involve E_r (area dwdz) while the left and right involve E_θ (area δ dz). If we include these faces, we then have

$$\int J \bullet dS = [\, J_z(\theta,z+dz) - J_z(\theta,z)]\, dw\, [\delta/(j+1)] + [\, 0 - 0]\, dwdz + [\, J_\theta(\theta+d\theta,z) - J_\theta(\theta,z)]\, [\delta/(j+1)]\, dz$$

[this is wrong because the top of the box is outside the conductor in the non-conducting dielectric and so the term shown in red above should be 0. Bottom of box is below skin layer so ≈ 0 there as indicated.]

This is then set equal to -jω n(φ,z) dw dz so we get

$$-j\omega\, n(\theta,z)\, dw\, dz = [\, J_z(\theta,z+dz) - J_z(\theta,z)]\, dw\, [\delta/(j+1)] + [0 - 0]\, dwdz + [\, J_\theta(\theta+d\theta,z) - J_\theta(\theta,z)]\, [\delta/(j+1)]\, dz$$

where write dw = adθ say.

$$-j\omega\, n(\theta,z)\, ad\theta\, dz = [\, J_z(\theta,z+dz) - J_z(\theta,z)]\, ad\theta\, [\delta/(j+1)] + [0 - 0]\, ad\theta\, dz + [\, J_\theta(\theta+d\theta,z) - J_\theta(\theta,z)]\, [\delta/(j+1)]\, dz$$

Now divide by dθ

$$-j\omega\, n(\theta,z)\, a\, dz = [\, J_z(\theta,z+dz) - J_z(\theta,z)]\, a\, [\delta/(j+1)] + [0 - 0]\, a\, dz + [\, J_\theta(\theta+d\theta,z) - J_\theta(\theta,z)]/d\theta\, [\delta/(j+1)]\, dz$$

Then divide by dz

$$-j\omega\, n(\theta,z)\, a = [\, J_z(\theta,z+dz) - J_z(\theta,z)]/dz\, a\, [\delta/(j+1)] + [0 - 0]\, a + [\, J_\theta(\theta+d\theta,z) - J_\theta(\theta,z)]/d\theta\, [\delta/(j+1)]$$

Then write as

$$-j\omega\, n(\theta,z)\, a = \partial_z J_z\, a\, [\delta/(j+1)] + 0\, a + \partial_\theta J_\theta\, [\delta/(j+1)]$$

$$-j\omega\, n(\theta,z) = \partial_z J_z [\delta/(j+1)] + 0 + \partial_\theta J_\theta\, (j+1)(\delta/a)$$

$$-j\omega\, n(\theta,z) = -j\beta_d\, J_z\, [\delta/(j+1)] + 0 + \partial_\theta J_\theta\, (j+1)(\delta/a) = 0 + [\delta/(j+1)]\, [\partial_\theta J_\theta\, (1/a) - j\beta_d\, J_z\,]$$

Note added 3/6/14. The corrected version of the above is (but I have already corrected it in place)

$$-j\omega\, n(\theta,z) = [\delta/(j+1)]\, [\partial_\theta J_\theta\, (1/a) - j\beta_d\, J_z\,]$$

This is the cylinder version of Case 1 "deep box" from "continuity at conductor surface", which says:

$$-j\omega\, n(x,0,z) = [\delta/(j+1)]\, [\partial_x J_x(x,0,z) + \partial_z J_z(x,0,z)]\ .$$

Now, I have claimed that $E_\theta = 0$ on the surface based on the idea that φ = constant on ring AND on the assumption that $A_x = A_y = 0$ (valid roughly for strong skin effect). This was one of my two Appendix D boundary conditions. If I assume this is true here not just at the surface but on average over depth δ (remember we have a deep box), then we must have $\partial_\theta J_\theta = 0$, and then I get this simple result

$$-j\omega\, n(\theta) = [\delta/(j+1)]\,[-j\beta_d\, J_z(a,\theta)] \qquad \Rightarrow J_z(a,\theta) = (j+1)(\omega/\delta\beta_d)\, n(\theta) = (j+1)(v_d/\delta)\, n(\theta,z)$$

This is the result I originally obtained, and then later decided was not valid, and now I think it is valid again! Typical of how things progress with me!

[5.14.14. I fancied up the work above in lines doc, but the above result is basically correct.]

Question: Given the above, does that not alter my charge pumping boundary condition? Suppose in the above picture I add a certain J_r coming up from the bottom. Then I would have

$$\int J \bullet dS = [\, J_z(\varphi,z+dz) - J_z(\varphi,z)]\, dw\, [\delta/(j+1)] - J_r\, dw\, dz = -j\omega\, Q_{enclosed} = -j\omega\, n(\varphi,z)\, dw\, dz$$

This is then set equal to $-j\omega\, n(\varphi,z)\, dw\, dz$ so we get

$$-j\omega\, n(\varphi,z)\, dw\, dz =$$

and then

$$[\, J_z(\varphi,z+dz) - J_z(\varphi,z)]\, dw\, [\delta/(j+1)] - J_r\, dw\, dz = -j\omega\, n(\varphi,z)\, dw\, dz$$

$$[\, J_z(\varphi,z+dz) - J_z(\varphi,z)]\, [\delta/(j+1)] - J_r\, dz = -j\omega\, n(\varphi,z)\, dz$$

$$\partial_z J_z(\varphi,z)\, [\delta/(j+1)] - J_r = -j\omega\, n(\varphi,z)$$

$$-j\beta_d\, J_z(\varphi,z)\, [\delta/(j+1)] - J_r(a-\delta,\varphi,z) = -j\omega\, n(\varphi,z)$$

Now what? This is my new charge pumping condition! But,

$$(\beta_d\, \delta) = 2\pi\, \delta/\lambda = 2\pi(\delta/\lambda) = \text{very small}$$

So maybe this scales down the large J_z and makes it be a similar size to J_r and then we must keep both terms! But something is still wrong with this box! If I pancake it down a whole lot, that should exclude all of the J_z current but I don't see that happening here.

Well, the J_r appearing in my last equations is really $J_r(a-\delta,\varphi,z)$ and this is still far from the surface. I think my charge pump condition then is still OK.

Note added 3/6/14. I show in "continuity at conductor surface" Case 2 and Case 3 that the charge pumping BC survives all this swirling around, with or without Debye surface currents! So the last conclusion just stated here is correct,

12. Implications of this new continuity fact.

Review of this section: Since $J_z(\varphi,z) = (j+1)(v_d/\delta)\, n(\varphi,z)$ from previous section in skin limit, I know $E_z(\varphi,z) = (j+1)(v_d/\delta\sigma)\, n(\varphi,z)$ on the surface. Then from the E,φ,A equation I can compute A_z everywhere on the surface and I end up with (using bipolar computation of n(θ))

$$-j\omega A_z(a,\theta) = V_1 \left[j\beta_d + (j+1)(v_d/\sigma\delta)\ \varepsilon/(R_1\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right]$$

which I will further process in Section 14 below.

I have shown that in the skin effect limit,

$J_z(\varphi,z) = (j+1)(v_d/\delta)\, n(\varphi,z)$ dim OK from above

and when I replace the z argument with the r argument this says,

$J_z(a,\varphi) = (j+1)\,(v_d/\delta)\, n(\varphi) = \sigma E_z(a,\varphi)$

Therefore

$E_z(a,\varphi) = (j+1)\,(v_d/\sigma\delta)\, n(\varphi)$

But now go back to this equation, where we now take x to be a point on the conductor surface

$-j\omega A_z(\mathbf{x}) = E_z(\mathbf{x}) - j\beta_d\varphi(\mathbf{x})$

$-j\omega A_z(a,\varphi) = E_z(a,\varphi) - j\beta_d\varphi(a,\varphi)$

But $\varphi(a,\varphi) = V_1$ since I assume we are talking conductor C_1. Then

$-j\omega A_z(a,\varphi) = E_z(a,\varphi) - j\beta_d V_1$

or

$-j\omega A_z(a,\varphi) = (j+1)\,(v_d/\sigma\delta)\, n(\varphi) - j\beta_d V_1$

Dimension check:

dim(LHS) = (1/sec)* (volt-sec/m) = volt/m from (1.3.1)

$dim(RHS_2)$ = (1/m)*volt = OK

$dim(RHS_1)$ = (m/sec)*(ohm-m)*(1/m) * (Cou/m^2) = (1/sec)*(ohm) * (Cou/m) = Amp-ohm/m = V/m so OK

So for the first time ever, I have an expression for $A_z(a,\varphi)$ on the surface of conductor C_1 . I also know

$$dQ_1(\xi_1,\theta) = \left[\frac{q}{2\pi}\, dz\, d\theta\right] \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \quad . \qquad \text{// dim RHS} = (\text{cou/m})\ \text{m} = \text{cou}, \qquad \text{// (10.27)}$$

But

$$\varphi(\xi) = -\frac{q}{2\pi\varepsilon}\xi \ , \qquad (10.16)$$

so

$$V_1 = \varphi(\xi_1) = -\frac{q}{2\pi\varepsilon}\xi_1 \qquad \Rightarrow \qquad \frac{q}{2\pi} = -\varepsilon\, V_1/\xi_1$$

and then

$$dQ_1(\xi_1,\theta) = -\varepsilon\, dz\, d\theta\, V_1/\xi_1 \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \qquad \text{// dim(RHS)} = (\text{farad/m})\ \text{m volt} = \text{coul}$$

But

$$dQ_1(\xi_1,\theta) = n_1(\theta)\, dA = n_1(\theta) R_1 d\theta\, dz \qquad \text{// 10.27}$$

which says

$$n_1(\theta) R_1 d\theta\, dz = -\varepsilon\, dz\, d\theta\, V_1/\xi_1 \frac{|sh\xi_1|}{ch\xi_1+\cos\theta}$$

$$n_1(\theta) R_1 = -\varepsilon\, V_1/\xi_1 \frac{|sh\xi_1|}{ch\xi_1+\cos\theta}$$

and then

$$-j\omega A_z(a,\theta) = (j+1)\,(v_d/\sigma\delta)\, n(\theta) - j\beta_d V_1$$

$$= -(j+1)\,(v_d/\sigma\delta)\,\varepsilon\, V_1/(R_1\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} - j\beta_d V_1$$

or

$$j\omega A_z(a,\theta) = V_1 \left[j\beta_d + (j+1)\,(v_d/\sigma\delta)\ \varepsilon/(R_1\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right]$$

So this is another first! I wonder what the relative sizes of the two terms might be?

Dimension check:

$$\dim[\text{first term}] = \dim(\beta) = 1/\text{m}$$

dim[2nd term] = m /sec* m-ohm * 1/m * (farad/m)*(1/m)
= 1 /sec* ohm * 1/m * (farad) = 1 /sec* sec * 1/m = 1/m

13. Dimension problems in bipolar doc

I realized just above that I had some bad notation (wrong in fact) in bipolar doc, and I will fix that soon, [DONE] the changes are listed in a separate working doc. It has to do with n versus Q.

Let's just review a few equations:

$$\varphi(\xi) = -\frac{q}{2\pi\varepsilon}\xi \ , \qquad (10.16)$$

LHS = volts q = Coul/m ε = farad/m
RHS = (Coul/m) /(farad/m) = Coul / Farad = volt so OK

Next look at (10.11)

$$n(\xi_1,u) = \varepsilon \frac{V}{h(\xi_1,u)} \frac{1}{\xi_2-\xi_1} \qquad \text{// surface charge density on the left conductor } C_1 \qquad (10.11)$$

RHS = (farad/m)*volts = Coul/m = linear surface charge, not 2D surface charge. WRONG

I see I have to make some fixes to bipolar doc on this issue. I have n standing for both linear charge and for 2D charge density. // Wrong, no fixes were necessary, all dimensions are correct, and variable n is indeed charge/m^2. I studied this carefully in a separate working doc. My mistake is on the above line where I forgot that h contains a! so

RHS = (farad/m)*volts*(1/m) = Coul/m^2 = correct for surface charge.

OK, I will create an edit list for updating bipolar with cleaner notation.

14. Resume on new A_z expression

Here I simplify my Section 12 result for A_z on the surfaces of the two cylinders. The result is the following:

$$A_z(R_1,\theta) = V_1(1/v_d)\ [\ 1\ -\ j\ (j+1)\ (\delta/2R_1) * (1/\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta}\ ]$$

The second term has the θ dependence. You see that in the skin depth limit ($\delta/2R_1$) << 1 and so the second term becomes small and we get - $A_z(R_1,\theta)$ = constant, consistent with King's assumption.

$$j\omega A_z(a,\theta) = V_1 \left[j\beta_d + (j+1)(v_d/\sigma\delta) \;\; \varepsilon/(R_1\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right] \qquad v_d = \omega/k = \omega/\beta_d$$

$$jA_z(a,\theta) = V_1 \left[j(\beta_d/\omega) + (j+1)\,(v_d/\omega\sigma\delta) \;\; \varepsilon/(R_1\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right] \qquad v_d = \omega/k = \omega/\beta_d$$

$$jA_z(a,\theta) = V_1 \left[j\,(1/v_d) + (j+1)\,(v_d/\omega)(1/(\sigma\delta)) \;\; \varepsilon/(R_1\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right]$$

$$jA_z(a,\theta) = V_1 \left[j\,(1/v_d) + (j+1)\,(1/(\beta_d\sigma\delta)) \;\; \varepsilon/(R_1\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right]$$

$$jA_z(R_1,\theta) = V_1 \left[j\,(1/v_d) + (j+1)\varepsilon/(\beta_d\sigma\delta R_1\xi_1) * \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right] \qquad \delta \equiv \sqrt{2/(\omega\mu\sigma)}$$

The second term is the E_z term, by the way, while the first term comes from the φ term. Again, I wonder about the relative size of these two terms! Another dim check

V_1 / v_d = volts sec/m = correct

$V_1\, \varepsilon / \beta_d\, \sigma\, \delta\, R_1$ = volt * (farad/m)*m*ohm-m*(1/m²) = volt * (farad/m)*ohm = volt sec/m

So at least both sides have the same dimensions

Now recall that $R_1 = a/|sh\xi_1|$, but that does not buy me much and then two meanings for a!

Here is the ratio of the two term coefficients

$$rat = (\text{first/second}) = \text{first} * (1/\text{second}) = \frac{\beta_d\sigma\delta R_1}{\varepsilon v_d} \qquad \delta^2 = 2/(\omega\mu\sigma) \qquad \omega\mu\sigma = 2/\delta^2$$

$$rat^2 = \frac{\beta_d^2\sigma^2\delta^2 R_1^2}{\varepsilon^2 v_d^2}$$

Now replace $\delta^2 = \dfrac{2}{\omega\mu\sigma}$ to get

$$rat^2 = \frac{2\beta_d^2\sigma^2 R_1^2}{\varepsilon^2 v_d^2 \omega\mu\sigma} = \frac{2\beta_d^2\sigma R_1^2}{\varepsilon(\varepsilon\mu) v_d^2\omega} = \frac{2\beta_d^2\sigma R_1^2}{\varepsilon\omega} = \frac{2\mu\beta_d^2\sigma R_1^2}{\varepsilon\mu\omega} = \frac{2\mu\beta_d^2 v_d^2\sigma R_1^2}{\omega}$$

$$= \frac{2\mu\omega^2\sigma R_1^2}{\omega} = 2\mu\omega\sigma R_1^2 = 2R_1^2\,(\mu\omega\sigma) = 2R_1^2\, 2/\delta^2 = 4\, R_1^2 / \delta^2 = (2R_1/\delta)^2$$

Amazing. Then

rat = $(2R_1/\delta)$ => (2nd coeff) = (first coeff) (1/rat) = $(1/v_d)(\delta/2R_1)$

so it would seem that

$$jA_z(R_1,\theta) = V_1 \left[j(1/v_d) + (j+1)\varepsilon/(\beta_d\sigma\delta R_1\xi_1) * \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right]$$

$$= V_1 \left[j(1/v_d) + (j+1)(1/v_d)(\delta/2R_1) * (1/\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right]$$

$$= V_1(1/v_d) \left[j + (j+1)(\delta/2R_1) * (1/\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right]$$

$$A_z(R_1,\theta) = V_1(1/v_d) \left[1 - j (j+1)(\delta/2R_1) * (1/\xi_1) \frac{|sh\xi_1|}{ch\xi_1+\cos\theta} \right] \qquad (*)$$

Now the relative sizes of terms is clearer. The second term is largest when $\cos\theta = -1$ so we have

$$A_z(R_1,\pi) = V_1(1/v_d) \left[1 - j (j+1)(\delta/2R_1) * (1/\xi_1) \frac{|sh\xi_1|}{ch\xi_1-1} \right]$$

Now for a very excellent conductor, and for some fixed ξ_1 , we argue that $(\delta/2R_1) << 1$ and then the second term goes away in (*) and we end up with

$$A_z(R_1,\theta) = V_1(1/v_d)$$

which is then a constant around the perimeter!!! I am surprised at this result. [5/14/14: but in lines doc this is now stated as Step 1 (3.7.5) for extreme skin effect, so I was basically on track.]

[So I have shown that for the extreme skin regime, the second term is tiny and then A_z = constant on the surface. This is correct.]

Go back a bit to

$$-j\omega A_z(a,\varphi) = E_z(a,\varphi) - j\beta_d V_1$$

The claim is that, in the extreme skin depth regime relative to wire radius, although $E_z(a,\varphi)$ has a rather violent shape going around the perimeter of the circle and is definitely not constant, its overall scale is so small that we can neglect this term relative to the last term so we get

$$-j\omega A_z(a,\varphi) \approx -j\beta_d V_1$$

$$A_z(a,\varphi) \approx -(\beta_d/\omega) V_1 = (1/v_d) V_1$$

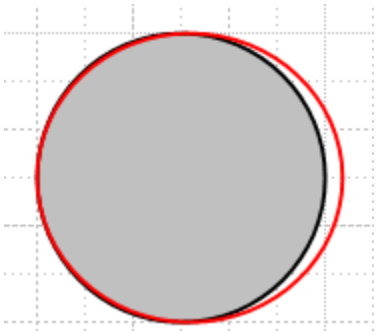
and then King's claim that A_z is constant on surface comes back to life! [correct]

What about the two Paradoxes? They are on the white board, and I think I see how they resolve!! But each one will need a write up. [I have now done this write-up in "the current asymmetry issue"]

Paradox 3. If A_z is constant round the perimeter, we know that a B field line must match the circle, and then there is no normal B component. But it seems that B strength would be strongest where J_z was strongest, so you would think the B strength varied, and therefore there is going to be some H_n which seems a contradiction. But perhaps this resolves the same as Paradox 1:

[I am assuming in the above paragraph that "B varies around circle => $H_n \neq 0$". But in the extreme skin effect limit, I show in "the current asymmetry issue Section 6 that even if B varies on the circle, you still have $H_n = 0$.]

(1) A_z is very close to constant on the circle, but not exact. Therefore, the B contour around the circle is not exact, so the actual contour is a slightly distorted circle, like the red B field line shown here,



I am just guessing this is now it is distorted. So the point is that yes, there really is some H_n component where we are distorted. But I claim the largest H_n should occur where $\partial_\theta H_\theta$ is largest. Well, OK, that gradient is 0 at the max point, so I think we are in like flint here.

A lot of stuff happened today!!!

15. Approximation Hierarchy

(a) When dealing with a "perfect conductor" one is in the extreme skin depth limit and $E_z = 0$ inside the conductor. In this case the EAφ equation just says

$$-j\omega A_z(\mathbf{x}) = E_z(\mathbf{x}) - j\beta_d\varphi(\mathbf{x}) = -j\beta_d\varphi(\mathbf{x})$$

Strictly this applies inside the conductor, but I think there would also be no E_z field outside the conductor either. In this limit then we have

$$-j\omega A_z(\mathbf{x}) = -j\beta_d\varphi(\mathbf{x})$$

$$\omega A_z(\mathbf{x}) = \beta_d\varphi(\mathbf{x})$$

$A_z(\mathbf{x}) = (\beta_d/\omega)\,\varphi(\mathbf{x})$ $\qquad\qquad$ $\beta_{d0} = (\omega/v_d)$

$A_z(\mathbf{x}) = (1/v_d)\,\varphi(\mathbf{x})$

This says that $A_z(\mathbf{x})$ is just a scaled copy of $\varphi(\mathbf{x})$ and this is King-compatible.

(b) Recall the King gauge condition

$$\beta^2 = \mu\varepsilon\omega^2 - j\omega\mu\sigma = \omega^2\mu(\varepsilon - j\sigma/\omega) = \omega^2\mu\,\xi \qquad \xi \equiv \varepsilon - j\sigma/\omega \quad . \qquad (1.5.1)$$

$$\text{div}\,\mathbf{A} = -\mu\varepsilon j\omega\varphi - \mu\sigma\varphi = -j\omega\mu(\varepsilon+\sigma/j\omega)\varphi = -j\omega\mu\xi\varphi = -j(\beta^2/\omega)\,\varphi \qquad \text{King gauge} \qquad (1.5.5)$$

In the dielectric, and ignoring transverse A_i, this last says

$$\partial_z A_z = -j(\beta^2/\omega)\,\varphi$$

or

$$-j\beta_d A_z = -j(\beta_d^2/\omega)\,\varphi$$

or

$$A_z = (\beta_d/\omega)\,\varphi = (1/v_d)\,\varphi(\mathbf{x})$$

This is the same as my part (a) result!

Conclusion: The assumption of a perfect conductor (extreme skin effect) results (EAφ equation) in $A_z(\mathbf{x}) = (1/v_d)\,\varphi(\mathbf{x})$ everywhere in space since $E_z = 0$ in this case. This is the same result as obtained from the gauge condition where you throw out the transverse contributions $\partial_x A_x + \partial_y A_y$. Perhaps the extreme skin effect limit squashes both E_φ and E_r since $\beta \to \infty$, and then there are no transverse currents, and then there is no transverse A_i ! I think this makes some sense.

[5.14.14: All of this conclusion is now to be found in lines doc in various places.]