Lagrange Doc Boltzmann Questions PhL 10.12.16

This doc has been rendered obsolete by my recent update of the balls in bins v2 doc. I will just add a few comments below.

I have found my little folder of "blackbody" docs.

Question 1:

My blackbody doc (and Zemansky p 255) argues that the ways to put N_1 identical particles into g_1 degenerate states is $g_1^{N1}/ N_1!$ and that this fact is supported in Zemansky (10.4) and does not require $N_1 < g_1$. Why does this differ from my Boson state count as presented in Lagrange doc? In that doc I claim that the number of ways of putting N_1 identical balls in g_1 boxes is $\binom{N+g-1}{N}$ which differs a lot from $\frac{g_1^{N_1}}{N_1!}$.

Zemansky is doing identical particles, and B >> N. His result (10-4) p 255 exactly matches my results 2' and 4' for Bose and Fermi cases. No need to continue here.

Plan A. Take a super simple example and just enumerate all the states in both cases, they can't both be right. Call things just N and g.

Try: N= 2 and g = 2. The two answers are $\frac{2^2}{2!} = 2$ and $\binom{3}{2} = 3$, so answers disagree.

Enumerate states the first way if balls are labeled, I see a total of 4 "ways"

box 1	box 2		
a	b	1	1
b	a	1	1
ab	0	2	0
0	ab	0	2

If the balls are identical the first two ways are the same, so the result is a total of 3 ways. This disagrees with the Zemansky method, so I now enumerate for that method:

box 1	box 2		
ab	0	2	0
a	b	1	1
b	a	1	1
0	ab	0	2

OK, for identicals the middle two states are identical. But Z says divide by 2 to get 4/2 = 1, which means that the first and last are treated as the same. Perhaps the boxes are also indistinguishable???

Very good. No, the boxes are always labeled.

Resolution: Review the argument one more time for $g_1^{N1}/ N_1!$ which is to say $g^N/N!$. Yes, g^N is the correct count for labeled balls going into the boxes. It is the justification for dividing by N! that is wrong. This is only justified if you can ignore states where more than one particle went into the same box. In Z's page 254 enumeration, he shows only 1 particle in a box.

I claim that Zemansky has not presented a clear argument although he has lots of words and on page 253 he does mention g >> N. He does not tie in this inequality with his page 254 discussion.

Restatement of Zemansky. Suppose g >> N, for example, 100 >> 3. Take the first ball and 100 choices, and so on so there are 100*100*100 = $g^N = 10^6$ ways to put in labeled balls. How many of these ways have 2 or 3 balls in the same box? Very few. If you wanted to prevent this from happening, you would do the Fermion count method which gives (g,N). I would do this as

g * (g-1) * (g-2) * .....(g-N+1]) / N! which is g! / [(g-N)! N!] = (g,N)

So Zemansky's argument is that we have g >> N so can ignore more than one ball in a box.

My presentation in Lagrange doc is much more accurate.

Question 2. Why is it that I have

$$\Omega(N_1,N_2...N_m) = \frac{g_1^{N_1}}{N_1!}\frac{g_2^{N_2}}{N_2!} \cdots \frac{g_m^{N_m}}{N_m!} .$$

where Zemansky and Wiki on Maxwell Boltzmann statistics have an extra M! overall on this thing.

See balls in bins, all cases are covered there.

Zemansky. Well I am wrong, page 255 shows the above with no M! extra factor.

Big Reif: I cannot find anything like the above equation in this book.

Livesey: Page 13 shows the above equation with all g's = 1and with an overall N! on top. So this is the situation without degeneracy of states.

Question 2a: How many ways can one partition N distinguishable particles into bins of N_i ?

See Appendix A and B in balls in bins for two answers to this question. It is only the non-ordered partition that gets used anywhere in that doc, it is in Appendix B.

Subquestion: Does position of a ball in a bin matter here? Yes, for a bin with N_i= 3 the way ABC differs from the way BAC. These are distinct "ways".

Subquestion: What does bin degeneracy g_i mean?

Fact: The number of ways you can partition N identical particles into m having N_i is this:

$$\text{ways} = \frac{N!}{N_1!N_2!N_3!...}$$

Proof: If the particles were labeled, the answer would be