Details of the Chapter 6 Exact Solution PhL 2/15/14
updated: 2/25/14

Title refers to the two-cylinders geometry.

[Final review 5.14.14. Here is a little summary:]

Summary

Section 1 shows the extreme skin effect ideal that $A_{zt} = \varphi_t$ and thus A_z (1/v)φ and I go around in a circle ending up with $E_z = 0$! I also realize that i(z) = v q(z) which is now in lines doc.

Section 2 suggests that we still have $A_{zt} = \varphi_t$ even if not low-loss, since transverse equations are the same. But on the other hand, this makes $A_z \neq$ constant, so probably this fact is useless. Very short.

Section 3 addresses The Z_s Paradox which is now all resolved in lines doc Ch 4. At the time I even wondered if somehow Z_s might really be constant for any conductor pair.

Section 4 uses Appendix D E_z expression to show that there will be some moments η_m (I don't know what they are at this point) and $Z_s(\theta)$ *really does* depend on θ for the two cylinders.

Section 5 calculates the two-cylinder moments η_m I guess for the first time!

Section 6 summarizes all my disasters resulting in my pulling lines doc off the web circa 2/20. There really work several big problems!

Reviewed this on 3/5/14. Two weeks have gone by, working every day, since I write this doc. About 8 days since I pulled lines doc from the web. Things are not resolved, but I claim to have made significant progress to that end. The TOC is the outline here.

Reviewed again on 5/12/14. The main problems are now all cleared up in into lines doc! The title of this doc only refers to item 5 below where I compute the η_m moments. This doc is *really* about why I had to pull lines doc from the web due to several big problems.

Doing this now due to a paradox noted in the edit log [concerns $Z_s(\theta)$!], but probably this is a good thing to do anyway! After all, this is the only non-trivial problem my method can solve!

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1. Comparing PDE's for φ and A_z : for truly lossless, I think φ_t and A_{zt} are the same.

[5/12/14 I think yes, these solutions are exactly the same for a perfect conductor]

OK, so what do we know about the solution of fat round conductors from Chapter 6 ??

1. The scalar potential is this:

$$\varphi(x,y,z) = \frac{1}{4\pi\varepsilon}\ q(z)\ \varphi_t(x,y)$$

$$\varphi_t(\mathbf{x}) = \ln(s_2^2/s_1^2) = 2\ln(s_2/s_1)$$

$$\varphi_t(C_1) = 2\ln(s_2/s_1)|^{C1} = -2B_1$$
$$\varphi_t(C_2) = 2\ln(s_2/s_1)|^{C2} = -2B_2\ . \qquad (6.3.1)$$

2. But what is the vector potential A_z? I have not made an ansatz for this as I have for φ. Could it have exactly the same form? I claim in (5.3.14) that A_z has the exact same asymptotic form. Suppose that were the case. Then I would have

$$A_z(x,y,z) = \frac{\mu}{4\pi}\ i(z)\ A_{zt}(x,y)$$
$$A_{zt}(\mathbf{x}) = \ln(s_2^2/s_1^2) = 2\ln(s_2/s_1)$$

It is true that the same *type* of BC applies,

$$\varphi_t(C_1) = K_1 \qquad \varphi_t(C_2) = K_2 \qquad K_1 - K_2 = K \qquad (5.1.10)$$
$$A_{zt}(C_1) = W_1 \qquad A_{zt}(C_2) = W_2 \qquad W_1 - W_2 = K \qquad (5.2.10)$$

If the solution is exactly the same, then $W_1 = K_1$ and $W_2 = K_2$. Is this really the solution?

How does this fit with the gauge condition,

$$\partial_z A_z(\mathbf{x}) = -\,j\,(\beta^2/\omega)\,\varphi(\mathbf{x}) \qquad (4.11.3)$$

$$-jk\,A_z(\mathbf{x}) = -\,j\,(\beta^2/\omega)\,\varphi(\mathbf{x})$$

$$k\,A_z(\mathbf{x}) = (\beta^2/\omega)\,\varphi(\mathbf{x})\ ?$$

But in low-loss we are supposed to have $k^2 = \beta^2$ from (5.3.8) so that seems to say

$$k\,A_z(\mathbf{x}) = (k^2/\omega)\,\varphi(\mathbf{x})$$

$$A_z(\mathbf{x}) = (k/\omega)\,\varphi(\mathbf{x}) = (1/v)\varphi(\mathbf{x}) \qquad v = \omega/k = \text{phase velocity in dielectric} \quad \text{// low-loss}$$

Question: What are the dimensions of things!!! Look at (1.3.1)

A = vector potential (tesla-m = amp-henry/m = volt-sec/m) **E** = volt/m
φ = scalar potential (volts) **B** = tesla .

so they have different dimensions, the difference is velocity. Fine.

Now go back a bit

$\varphi(x,y,z) = \frac{1}{4\pi\varepsilon}\ q(z)\ \varphi_t(x,y)$ = [Coul/m]/[farad/m] = volt OK

$A_z(x,y,z) = \frac{\mu}{4\pi}\ i(z)\ A_{zt}(x,y)$ = henry/m * amp = ohm-sec m^{-1} amp = volt sec/m OK

As I already state, φ_t and A_{zt} are both dimensionless. So then we would have as above, and everything is correct. So now I am sure that A_{zt} is the same as φ_t exactly!

3. What is E_z ?

E = - grad φ - ∂_t**A** . dim: volt/m and sec^{-1} volt sec/m

A = vector potential (tesla-m = amp-henry/m = volt-sec/m) **E** = volt/m
φ = scalar potential (volts) **B** = tesla .

E = - grad φ - jω**A**

$E_z(\mathbf{x}) = -\partial_z\varphi(\mathbf{x}) - j\omega A_z(\mathbf{x})$

$E_z(\mathbf{x}) = +jk\ \varphi(\mathbf{x}) - j\omega A_z(\mathbf{x})$ RHS = m^{-1} volt and sec^{-1} volt sec/m OK

$E_z(\mathbf{x}) = +jk\ [\frac{1}{4\pi\varepsilon}\ q(z)\ \varphi_t(x,y)] - j\omega[\frac{\mu}{4\pi}\ i(z)\ A_{zt}(x,y)]$

$E_z(\mathbf{x}) = +jk\ [\frac{1}{4\pi\varepsilon}\ q(z)\ 2\ \ln(s_2/s_1)] - j\omega[\frac{\mu}{4\pi}\ i(z)\ 2\ \ln(s_2/s_1)]$

$E_z(\mathbf{x}) = \{+(k/\varepsilon)\ q(z) - (\omega\mu)\ i(z)\}\ \ 2j\ \ln(s_2/s_1)](1/4\pi)$

$E_z(\mathbf{x}) = \{+(k)\ q(z) - (\omega\varepsilon\mu)\ i(z)\}\ \ 2j\ \ln(s_2/s_1)](1/4\pi\varepsilon)$

What do I make of this result?? Well recall that $\mu\varepsilon = 1/v^2$ but also $v = \omega/k$ so $1/v^2 = k^2/\omega^2$

$E_z(\mathbf{x}) = \{+k\ q(z) - (\omega/v^2)\ i(z)\}\ \ 2j\ \ln(s_2/s_1)](1/4\pi\varepsilon)$

$E_z(\mathbf{x}) = \{+k\ q(z) - (\omega/v^2)\ i(z)\}\ \ 2j\ \ln(s_2/s_1)](1/4\pi\varepsilon)$

How are q(z) and i(z) related to each other? [draw a cylinder picture, charge/sec across area A]

$i(z) = v\, q(z)$ C/sec = m/sec * C/m

How come this is not written down anywhere in lines doc??? I seem only to be interested in i(z) and V(z). This is a bad equation to omit! He omits it as well. So let's install it and see what happens:

[On 5/12/14 I added this above equation as (4.11.19a) to lines doc.]

Beware what v this actually is! Here it is the loss-less v.

$$E_z(x) = \{+k\, q(z) - (\omega/v^2)\, v\, q(z)\}\ 2j \ln(s_2/s_1)](1/4\pi\varepsilon)$$

$$E_z(x) = \{+k\, q(z) - k\, q(z)\}\ 2j \ln(s_2/s_1)](1/4\pi\varepsilon)$$

$$E_z(x) = 0$$

Interpretation? I have examined the exact 2 round wires solution. This solution assumes lossless so that we have the capacitor problem and potential theory in 2D. I compute E_z from φ and A_z and I find that $E_z = 0$ everywhere!!! Not just in the wires, but in the dielectric. That of course says $E_z = 0$ at the conductor surfaces, and that in turn says $Z_s = 0$ at the conductors, and that is consistent with "lossless", around the circle we go! [it all makes sense, really]

Misnomer: lossless really means $Z_s = 0$, it does not mean there is no loss in the dielectric, so this is a bad term to use and that should be repaired.

[On 5/12/14 I added a comment about low loss at the start of Section 5.4]

ok to here 2/25/14

Note added 3/5/14. Consider the current in one of the conductors this way

$$i(z) = dQ/dz * dz/dt$$

Now $dQ/dz = q(z)$, the charge per unit length. And dz/dt says how fast this charge is moving by, which is basically v_d . Therefore, $i(z) = q(z)\, v_d$ which is the result just found above. This seems to be a solid and general result for any shaped conductor! You can argue about the precision of v_d regarding losses.

Fact:

All of Chapter 6 operates in the loss-less limit of $Z_s = 0$, so you cannot use anything there to check on asymmetric Z_s !!

All of Chapter 5 after the point of setting $Z_s = 0$ to get the Laplace equation is also lossless. The whole 2D capacitor problem is based on lossless.

Only in Chapter 4 do I have mention $Z_s \neq 0$. So right now, that is the only place I can look for any information on this subject.

2. What happens to φ and A_z when losses are "turned on"?

Note added 3/5/14. I do have a certain inconsistency in today's thinking on this subject. With lossless, both the φ and A_z equations become Laplace equations and they both have the same solution which is the Chapter 6 solution. But when you "turn on the loss", you add a certain Helmholtz term to both the φ and the A_z equations as shown in lines doc here:

$$[\nabla_t^2 + (\beta^2 - k_A^2)]\, A_{zt}(x,y) = 0 \qquad (5.2.8)$$

$$[\nabla_t^2 + (\beta^2 - k_\varphi^2)]\, \varphi_t(x,y) = 0 \qquad (5.1.8)$$

where I later claim that

$$k_\varphi^2 = k_A^2 \equiv k^2 = -zy = -(R+j\omega L)(G+j\omega C) \qquad (5.3.4)$$

but OK, this does assume Chapter 4 stuff which right now is in question. But if the above three equations survive, then you would argue that the Helmholtz term is the SAME for both equations, and so the solutions should somehow stay equal, though different from the Laplace solution. So why do I assume φ stays the same, but A_z "changes slightly"? I am just *noting* this inconsistency for future reference.

So this question is unresolved.

3. Review of the Chapter 4 Logic Flow which leads to "the Z_S Paradox"

[This was the reason I pulled lines doc from the web! Call it the Z_s paradox. It even caused me to doubt the fact that J_z is asymmetric on the conductor perimeter, so in the next section I went and computed J_z for the round wire case and found that it is indeed asymmetric (proximity effect). It was not until 5/11/14 that I figured out how to resolve the Z_s paradox and end up with meaningful Z_s constants in the transmission line equations, in a manner compatible with my earlier discussions of active perimeters and such.]

Making another pass through Chapter 4, examining assumptions.

- assumption of separation of variables such as (4.1.2)
- assumption of balanced line as in (4.1.4)
- transmission line limit same as small β as in Section 4.3
- assumption *(3.8.10)* that φ is constant at a given z.
- assumption that transverse components A_x and A_y can be neglected so that $\mathbf{A} = A_z\hat{\mathbf{z}}$. (4..7.1)
- assumption about μ
- time dependence is $e^{j\omega t}$

It is with this long list of assumptions that we enter Section 4.11. Notice that "low loss" has NOT been assumed at this point [except I claim $\varphi(C_i)$ = constant], we have K and K_L separate. Everything we

know at this point I think appears in box (4.11.1). There are 6 equations in that box. Note that i(z) and q(z) do appear. Also NOT assumed is the wave dependence on z.

With the above listed assumptions, King gauge, (4.11.2) is undeniable.

The first line of (4.11.3) is nothing new, while the second line is new and uses the assumption that we can ignore A_x and A_y AND their derivatives. So a big assumption then going into (4.11.3). [I now know on 3/5/14 that this assumption is the same as extreme skin depth limit .]

Then (4.11.4) is nothing but a rewrite.

Then (4.11.5) is the evaluate and subtract thing for selected $\mathbf{x}_1$ and $\mathbf{x}_2$ points.

Then (4.11.7) I would write this way: [I am allowing that W = W(z) only here, for time being!]

$$E_{z12}(\mathbf{x}_1) - E_{z12}(\mathbf{x}_2) = -\partial_z V(z) - j\omega W(z)$$
$$\partial_z W(z) = -j(\beta^2/\omega) V(z) \quad . \qquad (4.11.7)$$

Then

$$E_{z1}(\mathbf{x}_1) = Z_{s1}(\mathbf{x}_1)\ i_1(z) \qquad E_{z2}(\mathbf{x}_2) = Z_{s2}(\mathbf{x}_2)\ i_2(z). \qquad (4.11.8)$$

so at this point both Z and E could vary azimuthally around a round conductor, say. For example, due to the form shown, if $E_{z1}(\mathbf{x}_1)$ is small at some azimuth, then $Z_{s1}(\mathbf{x}_1)$ is also small at that azimuth. Now let's maintain this notation and write (4.11.9):

$$[Z_{s1}(\mathbf{x}_1) + Z_{s2}(\mathbf{x}_2)]\ i(z) = -\partial_z V(z) - j\omega W(z)$$
$$\partial_z W(z) = -j(\beta^2/\omega) V(z)\ . \qquad (4.11.9)$$

[the Z_s paradox is visible above in 1st line. Right side function only of z, left side of $\mathbf{x}_1$ and $\mathbf{x}_2$]

Our set of assumptions above has then led us to this conclusion:

$$[Z_{s1}(\mathbf{x}_1) + Z_{s2}(\mathbf{x}_2)] = \text{value for any } \mathbf{x}_1 \text{ and any } \mathbf{x}_2 \text{ in the same z plane.}$$

In other words, the claim is that

$$[Z_{s1}(\mathbf{x}_1) + Z_{s2}(\mathbf{x}_2)] = f(z)$$

This is not really a claim, it is a fact resulting from all our work above including the assumptions made.

But I think we can obtain more. We can keep $\mathbf{x}_2$ fixed and let $\mathbf{x}_1$ vary to some new $\mathbf{x}_1'$. Then we have

$$[Z_{s1}(\mathbf{x}_1') + Z_{s2}(\mathbf{x}_2)] = f(z)$$

Subtract to get

$$Z_{s1}(\mathbf{x}_1) = Z_{s1}(\mathbf{x}_1')$$

but this in turn implies that, since the above is true for all points on the conductor perimeter.

$$Z_{s1}(x_1) = f_1(z)$$

The same then is true for the other, so we have

$$Z_{s2}(x_2) = f_2(z)$$

and these are stronger claims than just for the sum. Now go back to

$$E_{z1}(x_1) = Z_{s1}(x_1)\ i_1(z) \qquad E_{z2}(x_2) = Z_{s2}(x_2)\ i_2(z). \qquad (4.11.8)$$

These then read

$$E_{z1}(x_1) = f_1(z)\ i_1(z) \qquad E_{z2}(x_2) = f_2(z)\ i_2(z). \qquad (4.11.8)$$

and we then conclude that

$$E_{z1}(x_1) = E_{z1}(z) \qquad E_{z2}(x_2) = E_{z2}(z)$$

This is a very dramatic result. We know from box (1.1.50) that E_z is continuous at the surface. This tells us that

$$J_z(x_1) = J_z(z)$$

The current "just below the surface" is constant around the entire conductor! This seems extremely strange to me. It makes no comments about currents deeper down in the conductor.

This conclusion is extremely problematic for me. It says for example that all higher partial wave E_z fields must vanish, which is nonsense.

[Keep feet on the ground. Both E_z and J_z do vary in θ very strongly, they are not constant around the perimeter! And so $Z_s = Z_s(\theta)$ with this same strong variation. This paradoxical result arises due to earlier confusions which I will write up soon!]

4. Why I think $Z_S(\theta)$ varies strongly with θ for the two-cylinder problem.

Exercise #1 For Chapter 6, I could in theory compute the moments η_m from the charge distribution, and from that I could in theory compute E_z along the surface of one of the conductors. Here is my formula from (D.2.33) for E_z :

$$E_z(r,m) = (1/4)\ \eta_m\ I\ R_{dc}\ (a\beta')\ [\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}]$$

Setting r = a then gives

$$E_z(a,m) = (1/4)\,\eta_m\, I\, R_{dc}\, (a\beta')\left[\frac{J_m(x_a)}{J_{m+1}(x_a)} - \frac{J_m(x_a)}{J_{m-1}(x_a)}\right] \qquad x_a = \beta' a \qquad \beta'^2 = \beta^2 - \beta_d^2$$

Note that,

$$\left[\frac{J_m(x_a)}{J_{m+1}(x_a)} - \frac{J_m(x_a)}{J_{m-1}(x_a)}\right] = J_m(x_a)\left[\frac{1}{J_{m+1}(x_a)} - \frac{1}{J_{m-1}(x_a)}\right] = J_m(x_a)\,\frac{J_{m-1}(x_a) - J_{m+1}(x_a)}{J_{m-1}(x_a)\,J_{m+1}(x_a)}$$

$$= J_m(x_a)\,\frac{J'_m(x_a)}{J_{m-1}(x_a)\,J_{m+1}(x_a)}$$

so this is not identically zero. E_z is "small" because R_{dc} is very small.

Well, I already did this, and here is the result:

$$Z_s(\varphi) = (1/I) \sum_{m=-\infty}^{\infty} E_z(a,m)\, e^{jm\varphi} \qquad \text{// (D.1.3a)}$$

$$= (1/4)\, R_{dc} \sum_{m=-\infty}^{\infty} \eta_m \left[\frac{x_a J_m(x_a)}{J_{m+1}(x_a)} - \frac{x_a J_m(x_a)}{J_{m-1}(x_a)}\right] e^{jm\varphi} \qquad \text{// (D.2.33)}$$

where, (D.2.35)

$$\eta_m = N_m/N_0 = \frac{\omega a}{\beta_d I} \int_{-\pi}^{\pi} d\varphi\, n(\varphi)\, e^{-jm\varphi} \qquad \text{// (D.1.5b) and (D.2.31)}$$

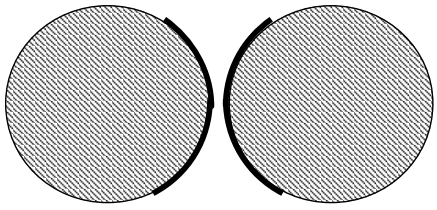
So this will give for sure a non-uniform $Z_s(\varphi)$ around the wire. This is a calculation I really could do using the Chapter 6 solutions since they determine surface charge $n(\varphi)$.

Back up Again. Here are some steps:

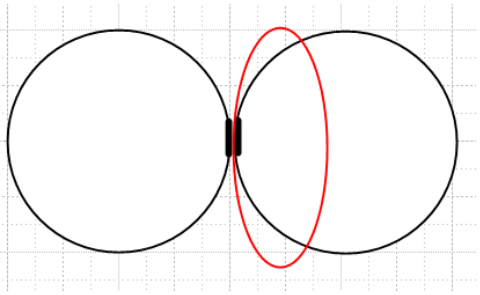
$\mathbf{E} = -\text{ grad }\varphi - j\omega\mathbf{A}$
$E_z(\mathbf{x}) = -\partial_z\varphi(\mathbf{x}) - j\omega A_z(\mathbf{x})$
$E_{z12}(\mathbf{x}) = -\partial_z\varphi_{12}(\mathbf{x}) - j\omega A_{z12}(\mathbf{x})$
$E_{z12}(\mathbf{x_1}) - E_{z12}(\mathbf{x_2}) = -\partial_z V(z) - j\omega W(z)$

I can ignore the 2nd equation in each pair, my problem is with the 1st equation of each pair!

Idea #1. I think several of my "mysteries" are tied together. For example, consider this situation,



and exaggerate it every more by putting the conductors VERY close together so there is only a tiny active area.



Since the current is so localized in the heavy dark region, I think a B field line will look like that shown in red, and this is NOT parallel to the conductor, so W(z) is NOT a constant on this surface as King always likes to say it is! This is really my "first proof" and here you see it totally falling apart. So this gets me back to the **E•B** = 0 business! Probably **B** is only parallel to the conductor surface "in the active region".

$$s_1^2 = (x+d)^2 + y^2 \qquad s_2^2 = (x-d)^2 + y^2 \tag{6.2.3}$$

$$\varphi_t(\mathbf{x}) = \ln(s_2^2/s_1^2) = \ln\left[\frac{(x-d)^2 + y^2}{(x+d)^2 + y^2}\right]$$

$$\partial_{\mathbf{x}}\varphi_t = \frac{(x-d)^2 + y^2}{(x+d)^2 + y^2}$$

```
phi := ln( ((x-d)^2 + y^2)/((x+d)^2 + y^2)  );
```

$$\phi := \ln\left(\frac{(x-d)^2+y^2}{(x+d)^2+y^2}\right)$$

```
diff(phi,x): simplify(%);
```

$$4\,\frac{d\,(x^2-d^2-y^2)}{(x^2-2\,x\,d+d^2+y^2)\,(x^2+2\,x\,d+d^2+y^2)}$$

```
diff(phi,y): simplify(%);
```

$$8\,\frac{y\,x\,d}{(x^2-2\,x\,d+d^2+y^2)\,(x^2+2\,x\,d+d^2+y^2)}$$

So OK, this should be the E field [ignoring A_z!], and I expect it to be large when d is small.

Update Feb 25, 2014. [updated again 3/8/14 after bipolar doc edits made]

5. Calculate the moments η_m for the two-cylinder problem!

[this whole calculation is now Appendix A in bipolar doc]

In bipolar doc I show that the angular charge densities on the two cylinders are given by

$$n_1(\xi_1,\theta) = dQ_1(\xi_1,\theta)/[d\theta dz] = \frac{q}{2\pi}\frac{|sh\xi_1|}{ch\xi_1+\cos\theta}$$

$$n_2(\xi_2,\theta) = dQ_2(\xi_2,\theta)/[d\theta dz] = -\frac{q}{2\pi}\frac{|sh\xi_2|}{ch\xi_2+\cos\theta} \qquad (10.28)$$

and each one is independent of the other one, what could be simpler! We know that

$$\int n_1(\xi_1,\theta)\, d\theta = q$$

Now from lines doc the moments are given by

$$N_m = (1/2\pi) \int_{-\pi}^{\pi} d\theta\, n_1(\xi_1,\theta)\, e^{-jm\theta}$$

In my case, n_1 is an even function of θ so only the cos(mθ) contributes and then

$$N_m = (1/2\pi)\frac{q}{2\pi}|sh\xi_1|\ 2 \int_0^{\pi} d\theta \frac{\cos(m\theta)}{ch\xi_1+\cos\theta}$$

$$= (1/2\pi)\frac{q}{2\pi}2\ [|sh\xi_1| \int_0^{\pi} d\theta \frac{\cos(m\theta)}{ch\xi_1+\cos\theta}]$$

Maple does not know this integral. It looks familiar, however. Consider

$$1.^6 \qquad \int_0^{\pi} \frac{\cos nx\, dx}{1+a\cos x} = \frac{\pi}{\sqrt{1-a^2}}\left(\frac{\sqrt{1-a^2}-1}{a}\right)^n \qquad [a^2<1, \quad n\geq 0] \qquad \text{BI (64)(12)}$$

$$|sh\xi_1| \int_0^{\pi} d\theta \frac{\cos(m\theta)}{ch\xi_1+\cos\theta} = |th\xi_1| \int_0^{\pi} d\theta \frac{\cos(m\theta)}{1+\cos\theta/ch\xi_1}$$

Let $a = 1/ch\xi_1 = sech\xi_1$ which is then $a < 1$.

$1 - a^2 = 1 - sech^2\xi_1 = th^2\xi_1$ Then $\sqrt{1-a^2} = th\xi_1$ so we get

$$\int_0^{\pi} d\theta \frac{\cos(m\theta)}{1+\cos\theta/ch\xi_1} = (\pi/\, th\xi_1)\,[\frac{th\xi_1-1}{sech\xi_1}]^m$$

But

$$\frac{th\xi_1-1}{sech\xi_1} = \frac{sh\xi_1/ch\xi_1-1}{1/ch\xi_1} = sh\xi_1 - ch\xi_1 = -e^{-\xi_1} \qquad \text{// says Maple}$$

So our integral is then

$$\int_0^{\pi} d\theta \frac{\cos(m\theta)}{1+\cos\theta/ch\xi_1} = (\pi/\ th\xi_1)\ (-1)^m \exp(-m\xi_1)$$

Then we have,

$$N_m = (1/2\pi)\frac{qdz}{2\pi} 2\ [|sh\xi_1| \int_0^{\pi} d\theta \frac{\cos(m\theta)}{ch\xi_1+\cos\theta}]$$

$$= (1/2\pi)\frac{qdz}{2\pi} 2\ [\ |th\xi_1|\ \ (\pi/\ th\xi_1)\ (-1)^m \exp(-m\xi_1)\ ]$$

$$= \frac{qdz}{2\pi} sign(\xi_1)\ (-1)^m \exp(-m\xi_1)$$

But the integral is really even in m from the very start, so the result is really

$$N_m = \frac{qdz}{2\pi} sign(\xi_1)\ (-1)^m \exp(-|m|\xi_1)$$

[this result is superseded by that shown in Appendix A: $N_m = \frac{q}{2\pi}\ (-1)^m \exp(-|m\xi_1|)$]

Good. Now if ξ_1 is very large, the m = 0 moment is $\frac{qdz}{2\pi} sign(\xi_1)$, but all the higher moments basically vanish, just as I wanted.

Note added: But the above says $N_0 = \frac{qdz}{2\pi} sign(\xi_1)$ which seems to conflict a bit with N_0 as it appears in lines doc which is

$$N_0 = (\beta_d/2\pi\omega a)\ I\ . \qquad (D.2.31)$$

Now suppose $\xi_1 = 0.25$ as in my bipolar doc example, then

$$\eta_m \equiv N_m/ N_0 = (-1)^m \exp(-|m|\xi_1)$$

$\eta_0 = 1$
$\eta_1 = -\exp(-.25) = -0.778$
$\eta_2 = +0.607$
etc.

Maple says,

```
xi1 := 0.25;
```

$$\xi 1 := .25$$

```
for m from 0 to 10 do
  print("m= ",m,  "etam = ", (-1)^m*exp(-m*xi1));
od;
```

"m= ", 0, "etam = ", 1
"m= ", 1, "etam = ", -.7788007831
"m= ", 2, "etam = ", .6065306597
"m= ", 3, "etam = ", -.4723665527
"m= ", 4, "etam = ", .3678794412
"m= ", 5, "etam = ", -.2865047969
"m= ", 6, "etam = ", .2231301601
"m= ", 7, "etam = ", -.1737739435
"m= ", 8, "etam = ", .1353352832
"m= ", 9, "etam = ", -.1053992246
"m= ", 10, "etam = ", .08208499862

I was wrong in lines doc! I thought this would have a strong m = 1 only, but it is a slowly converging series involving a lot of moments.

Let's try to reconstruct n(θ) from all the moments.

$$n(\varphi) = \sum_{m=-\infty}^{\infty} N_m e^{jm\varphi} \qquad \text{(D.1.5a)}$$

$$= N_0 + \Sigma_{m=-\infty}^{-1} N_m e^{jm\varphi} + \Sigma_{m=1}^{\infty} N_m e^{jm\varphi}$$

$$= N_0 + \Sigma_{m=1}^{\infty} N_{-m} e^{-jm\varphi} + \Sigma_{m=1}^{\infty} N_m e^{jm\varphi}$$

$$= N_0 + \Sigma_{m=1}^{\infty} N_m e^{-jm\varphi} + \Sigma_{m=1}^{\infty} N_m e^{jm\varphi}$$

$$= N_0 + \Sigma_{m=1}^{\infty} N_m (e^{-jm\varphi} + e^{jm\varphi})$$

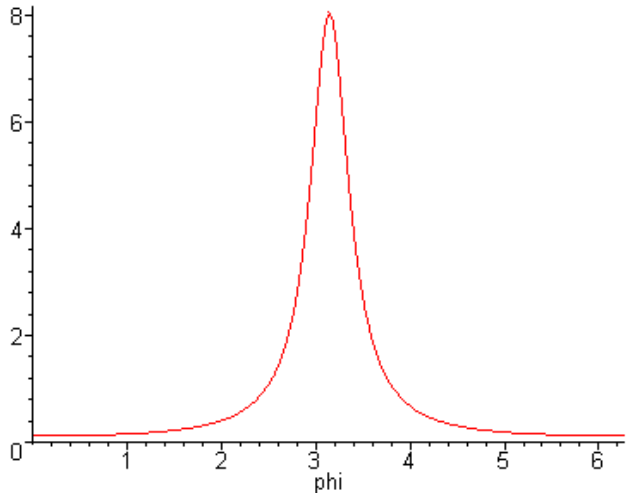
$$= N_0 + \Sigma_{m=1}^{\infty} N_m 2\cos(m\varphi)$$

so

$$n(\varphi)/N_0 = 1 + 2 \Sigma_{m=1}^{\infty} \eta_m \cos(m\varphi)$$

$$= 1 + 2 \Sigma_{m=1}^{\infty} (-1)^m \exp(-|m|\xi_1)\cos(m\varphi)$$

```
f := 1 + 2*sum((-1)^m * exp(-m*xi1)*cos(m*phi),m=1..30):
plot(f,phi=0..2*Pi);
```



and this looks exactly right for that value of $\xi_1 = 0.25$.

The reason there are many is that the alternating sines cancel out near $\theta = 0$ and are additive at $\theta = \pi$.

Conclusions: I think everything makes complete sense:

(1) If conductors are closely spaced and charge is localized, there will be lots of moments with alternating signs to give a peak like that shown above.

(2) If conductors are more widely spaced (larger ξ) then the m = 1 will be the main correction to m = 0

I could compute $Z(\varphi)$ using the above formula, it will be some complicated mess, but it won't be constant around the conductor, that is for sure. It will have φ dependence. [It will be $Z_s(\theta) = E_z(\theta)/ i(z)$]

The moments control J_z at the surface, which is E_z at the surface, and so I think you can compute the internal J_z from the moments without too much work using lines doc.

$$E_z(r,m) = (1/4)\,\eta_m\, I\, R_{dc}\, (a\beta')\, [\frac{J_m(x)}{J_{m+1}(x_a)} - \frac{J_m(x)}{J_{m-1}(x_a)}] \qquad = K_{m,a}\,\eta_m$$

$$E_z(r,\varphi) = \sum_{m=-\infty}^{\infty} E_z(r,m)\, e^{jm\varphi} = \Sigma_m\ K_{m,a}\,\eta_m\, e^{jm\varphi}$$

It is not totally obvious that $E_z(r,\varphi)$ will also have a peak at $\varphi = \pi$, but that will probably be true.

6. Status Report and Pull lines doc from the Web

I have multiple problems now that I think are all related, and they are big enough that I decided to pull my lines doc paper from the web for the time being until I can effect repairs. Here is a list of some of the problems:

(1) I have a logical failure in Chapter 4 as described above [the Z_s paradox], and I suspect that the problem is either that W(z) on conductor cross sections is invalid, or that I need some approximation hierarchy for what to do after I do the loss-less analysis. [both V(z) and W(z) are slightly invalid]

(2) If the B field lines are not tangent to the conductor surface, then I think W(z) cannot be constant on the surface, so this whole underpinning of lines doc wobbles. [correct, so must stick to large ω only]

(3) This all ties in with the E • B = 0 problem which I did a half-baked rescue of.

[As of 5.12.14 I have resolved the above three problems and installed into lines doc.]

(4) I had the general feeling that the asymmetry of current in a conductor matched that asymmetry of the charge density on the surface and may have even stated that in lines doc. I suspect that this association of the two asymmetries is only valid at "high frequency" where the skin effect is active. Probably at low frequency you could have a huge charge density asymmetry and a completely uniform E_z in the round wire or other conductor. [I will investigate this item]

(5) I am still unhappy about the question of "where does the surface charge come from"? I made that into a reader exercise because I could not answer it! [I will investigate this item at some point]

So my current version of lines doc is then only a "working platform" which at least organizes certain pieces of EM information, but it is not ready for prime time because there are too many fudged and unanswered questions. I kept sweeping things under the rug to get it done, and now those things have come back to roost. But I do think I have the tools to approach the issues, and in the end I think I can get lines doc into something worth while. Right now it is worthless and embarrassing. In particular, I think I can compute a lot of things for the parallel cylinders based on the bipolar coordinates doc.