Sound Waves PhL 7.21.11

I found a nice compact discussion of sound waves here

http://mysite.du.edu/~jcalvert/waves/soundwav.htm#C

and I printed it out as a PDF and am now reading the thing doing underlining.

Plane Waves. Sound can be regarded as a longitudinal density wave, density $\rho(x,t)$ where we have a plane wave in the x direction. The quantity $[\rho(x,t)-\rho_0] / \rho_0 = \Delta\rho/\rho_0$ is called the **condensation**. The medium has a displacement he calls $\xi(x,t)$. This is displacement from rest position of air particles due to the sound wave. Particle velocity is then $v(x,t) = \partial_t\xi(x,t)$ in the sense of fluid dynamics (?). The other derivative $\partial_x \xi(x,t)$ is called the **dilatation**, but I don't understand what he says about it, and this is not a common web term, so lets try to skip it and move on. Condensation if $\Delta\rho > 0$ and rarefaction if $\Delta\rho < 0$.
[See Thuras notes for explanation of missing items here.]

Aside on Diffusion: What do I know about "the diffusion equation"? The answer is: not much. I am now looking for notes on this subject. Stak mentions it various times but only in passing. I have no physics notes on this topic at all! My fluid book has nothing really on diffusion. Reif on page 484 gives a good simple discussion. The notion of diffusion arises when you have significant variation of the density of some solute in a solution. In his example, that solute is a set of radioactively tagged molecules among non-tagged molecules. The first claim is a bit like heat flow and just says that we expect $J_z = -Ddn_1/dz$ where n_1 is the tagged particle density. For heat flow D would be the thermal conductivity and n_1 would be temperature and J_z would be heat flow. So this brings in the first derivative of the density $\partial_z n_1$. The second ingredient is conservation of mass

$\nabla\bullet\mathbf{J} = -\partial_t\rho$ $\quad m^{-1}\ kg\ sec^{-1}m^{-2} = kg\ sec^{-1}m^{-3}$ check

which he treats in 1D. We of course also claim in general that

$\mathbf{J} = -D\,\nabla n.$ $\quad kg\ sec^{-1}m^{-2} = -[kg\ m^2\ sec^{-1}]\ m^{-1}m^{-3}$ defines D units

So combine these and we have $D = [kg\ m^2\ sec^{-1}]$

$\nabla\bullet[-D\,\nabla n] = -\partial_t\rho = -D\nabla^2 n$

and now we have our famous second derivative object acting on n. Then $\rho = mn$ so we get

$\partial_t n = (D/m)\nabla^2 n$ $\quad sec^{-1}m^{-3} = [\,m^2\ sec^{-1}\,]\,m^{-2}\,m^{-3}$ check

$\Rightarrow (\partial_t - [D/m]\nabla^2)n(\mathbf{r},t) = 0$ $\quad (\ sec^{-1} - [m^2\ sec^{-1}]\,m^{-2})\,m^{-3} = 0$ check

and we thus end up with the Stakgold heat equation applied here to diffusion.

Now comes an obvious question: does this diffusion process have anything to do with sound wave propagation? The air molecules are uniformly distributed, there is nothing to diffuse, really, so I think the answer is no, it does not.

On the other hand, we are claiming in a sound wave that we have a variation in our primary n(x) density, so would this not cause some diffusion current in the sense of $\mathbf{J} = -D\,\nabla n$? And mass must be conserved, so we still have $\nabla\bullet\mathbf{J} = -\partial_t\rho$. If I multiply the heat equation above by particle mass, we get

$\partial_t\rho = (D/m)\nabla^2\rho$ $\qquad$ $\rho = nm$ $\qquad$ $\sec^{-1}\,\mathrm{kg}\,\mathrm{m}^{-3} = [\mathrm{m}^2\,\sec^{-1}]\,\mathrm{m}^{-2}\,\mathrm{kg}\,\mathrm{m}^{-3}$ chk

so this would seem to relate these two derivatives of ρ at any point in spacetime. Moreover as in my little comparison doc we can define the velocity field by

$\mathbf{v} = \mathbf{J}/\rho$. $\qquad$ $\mathrm{m/sec} = \mathrm{kg}\,\sec^{-1}\mathrm{m}^{-2} / [\,\mathrm{kg}\,\mathrm{m}^{-3}]$ $\qquad$ check

Now consider

$\nabla\bullet\,\mathbf{v} = \nabla\bullet[\mathbf{J}\,(1/\rho)] = \nabla\bullet\,\mathbf{J}\,(1/\rho) + J\bullet\nabla(1/\rho) = -(\partial_t\rho)/\rho + J\bullet\nabla(1/\rho)$

$\mathrm{m}^{-1}\,\mathrm{m}\,\sec^{-1} = \sec^{-1} + \mathrm{kg}\,\sec^{-1}\mathrm{m}^{-2}\;\mathrm{m}^{-1}\,\mathrm{m}^3/\mathrm{kg}$ check

Meanwhile, just doing random things here, we should have

$\mathbf{J} = -D\,\nabla n = -(D/m)\,\nabla(\rho)$ $\qquad$ $\mathrm{kg}\,\sec^{-1}\mathrm{m}^{-2} = [\mathrm{m}^2\,\sec^{-1}]\mathrm{m}^{-1}\,\mathrm{kg}\,\mathrm{m}^{-3}$ $\qquad$ check

Then

$J\bullet\nabla(1/\rho) = -(D/m)\,\nabla(\rho)\bullet\nabla(1/\rho) = (-D/m)\,\partial_i\rho\;\partial_i\rho^{-1}$

$= (-D/m)\,\partial_i\rho\;[\,-\rho^{-2}\,\partial_i\rho\,] = (D/m\rho^2)\,|\nabla\rho|^2$

$\sec^{-1} = [\mathrm{m}^2\,\sec^{-1}]\;\mathrm{m}^{-2}$ $\qquad$ check

So I think I have just derived this result

$\nabla\bullet\,\mathbf{v} = -(\partial_t\rho)/\rho + (D/m\rho^2)\,|\nabla\rho|^2$

I install now $\partial_t\rho = (D/m)\nabla^2\rho$ in the first term,

$\nabla\bullet\,\mathbf{v} = [-(D/m)\nabla^2\rho]/\rho + (D/m\rho^2)\,|\nabla\rho|^2$

$= (D/m\rho^2)\,\{|\nabla\rho|^2 - \rho\nabla^2\rho\,\}$

Is there a simpler way to write this? VI says

$$\nabla^2(\rho^2) = 2\rho\nabla^2\rho + 2\ |\nabla\rho|^2$$

Well, suppose things vary only in the x dimension. Then

$$\partial_x v = (D/m\rho^2)\ \{(\partial_x\rho)^2 - \rho(\partial_x{}^2\rho)\ \}$$

Later comments on Diffusion. I think the correct answer here is that yes, diffusion goes on as a sort of separate process when you have sound waves. But the time constants are so different, that in a sound wave there is never time for any diffusion to take place, so diffusion is irrelevant. I will leave my flailing equations above in place in case they later prove of use.

My Thuras notes have explained everything Calvert states in his Plane Waves section.

Properties of Sound Waves. Again, see Thuras notes for details. He states the conditions under which the wave equation is valid, all of which I have shown in my Thuras notes. I think he means the ideal adiabatic gas law here. He then quotes lots of nice real world numbers for c. If we throw in the ideal gas law, we can write $c = \sqrt{\gamma RT/M}$ where M is the molecular mass in proton units, so this shows the famous square root dependence on Kelvin temperature.

Sine Waves and Power. Here he shows a little series of wave equations, I will do them here:

$$\xi(x,t) = A\, e^{-i(kx-\omega t)}$$

$$u = \partial_t\xi = i\omega\xi$$

$$\Delta = \partial_x\xi = -ik\xi$$

$$s \approx -\Delta = ik\xi$$

$$\Delta p = \gamma p_0 s = ik\xi\ \gamma p_0 = i\omega\xi\ (k/\omega)\gamma p_0 = u\ (k/\omega)\gamma p_0 = u\ \gamma p_0/c \equiv u\ r = ru$$

Here he defines a new item $u = \gamma p_0/c$. We have Δp (his p) = ru. You can think of this as a linear frictional resistance formula F = k x velocity, so therefore he refers to r as the wave resistance, or the acoustic impedance. In other words, we have $u = \Delta p/r$ so for a given Δp, how much displacement velocity u do we get? Small resistance means a lot of velocity. I don't think this implies and energy loss.

What is the meaning of "power in a wave" ? If you have a sound wave moving to the right, is energy moving to the right? It must be. The driver makes a compressed zone of the air by doing work on it, and that compressed zone moves to the right and later does work on an absorber.

Here is one idea for computing the power based on the Thuras picture. What is the work needed to push the little band from its rest position to its position on the right? $dW = Fd\xi$ and $F = \Delta p$ the overpressure. so

we would have W (push band from $\xi = 0$ to $\xi = \xi_0$) $= \int_0^{\xi_0} d\xi\, \Delta p = r \int_0^{\xi_0} d\xi\, u$. Let's try this with real functions:

$\xi(x,t) = \xi_0 \sin(kx-\omega t)$
$u = \partial_t \xi(x,t) = \xi_0 (-\omega) \cos(kx-\omega t)$

Let's sit at a fixed value of x and let a quarter cycle go by during which time this compression occurs. Then maybe we can say

$$W(1/4 \text{ cycle}) = r \int_0^{\xi_0} d\xi\, u = r \int_0^{T/4} dt\, (d\xi/dt)\, u = r \int_0^{T/4} dt\, u^2$$

$$= r \int_0^{T/4} dt\, \xi_0^2 \omega^2 \cos^2(kx-\omega t) = r \xi_0^2 \omega^2 \int_0^{T/4} dt \cos^2(kx-\omega t)$$

where $T = 2\pi/\omega$. Let $z = kx-\omega t$ so $dz = -\omega dt$ and then $z=kx$ and $z=kx-\omega T = kx-\omega(2\pi/\omega) = kx-2\pi$ so our integral is then

$$\int_0^{T/4} dt \cos^2(kx-\omega t) = \int_{kx}^{kx-2\pi} (-dz/\omega) \cos^2 z = -(1/\omega) \int_{kx}^{kx-2\pi} dz \cos^2 z$$

$$= (1/\omega) \int_{kx-2\pi}^{kx} dz \cos^2 z = (1/\omega) \int_0^{2\pi} dz \cos^2 z = (\pi/\omega)$$

So we find then that

$$W(1/4 \text{ cycle}) = r \xi_0^2 \omega^2 (\pi/\omega) = \pi r \xi_0^2 \omega$$

Now this work was done in time T/4, so the power must be

$$\text{Power} = \pi r \xi_0^2 \omega / (T/4) = 4\pi r \xi_0^2 \omega / T = 4\pi r \xi_0^2 \omega\, \omega / 2\pi = 2r \xi_0^2 \omega^2$$

Here is another way to do it. Start again with

$$dW = F d\xi = \Delta p\, d\xi$$

$$\text{power} = dW/dt = \Delta p\, \partial\xi/\partial t = \Delta p\, u = r u^2$$

He says power = P = pu*/2 and this just relates to the complex phasor notation.

Now he does a real world power example. Speaker radiating an actual 100W which is a very loud loud speaker. So P = 100W. We know that r = 42.6 cgs, so he is able then to find u = 1629 cm/sec. The condensation s = u/c = .05 and $\Delta p/p$ = .007. He computes all the numbers, a great example. But he does not actually say what the SPL would be for this plane wave (plane near the speaker I guess).

Velocity Potential. Another complete tour de force section! He ties us in with the "other way" of doing things, and notes that Eulerian is harder than Lagrangian for 1D plane waves, but easier in 3D. This would be a section to be carefully digested along with my Fluid book! The point is that you obtain a wave equation for the potential φ, and then $\mathbf{v} = +\nabla\varphi$. Since you obtain a vector velocity, they refer oddly to φ as the "vector potential". And he says that $\varphi = 0$ is the BC for an "open end" and Neumann for a closed end, so I wonder what on earth Stakgold's u function is in sound theory?

I finished reading this PDF, many interesting topics. The guy has a site I might want to emulate, and I will study it. The search engines seem to do it well.