

EM

Mass

Shifts

Feynman & Speisman
(1954)

Cini, Ferrari & Gatto
(1958)

Cottingham
(1963)

Coleman & Glashnow
(1963)

Coleman & Schnitzer
(1964)

Suzuki & Zachariasen
(1966)

Harar
(1966)

Ball & Zachariasen
(1968)

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Feynman & Speisman
(1954)

Proton-Neutron Mass Difference

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(Received February 23, 1954)

SUPPOSE all deviations from isotopic spin symmetry are due solely to electromagnetic effects. Then such things as the mass difference of charged and neutral π mesons, and the neutron-proton mass difference would have to be just electrodynamic.

We have investigated this point and have found that it is a reasonable possibility. For particles of zero spin, like π mesons, assumed to be elementary particles, the self-energy is quadratically divergent. If the photon propagation function¹ $1/k^2$ is cut off by a convergence factor $C(k^2) = [A^2/(A^2 - k^2)]^2$, the resulting energy is about $3e^2A^2/8\pi m$, where A is the cut-off energy and m is the π -meson mass, and we assumed $A \gg m$. This gives the observed mass difference of about 11 electron masses for a cut-off A of about 1.0 proton masses.²

It is usually assumed that the negative value of the proton-neutron mass difference speaks against an ultimate electromagnetic explanation. The following calculation shows that this is an unwarranted assumption. The particles are not simple and there is uncertainty as to the correct law of coupling to the electromagnetic field. But for low energy, the proton can be represented by the Dirac equation with an additional Pauli term to represent the anomalous moment. Since we do not know to how high an energy this may be a reasonable approximation, we have tried providing the moment coupling term with a cut-off factor of its own. We write for the proton self-energy

$$\Delta M = (e^2/\pi i) \int \left[\gamma_\mu - \frac{\mu}{4M} (\gamma_\mu k - k \gamma_\mu) G(k) \right] (\not{p} - k - M)^{-1} \times \left[\gamma_\mu + \frac{\mu}{4M} (\gamma_\mu k - k \gamma_\mu) G(k) \right] k^2 d^4k C(k)$$

in the notation of reference 1. We used $G(k) = -\lambda^2(k^2 - \lambda^2)^{-1}$ to cut the moment coupling off at energies about λ , and $C(k) = -A^2(k^2 - A^2)^{-1}$ to cut off the photon propagation function at energy A . The expression for the neutron is the same, except that the γ_μ coupling terms are omitted and the value of μ , the anomalous moment in nuclear magnetons, is -1.91 instead of 1.79 for the proton. M is the nucleon mass.

For the proton the term for $\mu=0$, representing coupling of current with current, is positive, as is also the term in μ^2 . But the cross term, linear in μ , is negative and quite large if the moment is not cut off too soon. Thus the proton-neutron mass difference can easily turn out negative. For example, if A and λ are both taken at $1.4M$, the experimental value of -2.5 electron masses results for this difference (in this case for the neutron ΔM is roughly 1.5 , for the proton -1.0 electron masses). No small difference of large numbers is involved. If the cut-off λ is reduced below about $0.75M$, a negative mass difference cannot be obtained. For $\lambda=1.0M$, $A=4M$ gives the experimental result.

The high cutoff for the anomalous moment implies that the charge responsible for the moment must be spread over only a small distance (of order $\hbar/Mc$). This is also suggested by the relatively small changes that the nucleon moments undergo when nucleons form nuclei.

The cutoff for the propagation function may be interpreted in two ways. Firstly, electrodynamics may fail at high energies, the failure being represented in a crude way by the cutoff. If this is so we could guess from our results that the failure occurs at energies in the neighborhood of the nucleon mass. Another possibility is that the electrodynamics is correct, but the cutoff represents, roughly, the error committed in assuming that the particles are elementary. For example, in the case of the π meson, we have assumed the π meson in virtual states acts as a simple particle. But for energies as high as M , strongly coupled virtual nucleon pairs may be formed. They, rather than failure of electrodynamics, may provide the convergence at energies of order M . In a like

manner, the complex of virtual mesons presumed to be associated with nucleons may have the effect that at sufficiently high energy the electromagnetic coupling of neutron and proton may be nearly the same, so that the integral representing the difference of their masses may converge without modification of electrodynamics. In this way, the presumed convergence of the mass differences might tell us something about the character of coupling with the electromagnetic field at high energy.

We conclude that all of the deviations from isotopic spin symmetry could be due solely to coupling with the electromagnetic field.

¹ R. P. Feynman, Phys. Rev. 76, 769 (1949). We use the notation in this reference.

² This result was given by one of us (RPF) at the International Conference in Theoretical Physics, Paris, 1950 (unpublished).

Polarization of Elastically Scattered Nucleons from Nuclei*

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(Received February 23, 1954)

NUCLEONS of low or moderate energy which are elastically scattered from nuclei should be partially polarized¹ by the strong spin-orbit potential underlying the predictions of the shell model of the nucleus. This spin-orbit potential is a consequence of the collective action of many nucleons on the particular nucleon. Thus for incident nucleons whose wavelength is greater than the nuclear spacing ($E \lesssim 50$ Mev), it would be expected that the spin-orbit potential of the shell model would make itself felt. For progressively higher energies the incident nucleon begins to see only one nucleon at a time and while a spin dependence of the elastic scattering can still be expected, it would be more a reflection of the individual nucleon-nucleon interactions than of the spin-orbit potential of the shell model. It will be supposed that even at these higher energies the spin dependence has the form of the usual spin-orbit potential. In either case this spin dependence of the elastic scattering can be investigated phenomenologically by treating the interior of the nucleus in terms of a spin-dependent complex index of refraction²—an obvious generalization of the optical model of the nucleus.³

For low or moderate energies there is no suitable approximate method for treating the elastic scattering—a phase-shift analysis is necessary. Also, at high energies any polarization calculations using conventional approximation methods⁴ are made uncertain by the direct dependence of the polarization on the phase of the scattered wave. A phase-shift analysis for various energies is therefore being undertaken on the Univac at the University of California Radiation Laboratory at Livermore in collaboration with S. Fernbach.

An estimate for small angles of scattering, though rough at best, may be readily obtained by making several simplifying assumptions. The magnitude of the polarization is given by

$$P = \left(\frac{AB^* + A^*B}{d\sigma/d\Omega} \right) \sin\theta. \quad (1)$$

Here A and B represent the scattering amplitudes corresponding to the spin-independent and spin-dependent parts of the interaction, respectively. The known experimental value for the differential cross section $d\sigma/d\Omega$ may be used. The amplitude, B , for spin-dependent scattering may be estimated by using the Born approximation. Then only the imaginary part of A contributes to P . For small angles this is approximately proportional to the total cross section.

For 300-Mev neutrons incident on carbon, for example, a square-well spin-orbit interaction ($R=1.4A^{1/3} \times 10^{-13}$ cm) of 2-Mev depth gives a polarization of 40 percent at five degrees. Though this is probably an overestimate, it suggests that the existence of a small

spin-orbit in polarizations more phase large diffract polarized pa

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† W. Hecke Laboratory R
‡ This meth scattering by
§ Fernbach.
¶ For exam always gives a

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The cross where the fin as the initial old energy 2

Since only lowest order symmetrical

$\phi(p)$ a result in Here $E=2A$ particle in t

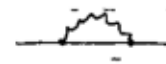
The (p,p) principle, sin state of zero of the final production b there are a gr possibility of states. These predominant

The (n,p) $\phi(n)$

I wish to problem.

* Based on doctor's deg
† Now at L
‡ C. W. He-
§ M. Taker
¶ Units $A=$ fine structure

Feynman and Speisman, (Caltech 1954.)

- ① I presume this is the first paper ever to compute  as the em mass shift of a particle. Here they consider a pion with (I presume) the obvious Klein-massimal coupling. The calculation is done in BD-I p 164 for an electron. In either case result is positive, i.e., π^+ heavier than π^- .

$$\delta m_{\pi^+} = \frac{3e^2 \Lambda^2}{8\pi m}$$

$$\delta m_e = \frac{3dm}{4\pi} \ln \left[\frac{\Lambda^2}{m^2} \right]$$

In either case you are IR OK, but need a UV cutoff Λ . Now FS show that if $\Lambda = 1.0 m_p$, you get right answer for the $\pi^+ - \pi^0 = 4.6 \text{ MeV}$, roughly.

- ② Note that $K^+ - K^0 = -4 \text{ MeV}$, so is more like $p-n$ situation. To explain the fact that $p-n < 0$, FS include a " μ " type term along with the Dirac coupling. Each mag-moment term is cutoff at Λ , but photon still cuts off at Λ . Since $\mu_p \neq \mu_n$, they are able to get the sign reversal.

- ③ Very simple piece of work, but they were the first to do it!

Cini, Ferrari & Gatto
(1958)

versal by about 16°C, in contrast to the Te-Se alloy mentioned above, for which the reversal temperature was apparently lowered about 5°C. The impurity concentration of the two samples of Fig. 1, as determined from the Hall coefficient R at 77°K and the approximate formula $R = 1/pe$ (p = carrier density, $e = 1.6 \times 10^{-19}$ coulomb), is 7.2×10^{14} carriers/cm³ for sample 1 and 2.2×10^{15} carriers/cm³ for sample 2. In addition, the upper reversal temperature for a third sample with $p = 7.2 \times 10^{16}$ was found to occur at 498°K and this demonstrates the shift of the upper reversal temperature with impurity concentration. Long¹² has computed that the hydrostatic pressure used in this experiment causes the energy gap to decrease by 0.032 eV, and this in turn is responsible both for the decrease in the lower reversal temperature which he reported and for the decrease in Hall coefficient with pressure in the region below the "cross-over" point at about 217°C on both pairs of curves in the present experiment. However, the cause of the upper reversal shift still remains unknown.

¹P. I. Wold, Phys. Rev. **7**, 169 (1916).

²Fukuroi, Tanuma, and Tobisawa, Science Repts. Research Insts., Tôhoku Univ. **A1**, 373 (1949).

³V. E. Bottom, Science **115**, 570 (1952).

⁴A. Nussbaum, Ph. D. thesis, University of Pennsylvania, 1954 (unpublished).

⁵H. Fritzsche, Science **115**, 571 (1952).

⁶H. B. Callen, J. Chem. Phys. **22**, 518 (1954).

⁷R. Gáspár, Acta Phys. Acad. Sci. Hung. **7**, 289 and 313 (1956).

⁸G. Dresselhaus, Phys. Rev. **105**, 135 (1957).

⁹A. Nussbaum, Phys. Rev. **94**, 337 (1954).

¹⁰T. J. Davies, J. Appl. Phys. **28**, 1217 (1957).

¹¹H. Roth, as reported at the International Conference on Semiconductors, Rochester, August, 1958 (unpublished).

¹²D. Long, Phys. Rev. **99**, 388 (1955).

¹³D. Long, Phys. Rev. **101**, 1256 (1956).

NEUTRON-PROTON MASS DIFFERENCE BY DISPERSION THEORY

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(Received December 3, 1958)

An attempt to explain the neutron-proton mass difference by introducing in the divergent inte-

grals of the second-order electromagnetic self-mass a fundamental length has been made by Feynman and Speisman.¹ They show that the proton can in fact turn out lighter than the neutron, in spite of its electrostatic energy, if the anomalous moments of the nucleons are introduced in the interaction and the integrals are cut off at sufficiently high energy. Since then it has been realized that the electromagnetic self-masses of the nucleons might be finite even in a microscopic causal theory if the electromagnetic form factors vanish sufficiently rapidly at high momenta, as indicated by certain consistency requirements.² With this assumption Wick and Sorensen³ have attempted a calculation of the nucleon mass difference on the basis of a formalism developed by Low.⁴ Their calculation yields a negative result, giving a proton heavier than the neutron. No direct comparison can, however, be made between the two calculations. In FS the relativistic Born approximation to the self-energy is used, while WS are essentially led to a Born approximation formula with form factors, in which only positive-energy states are kept.

For this reason we have re-examined the problem by using as a basis the expression of the Compton scattering amplitude derived with a dispersion relation approach. The single-nucleon contribution is in our case simply the FS expression with their arbitrary cutoffs replaced by the electromagnetic form factors of the nucleons. We find that the mass difference that one obtains by extrapolating at high momenta the experimental form factors is wrong in sign and in magnitude. The reason is that the radii of the experimental distributions correspond to cutoffs considerably lower than those used in FS. We also find that the correct mass difference can be obtained, from the single-nucleon contribution and without contradicting the Stanford data, with a rather pathological neutron charge distribution, concentrated at small distances.⁵ It may well be, on the other hand, that the main effect comes from the many-particle intermediate states.

We start, as in FS, with the expression:

$$\langle p' | S_2 | p \rangle = -i\delta m(2\pi)^4 \delta(p-p') \bar{u}(p') u(p) \\ = \frac{1}{2} \delta(p-p') \int \frac{d^4 k}{k^2 - i\epsilon} \phi(k), \quad (1)$$

where

$$\phi(k) = i \int d^4 x e^{ikx} \langle p | T(j_\mu(\frac{1}{2}x), j_\mu(-\frac{1}{2}x)) | p \rangle, \quad (2)$$

with $j_\mu(x)$ the Heisenberg current operator. In

Cini, Ferrari and Gatto (1958)

1. Basically, they repeat exactly the FS calculation of the p-n mass difference. Recall that this calculation is based on approximating the proton-photon Compton amplitude by the nucleon pole term. In QED this is justified because coupling weak, but maybe not so for proton. Anyway, the difference in CFG from FS is that CFG use actual experimental form factors for the neutron and photon. I.e., they use the forms (8) in the formula (9). Eg, the neutron has identically zero charge form factor because one guesses it has no charge anywhere in it. The other three form factors go like $1/q^4$ for large q , a fact I recall from Feynman. Since the data set the scale for these form factors, you get a definite answer.

2. The answer is the wrong sign! Comparing to FS, you see that the experimental form factor cuts off roughly at _____ which is less than the artificial cutoff's used by FS. It just doesn't work! If you insist that the nucleon-pole term work, you have to make the form factors have large k^2 meat to them, and that in turn implies small scale structure to the charge and magnetic moment distributions in the nucleons. Authors make a little model where the neutron charge form factor is doctored up in some way to make things work, but seems artificial.

3. They seem to suggest that the answer lies in not using just the pole term for the Compton amplitude.

Cottingham
(1963)

The Neutron Proton Mass Difference and Electron Scattering Experiments

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In this work equations are derived that relate, to first order in e^2 , the neutron proton mass difference to information directly obtainable from electron scattering experiments. The equations are derived under two principal assumptions. The first that the mass difference is electromagnetic in origin. The second relates to the convergence properties of the theory.

I. THE ELECTROMAGNETIC SELF-ENERGY OF THE NUCLEON

In perturbation theory, the electromagnetic self-energy of a nucleon can be formally expressed, to first order in e^2 , as (1)

$$\Delta M = \frac{i}{(2\pi)^2} \frac{1}{2} \int_{-\infty}^{+\infty} \frac{T_{\mu\nu}(q, q^0) g^{\mu\nu}}{q^2 - q^{02} - i\epsilon} d^4 q \quad (1.1) \quad \text{OK}$$

where $g^{\mu\nu}$ is the metric tensor

$$T_{\mu\nu}(q, q^0) = i\pi \int_{-\infty}^{+\infty} e^{-iq \cdot x + i q^0 x^0} \left[\langle n | P(j_\mu(x), j_\nu(0)) | n \rangle + \langle n | P(j_\nu(x), j_\mu(0)) | n \rangle \right] d^4 x \quad (1.2) \quad \text{OK}$$

and $j_\mu(x)$ is the Heisenberg current operator. $\langle n | P(j_\mu(x), j_\nu(0)) | n \rangle$ is the expectation value of the time ordered product for the single nucleon state (at rest). Formally, $\eta^{(\mu)} \eta^{(\nu)} T_{\mu\nu}(q, q^0)$ is the forward Compton amplitude for the scattering of a virtual photon with three-momentum q , energy q^0 , and polarization vector η^μ from a nucleon at rest.

There are two independent scalar invariants associated with this amplitude. They are defined to be

$$\begin{aligned} q^2 &= |q|^2 - q^{02} = g^{\mu\nu} q_\mu q_\nu \\ q^0 &= -\frac{1}{M} q \cdot l = -\frac{1}{M} g^{\mu\nu} l_\mu q_\nu \end{aligned} \quad (1.3)$$

where l_μ is the four-momentum of the nucleon and M is the nucleon mass. The calculations are performed in the "laboratory" system in which $l = (0, M)$. q^0 is the photon energy in this system.

THE NEU

There are two gauge invariance. They are defined here the four-vector q .

$$D_{1,\mu\nu} = q^2$$

$$D_{2,\mu\nu} = q$$

$T_{\mu\nu}(q, q^0)$ can be expressed in terms of coefficients being functions

$$T_{\mu\nu}(q, q^0)$$

From Eq. (1.2), $T_{\mu\nu}(q, q^0)$

$$\begin{aligned} T_{\mu\nu}(q, q^0) &= i\pi \int_{-\infty}^{+\infty} e^{-iq \cdot x} d^4 x \\ &+ i\pi \int_{-\infty}^{+\infty} e^{-iq \cdot x} d^4 x \end{aligned}$$

where $\sum | \kappa \rangle \langle \kappa |$ is a sum over all states. If we now make the transition

where P is the total four-momentum over x in Eq. (1.6), we obtain

$$T_{\mu\nu}(q, q^0) = \frac{(2\pi)^4}{2} \sum \langle n | j_\mu(x) j_\nu(0) | n \rangle \left[\frac{\delta^2(x - 0)}{k^0 - M} \right]$$

where (k, k^0) is the four-momentum of the photon.

The delta function of the integration over three-space $d^3 x$

Consider $T_{\mu\nu}(q, q^0)$ as a function of q^0 . It is seen to be an analytic function of q^0 and cuts just below the positive real axis. The poles are the single nucleon states.

$$q^0 =$$

1. C writes the usual formula for dM in terms of the object T_{uv} . Then T_{uv} is written as a d^4x integral of $(p/j_u(x) \cdot j_v(0)/p)$ expo, but symmetrized. Because currents must be conserved, your candidate tensors must be gauge invariant, and the usual two tensors are written in (1.4). The structure functions are then called t_1 and t_2 . When the complete d^4x integration is performed, you get (1.7) which shows some energy poles and the feynmanlike $\text{SUM } (n/j_u(0)/k)(k/j_v(0)/n)$. The thing to note here is that now both currents are evaluated at $x=0$. Taking the imaginary part of (1.7) then tells you: $\text{Im } T_{uv} = \text{SUM } j_u \cdot j_v \cdot \delta^4$, which now looks like unitarity. I had been wondering where the pole came from, now I see: it comes from the dq^0 integration and the momentum translation deal.

2. So if you contract u on v , (1.7) gives the integrand of the dM formula. As a function of q_0 this thing has poles and cuts; the photon propagator also makes two poles. But things are such that you can rotate the contour CC to the vertical, thus getting (1.8) where d^4q is now euclidean. Now here is the point: in (1.8) you have $T_{uu}(q; iq_0)$ so energy is always imaginary. Thus, the q^2 is exclusively spacelike. Thus, you don't need timelike information (like from $e+e^-$) to do this integral.

3. Now simply substitute (1.5) into (1.8) to get dM as an integral over spacelike q^2 and over q^0 of t_1 and t_2 . Next, disperse t_1 and t_2 and note their pole contributions (involving form factors). Note that the cut disc $\text{Im} t_i$ is called h_i . Finally, if you shove these dispersions into (1.9), you get final result (1.13) through (1.15). I.e., you get the mass difference in terms of a spacelike q^2 integral of form factors (which we later get from elastic scattering), and as a double integral of the Imag part of the structure functions, which we hopefully get from inelastic (deep) scattering. It is nice to have clean formulas at last. There seems to be a clear experimental goal now.

II. Electron-Nucleon Scattering. Here, in formula (2.4), we see the deep inelastic DCS written in terms of the familiar lepton trace and the feynmanic current stuff which we realize is just $\text{Im} T_{uv}$, as noted above. Thus, you can write the DCS in terms of h_1 and h_2 (it is these which must connect to W_1 and W_2). This is (2.6). Can be reparametrized in terms of transverse and long polarized off shell photon cross sections if you like, as on page 429. Similarly, but restricting to the elastic ~~simplex~~ events, you can write elastic DCS as in (2.8), from which you can get the form factors f_1 and f_2 . So if someone would go do these experiments, you could find all these functions and plug them into (1.13) through (1.15) and get a true number for the n - p mass difference. This calculation would then include the effect of all intermediate states, and would therefore be better than the purely pole

models of earlier papers. C obviously eagerly awaits experiments.

III. Possible subtractions. The catch is of course that when you disperse your functions t_1 and t_2 , depending on the large q^0 behavior, there may have to be subtraction constants also in the dispersion relation. Apparently there is a way to show that t_2 cannot have such a constant, but t_1 could. So in the end, data may show that the t_1 integration fails to converge. Then you are cooked unless you can find a way to get that subtraction constant.

Comparison to CFG paper: CFG know that you should think about inelastic states and about subtractions, but in the end they only did the elastic form factor thing. By inserting only the pole term of their dispresl (5) into dM formula, and assuming no subtractions (polynomials) they got (7), and then they show based on elastic data that you get the wrong sign. Cottingham did much more: he included the inelastic stuff and went on to show how you could get the inelastic data from inelastic e-p scattering. I wonder: is Cottingham one of the pioneers of this idea of doing deep-inelastic experiments and connecting that data to dM, or was that whole program well-known at the time?

Cottingham:

1) Zee, PR 3C, 127 (1972) review App. A.

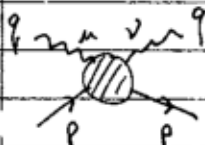
Treat proton as a particle. Consider this contribution to δm .



low hadron pert theory.



"to all orders" in strong.



= off-shell "Compton" amplitude. Use LSZ to get

$$= T_{\mu\nu}(p, q) = \langle p | T(\bar{\psi} \gamma_\mu \psi) | q \rangle.$$

I.e., pull out photon states + somehow get currents. See Feynman!

Neutron does not have this diag, so

$$\delta m = m_p - m_n = \int d^4 q \Delta_{\mu\nu}^{\text{photon}}(q) \cdot [T_{\mu\nu}^p - T_{\mu\nu}^n].$$

Key. B2 in n this PR 177, (1969) 2264

Relation to "form factor"?

Cottingham original ref: AP 25, 424 (1963)

① How does one get (1.2) from the LSZ reduction formulas? The Compton amplitude starts out as: $\langle p'q' | p q \rangle$, i.e., regular S-matrix element. Look at BD p 165 to learn

$$\langle p'q' | p q \rangle = -(i)^1 \int d^4x_1 \frac{1}{\sqrt{(2\pi)^3 2E_1}} e^{i p' \cdot x_1} \square_{x_1} \langle p_2 q_2 | A_{\mu_1}(x_1) | p_1 \rangle \epsilon^{\mu_1}(q_1, \lambda) \quad \begin{array}{l} \text{pull} \\ \text{integ.} \\ \text{norm} \\ \text{expt. operatn} \\ \text{photn field at } x_1 \\ \text{pol.} \end{array}$$

Now pull both photons:

$$\langle p_2, s_2; q_2, \epsilon_2 | p_1, s_1; q_1, \epsilon_1 \rangle = (i)^2 \int d^4x_1 d^4x_2 \frac{1}{\sqrt{(2\pi)^3 2E_1}} \frac{1}{\sqrt{(2\pi)^3 2E_2}} e^{-i q_1 \cdot x_1} e^{i q_2 \cdot x_2} \times \epsilon^{\mu_1}(q_1) \epsilon^{\mu_2}(q_2) \cdot \square_{x_1} \langle p_2, s_2 | [A_{\mu_1}(x_2) A_{\mu_2}(x_1)] | p_1, s_1 \rangle \square_{x_2}$$

where

$$T\{A_\nu(x_2) A_\mu(x_1)\} = \theta(t_2 - t_1) A_\nu(x_2) A_\mu(x_1) + \theta(t_1 - t_2) A_\nu(x_1) A_\mu(x_2).$$

Now, how do we apply $\square_{x_1} = (\partial_\mu^2) \big|_{x_1} = (\frac{\partial^2}{\partial t_1^2} - \vec{\nabla}_{x_1}^2)$?

Recall that $\square A_\mu = j_\mu$ so ignoring those thetas, you get

$$\text{amp} = \int d^4x_1 d^4x_2 \frac{e^{-i q_1 \cdot x_1}}{\sqrt{2E_1}} \frac{e^{i q_2 \cdot x_2}}{\sqrt{2E_2}} \epsilon^{\mu_1}(q_1) \epsilon^{\mu_2}(q_2) \langle p_2 | T\{J_\mu(x_1) J_\nu(x_2)\} | p_1 \rangle$$

$$\text{But } \square_\mu \theta(t_2 - t_1) = (\partial_\mu^2 - \vec{\nabla}^2) \theta(t_2 - t_1) = \partial_\mu^2 \theta - \theta \cdot \nabla^2$$

$\partial_{x_1} \theta = \delta(t_1 - t_2)$ $\partial_{x_1}^2 \theta = \delta'(t_1 - t_2)$, so pick up a term like so:

$$\int d^4x_1 d^4x_2 \frac{e^{-i q_1 \cdot x_1}}{\sqrt{2E_1}} \frac{e^{i q_2 \cdot x_2}}{\sqrt{2E_2}} \epsilon^{\mu_1}(q_1) \epsilon^{\mu_2}(q_2) \langle p_2 | [\partial_\mu \theta(t_2 - t_1) A_\nu A_\mu + \partial_\mu \theta(t_1 - t_2) A_\nu A_\mu] | p_1 \rangle$$

Coleman & Glashnow
(1963)

$$\text{Im} \left[\phi^* \frac{d\phi}{d(kr)} \right] = 1, \text{ for all } r. \quad (\text{A6})$$

Now, from Eq. (4.14) we write $S = \eta e^{2i\delta}$ in terms of $f_{\text{eff}} = f_r + if_i$

$$S = - \left(\frac{f_{\text{eff}} \phi^* - \beta \phi'^*}{f_{\text{eff}} \phi - \beta \phi'} \right), \quad (\text{A7})$$

where the prime denotes differentiation with respect to r , $\beta \equiv kr_0$ and ϕ , ϕ' , ϕ^* , ϕ'^* are evaluated at $kr = \beta$.

$$\therefore S = \frac{A^* + i\phi^* f_i}{A + i\phi f_i}, \quad (\text{A8})$$

where $A \equiv f_r \phi - \beta \phi'$.

$$\therefore |S|^2 = \frac{|A|^2 + f_i^2 |\phi|^2 - 2f_i \text{Im} A \phi^*}{|A|^2 + f_i^2 |\phi|^2 + 2f_i \text{Im} A \phi^*}. \quad (\text{A9})$$

But

$$A \phi^* = f_r |\phi|^2 - \beta \phi' \phi^*.$$

$$\begin{aligned} \therefore \text{Im} A \phi^* &= -\beta \text{Im} \phi^* \frac{d\phi}{d(kr_0)} \\ &= -\beta \quad [\text{from (A4)}]. \end{aligned} \quad (\text{A10})$$

Thus, from (A9) and (A10),

$$\begin{aligned} |S|^2 &= \frac{|A|^2 + f_i^2 |\phi|^2 + 2\beta f_i}{|A|^2 + f_i^2 |\phi|^2 - 2\beta f_i} \\ &\leq 1, \text{ if and only if, } \text{Im} f_{\text{eff}} \leq 0; \end{aligned} \quad (\text{A11})$$

we know that $|S|^2 \leq 1$, if and only if, f is Hermitian. Thus, f is Hermitian, if and only if,

$$\text{Im} f_{\text{eff}} \leq 0. \quad (\text{A12})$$

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Departures from the Eightfold Way: Theory of Strong Interaction Symmetry Breakdown*

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AND

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We consider the three kinds of departure from exact unitary symmetry: medium-strong interactions which leave only isospin and hypercharge as good symmetries, electromagnetism, and weak interactions. We postulate the existence of an octet of scalar mesons that give the possibility of symmetry-breaking tadpole diagrams. Our fundamental dynamical assumption—that symmetry-violating processes are dominated by symmetry-breaking tadpole diagrams—gives an immediate explanation of the success of two empirical laws: the Gell-Mann-Okubo mass formulas and the nonleptonic $\Delta I = \frac{1}{2}$ rules. Moreover, including tadpole diagrams and some other electromagnetic corrections, we calculate the six electromagnetic mass splittings of mesons and baryons in terms of a single unknown parameter correctly to within 0.5 MeV.

I. INTRODUCTION

WE assume that the fundamental interactions of elementary particles fall into the following classes, arranged in order of diminishing strength (we omit gravity):

1. Very-strong interactions, invariant under the transformations of "the eightfold way"—the symmetry

scheme of Gell-Mann¹ and Ne'eman,² based on the group SU(3).

2. Medium-strong symmetry-breaking interactions, invariant under only the isospin-hypercharge subgroup of SU(3). Whether these interactions are introduced at the beginning, or whether they arise by some kind of spontaneous symmetry breakdown is immaterial to our discussion.

3. Electromagnetism.

4. Weak interactions.

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¹ M. Gell-Mann, California Institute of Technology Synchrotron Report CTSL-20, 1961 (unpublished); Phys. Rev. 125, 1067 (1962).

² Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

Coleman and Glashow: Mass Splittings in terms of Tadpoles (1963)

1.-The goal of this paper is to unify various schemes which have been invented to explain mass splittings. Eg, the usual scheme of an L_{ms} medium-strong symmetry breaking lagrangian which transforms as F_8 is called octet dominance and leads to the Gell*mann Okubo mass formulas. There is a similar rule which leads to certain electromagnetic splitting sum rules. Also, there are certain weak interaction rules. I am ignoring the parts of this paper which pertain to weak interactions, at least for now.

2. The proposal is this: assume there exist an octet of scalar mesons (by the way, none of the usual mesons are scalars). For such a meson, you can have "tadpole" feyman diagrams which give non-zero contributions. Eg, in a mass calculation you would include  along with the standard diagrams like . The point is that a scalar meson has vacuum quantum numbers as far as parity and spin are concerned, so you can have such a meson with momentum $q = (0, 0, 0, 0)$ dissolve into nothing. Of course, if you postulate as CG do that these mesons form an octet, then the fact that they can dissolve into vacuum represents an $SU(3)$ breakage. Ie, these scalar mesons are the means by which unitary symmetry breaking occurs. Ie, you assume a non-zero VEV for such mesons. Obviously, by the way, this whole idea should be and no doubt has been reworked in terms of SSB gauge theories.

3. Now, although giving non-zero VEV's breaks SU_3 , the Yukawa couplings (interaction lagrangian involving these scalars) are assumed to be su_3 scalars, as in (2). It turns out as usual that there are two scalar couplings, with constants d and f , when you couple to baryons. So, the information contained in (1) and repeated in (2) leads to the formulas (3) for self-masses in the one-tadpole approximation. Note that m_0 and u_0 are masses in limit of no tadpole contributions. There are 13 masses given in terms of 6 parameters, hence there will be 7 sum rules, as seen on p 675.

4. Results: all known rules are duplicated in a unified way, and some new rules are found. All rules are reasonably obeyed. Similarly, the decuplet rule comes out, (11).

4. In Section (4) the focus is on the em mass splittings. Eg, the n - p splitting. As you see in the chart page 676, the usual calculation with form factors gives the wrong sign (see nontadpole or  column). The tadpole graph is different for p and n in the right way to give an overriding right-sign contribution. In fact, all 6 splittings listed come out with right sign and to maybe 30% or better. Notice that for the pion there is no tadpole contribution (see result (8)), so this explains why the conventional calculation gives the right answer in this case.

A simple interpretation of this tadpole stuff is shown in figure 3(c). A scalar meson can couple directly to a photon (over)

and thus you get t-channel pole contributions to the Compton amplitude . These things contribute to the cut part of the dispersion relation, or maybe to the subtraction constant, and such terms are usually ignored in the "conventional" calculation. They don't say much about Regge theory, but obviously these exchanges can be Regge-interpreted.

5. Comment: it is claimed that because you are doing a complete d^4q integration to get self mass, only a spin-zero exchange in t-channel can contribute. I will have to think about that.

6. Section V on weaks I have not read.

7. Remaining remarks: why is GMO formula so accurate? You would expect that the medium-strong tadpole η' single-tadpole approx is not so good, and you would expect maybe multi-tadpole contributions would be significant. But perhaps somehow these have already been included by using experimental masses... Another point I glossed over is the fact that the scalar meson coupling has to be universal, ie, coupling of scalars to the 0^- , 0^+ and 1^6 mesons is the same. This fact was needed to get various sum rules.

8. Pignotti is quoted privately as conjecturing about scalars = ghost regge effect. This idea later carried by Fred and Jim.

Coleman & Schnitzer
(1964)

Departures from the Eightfold Way. II. Baryon Electromagnetic Masses

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The leading nontadpole contributions to the electromagnetic mass splittings of the baryons are calculated. Best fits to experiment are used for the nucleon form factors; the strange baryon form factors are obtained from these by unitary symmetry. Account is taken of the breakdown of unitary symmetry by using experimental masses in intermediate states and by including the effects of θ - ω mixing. These calculations and the assumption of tadpole dominance are used to fit the six mass splittings of baryons and pseudoscalar mesons with one free parameter. All of the splittings are in good agreement with experiment except for the kaon splitting, which has the right sign but only $\frac{2}{3}$ of the proper magnitude. The results of a preliminary version of this calculation were reported in the first paper in this series.

I. INTRODUCTION

SEVERAL years ago, Cini, Ferrari, and Gatto¹ obtained a formula for the electromagnetic self-mass of the nucleon in terms of its electromagnetic form factors.² The formula is represented diagrammatically by Fig. 1, where the blobs represent form factors, and analytically by the expression

$$i\bar{u}u = \frac{-ie^2}{(2\pi)^4} \int \frac{d^4k}{k^2 + i\epsilon} \left[F_1(k^2) + \frac{i}{2m} F_2(k^2) \sigma_{\mu\nu} k_\nu \right] \\ \times \frac{p_\lambda \gamma_\lambda + k_\lambda \gamma_\lambda + m}{k^2 + 2p \cdot k + i\epsilon} \left[F_1(k^2) - \frac{i}{2m} F_2(k^2) \sigma_{\mu\nu} k_\nu \right] u, \quad (1)$$

where m is the mass of the nucleon, e is its charge, and F_1 and F_2 are the form factors, normalized such that $F_1(0)$ is one and $F_2(0)$ is the anomalous moment in nuclear magnetons.

Equation (1) is obtained by writing the self-mass in terms of nucleon-photon scattering (with unphysical photons), and then approximating the scattering amplitude by the contribution from one-nucleon intermediate states (pole term). It has been often applied to calculations of the neutron-proton mass difference, with a notorious lack of success.^{1,3} Several reasons have been advanced for this failure:

(1) Equation (1) neglects the contributions from intermediate states heavier than one nucleon. Perhaps these contributions are large.⁴

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† Partially supported by the National Science Foundation.

¹ M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters 2, 7 (1959).

² A closely related formula had been obtained earlier from a somewhat different viewpoint by R. Feynman and G. Speisman, Phys. Rev. 94, 500 (1954).

³ S. Sunakawa and K. Tanaka, Phys. Rev. 115, 754 (1959).
⁴ W. N. Cottingham, Ann. Phys. (N. Y.) 25, 424 (1963). Of course, the contribution from the next state, the 3-3 resonance, vanishes because of isospin symmetry; but still higher resonances may make nonvanishing contributions. The contributions of other

(2) Equation (1) is critically dependent on the behavior of the form factors at high momentum transfers, of which we know nothing. Indeed, if we use form factors with hard cores, the formula diverges quadratically. In order to obtain convergent results, we must use a cutoff; widely different mass differences may be obtained, depending on the choice of cutoff.⁵

(3) Equation (1) neglects the contributions from possible scalar meson tadpole diagrams.⁶ This is not, as might appear, a special case of the intermediate-state criticism stated above; the scalar tadpoles simply add a constant to the unphysical photon-nucleon scattering amplitude, and make no contribution to the absorptive part.

This third viewpoint was proposed in the first paper in this series, and it is the one we adhere to here. The main body of this paper (Sec. II) is a calculation of the leading nontadpole contributions to the electromagnetic mass splittings, not only of the nucleon, but of all the baryons, using Eq. (1). Experimental best-fit form factors are used for the nucleon,⁷ and unitary symmetry is used to obtain the strange baryon form factors from these.⁸ Account is taken of departures from unitary symmetry by using physical masses for intermediate states and by correcting the form factors for ϕ - ω mixing.⁹

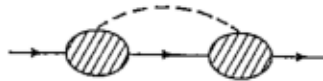


FIG. 1. The leading nontadpole contribution to the electromagnetic mass splittings of the baryons. The blobs represent electromagnetic form factors.

members of the $3/2^+$ decuplet to other baryon splittings do not vanish because of symmetry; preliminary calculations indicate that they may be significant [R. Socolow (private communication)].

⁵ L. Pande, Nuovo Cimento 26, 1063 (1962).
⁶ S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964).
⁷ G. Hand, D. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).
⁸ S. Coleman and S. Glashow, Phys. Rev. Letters 6, 423 (1961).
⁹ S. Coleman and H. Schnitzer, Phys. Rev. 134, B863 (1964).

Form Factors.

$$F_{13} = \frac{a}{q^2 - A} - \frac{b}{q^2 - B} = \frac{q^2(q^2 - B)a - b(q^2 - A)}{(q^2 - A)(q^2 - B)}$$

$$= \frac{q^2(a - b) + (Ab - aB)}{q^4 - q^2(A + B) + AB} \Rightarrow \frac{(a - b)}{q^2} \text{ as } q^2 \rightarrow \infty.$$

So all these F_i 's go as $1/q^2$. Look at (2a)

$$G_E(q^2) \sim \frac{C_1}{q^2} + \frac{q^2}{4m^2} \frac{C_2}{q^2} \rightarrow \text{const} + \frac{C_1}{q^2}$$

$$G_M(q^2) \sim \frac{C_2}{q^2}$$

Is this the current exper form? Feynman (1970) p 115 says

$$G_E \sim G_M \sim \frac{1}{(1 + \frac{q^2}{.71})^2} \sim \frac{C_1}{q^4} + \text{no constant.}$$

Seems to disagree! Why?

1. This short paper is a followup of the tadpole paper of Ref. 6. Here they concentrate on the conventional form-factor calculation of the (nontadpole) one-pole piece. They use the old formula, here stated as (1) for baryons, and the game is (a) to use the experimental form factor data and integrate and see what you get ; (b) to use SU3 to guess what the corresponding form factors are for strange baryons.
2. One technical aspect of the paper involves the idea of "mixing", I don't know what this means, except it involves correction to the SU3 predictions of things. There is ref 's 8 and 9 on this if I care to learn about it. The results are given in Table I and II. The numbers differ by order of 1 MeV depending on whether or not you incorporate "mixing". Their point here is simply that you probably cannot trust these calculations to better than 1 MeV since different calculations differ by this much.
3. Remember that Ξ^- and Ξ^+ are not antiparticles, so their predicted splits from the Ξ^0 are not the same. (nor are their experimental splits). I.e., the Ξ^+ and Ξ^- do not have the same mass!
4. The meson calculations were done by Socolov in a Harvard thesis. CS have only done the baryon calcs.
5. Page 224 shows the experimental form factors for F_1 and F_2 ; these seem to disagree in asymptotic form from what Feynman gives in his 1970 book. How to resolve this (see attached sheet for comparison)
6. Finally, Table III gives the tadpole and non-tadpole stuff and compares to exper. This table supercedes that in Ref 6 because they made some numerical errors. All signs are right, agreement is good.
7. Now I know what mixing means. Remember that vector meson dominance gives a theory of form factors. So you get a simple pole for each vector that couples. This the isoscalar form factor will have a ~~PHOTON~~ ω and ϕ contribution. Thus, the vector dominance says asymptotic FF behavior is $1/q^2$, not $1/q^4$.
Feynman says: VDM always predicts FF go as $1/q^2$. Exper, this is true for the pion FF, but Feyn says the nucleon FF go as $1/q^4$. This is the fact being claimed differently here.

Suzuki & Zachariasen
(1966)

scattering⁷ as for proton-proton scattering⁸ at lower energies. In view of the similarity of proton-nucleus and proton-proton scattering at high energies demonstrated in the present paper, it is reasonable to assume that the ratio $\text{Re}T(0)/\text{Im}T(0)$ is negative at high energies also for proton-nucleus scattering, which gives a repulsive potential $V_0 = +12$ MeV.

The way in which the potential scattering has been introduced is only one of many possible ways, the essential point being that without potential scattering, $V_0 = 0$, we should have pure shadow scattering, $h_R = 0$ and $h_I = \frac{1}{2}[1 - (1 - \rho)^{1/2}]$. In order to investigate this point we also made an attempt to fit the experimental results using a complex potential, i.e., we put

$$h = (i/2)(1 - e^{-\eta - i\chi}), \quad (11)$$

where $\chi = 2V_0\beta^{-1}(R^2 - b^2)^{1/2}$ describes potential scattering while η is given by shadow scattering only, $e^{-2\eta} = 1 - \rho(b)$. The values of μ_0 and A are as before, but we had to choose $R_0 = 1.00$

fm, $R = 1.32A^{1/3}$ fm, and $|V_0| = 22$ MeV. The results are shown as dashed curves in Figs. 1 and 2 for Be, Cu, and Pb. The agreement with experimental results is approximately as good as with our first model.

¹G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillenthun, J. P. Scanlon, and A. M. Wetherell, Nucl. Phys. **79**, 609 (1966).

²L. van Hove, Nuovo Cimento **28**, 798 (1963).

³W. N. Cottingham and R. F. Peierls, Phys. Rev. **137**, B147 (1965).

⁴L. J. Cook, E. M. McMillan, J. M. Petersen, and D. C. Sewell, Phys. Rev. **75**, 1352 (1949).

⁵H. A. Bethe and P. Morrison, *Elementary Nuclear Theory* (John Wiley & Sons, Inc., New York, 1961), pp. 7-11.

⁶D. Harting et al., Nuovo Cimento **38**, 4640 (1965).

⁷F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).

⁸G. Bellettini, G. Cocconi, A. N. Diddens, E. Lillenthun, J. Pahl, J. P. Scanlon, J. Walters, A. M. Wetherell, and P. Zanella, Phys. Letters **14**, 164 (1965).

SIGN OF THE PROTON-NEUTRON MASS DIFFERENCE AND A GHOST SCALAR MESON*

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(Received 3 October 1966)

A new interpretation of the proton-neutron mass difference has been given independently by Pagels¹ and Fried and Truong.² In this note, it is shown that, if this model works, it implies a ghost scalar meson with $T = 1$ and $G = -1$. Use is made of the technique of spontaneous symmetry-breaking theory.³ By identifying the ghost with that of the Regge-pole model, one can crudely estimate the magnitude of the feedback by means of a linear extrapolation.

The physical mass of the proton, in the absence of electromagnetic interactions, may be written

$$m_p = m_0 + \Sigma_p(p', m_{0p}, m_{0n})|_{p'=m_p}, \quad (1)$$

where Σ_p is the unrenormalized proper self-energy part. Its dependence on the bare masses of the proton and neutron are indicated explicitly. It is also a function of the bare masses of other strongly interacting particles and of the bare strong-interaction coupling constants.

Now let us introduce electromagnetism. The proton mass will shift to $m_p + \delta m_p$, the func-

tion Σ_p will become $\Sigma_p + \delta\Sigma_p$, and we may write

$$\delta m_p = \frac{\partial \Sigma_p(p', m_{0p}, m_{0n})}{\partial p} \bigg|_{p'=m_p} \delta m_p + \delta \Sigma_p(p')|_{p'=m_p}. \quad (2)$$

This may be rewritten in the form

$$\delta m_p = Z_p \delta \Sigma_p(p')|_{p'=m_p}, \quad (3)$$

using the fact that the proton wave-function renormalization constant is

$$Z_p = \left[1 - \frac{\partial \Sigma_p(p', m_{0p}, m_{0n})}{\partial p} \bigg|_{p'=m_p} \right]^{-1}. \quad (4)$$

Similar equations can, of course, be written for the neutron, and we note that to lowest order in the fine-structure constant we may take $Z_p = Z_n = Z$.

The physical effect exploited by Pagels and by Fried and Truong is the feedback on the elec-

Harar
(1966)

$$\Delta M \approx$$



and

$$T_{\mu\nu}(q^2, \nu) \approx \text{diagram} \text{ in Regge as } \nu \rightarrow \infty$$

SUPERCONVERGENT DISPERSION RELATIONS AND ELECTROMAGNETIC MASS DIFFERENCES*

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(Received 9 November 1966)

Using dispersion relations and the convergence properties of t -channel isospin amplitudes, we show that (a) $\Delta I=2$ electromagnetic mass differences should be correctly obtained by summing the self-energy contributions of a few low-lying states; (b) $\Delta I=1$ mass differences cannot be obtained in this way, and a subtraction term is always necessary; (c) the subtraction term has the correct sign for explaining the proton-neutron mass difference.

The problem of computing the electromagnetic mass differences between particles in a given isomultiplet has always been one of the greatest puzzles of elementary-particle physics. It is well known that the simple, naive calculations which include only the contributions of a few low-lying states to the self-energy diagram lead in most cases to totally wrong results (including the notorious wrong signs for the proton-neutron and K^+-K^0 mass differences). On the other hand, the same simple approach gives the correct sign and magnitude in a few other cases (such as the $\pi^+-\pi^0$ difference). In this paper we propose simple, reasonable assumptions on the energy dependence of t -channel isospin amplitudes for forward Compton scattering and, using these assumptions, we reach the following conclusions:

(a) All $\Delta I=2$ mass differences should be correctly obtained when we approximate the self-energy diagram by the contributions of a few low-lying states.

(b) There is no reason to expect that the same

simple approximation will give the correct order of magnitude or even the correct sign for the $\Delta I=1$ mass differences.

(c) A consistent calculation of the $\Delta I=1$ terms must include an additional "subtraction" term. We show that this term has the correct sign and, roughly, the correct order of magnitude required by the experimental masses.

We also demonstrate that the statements (a)-(c) are correct for all six electromagnetic hadron mass differences which are experimentally known, and we propose further experimental tests of our assumptions.

In perturbation theory, the electromagnetic self-energy of a hadron is given by^①

$$\Delta M = -\frac{1}{8\pi^2} \int_{-\infty}^{+\infty} \frac{T_{\mu\nu}(q^2, \nu) g^{\mu\nu}}{q^2 - i\epsilon} d^4q, \quad (1)$$

where $\epsilon^\mu \epsilon^\nu T_{\mu\nu}(q^2, \nu)$ is the forward amplitude for Compton scattering of a virtual photon with mass q^2 , energy $q^0 = \nu$, and polarization ϵ^μ from a hadron with momentum p and mass M ($M\nu = -p \cdot q$). We write $T_{\mu\nu}$ as

$$T_{\mu\nu}(q^2, \nu) = t_1(q^2, \nu)[q^2 g_{\mu\nu} - q_\mu q_\nu] + t_2(q^2, \nu)[\nu^2 g_{\mu\nu} + \frac{q^2}{M^2} p_\mu p_\nu + \frac{\nu}{M}(p_\mu q_\nu + p_\nu q_\mu)]. \quad (2)$$

Cottingham^② has shown that by rotating the integration contour in (1) from the real to the imaginary axis in the complex ν plane, one can express ΔM in terms of scattering amplitudes for spacelike photons, allowing us, in principle, to use experimental electron scattering data in order to compute the integral. Substituting $\nu - i\nu$ and integrating over the angular variables in (1), we find

$$\Delta M = -\frac{1}{4\pi} \int_0^\infty \frac{dq^2}{q^2} \int_{-q}^{+q} d\nu (q^2 - \nu^2)^{1/2} [3q^2 t_1(q^2, i\nu) - (q^2 + 2\nu^2) t_2(q^2, i\nu)]. \quad (3)$$

We can now write, for t_1 and t_2 , fixed- q^2 dispersion relations in ν and compute the t_i 's in terms of their absorptive parts.^③ The main obstacle at this point is, of course, the question of possible subtractions in the dispersion relations, since only in the case of no subtractions can we hope that the contribution of the

first few low-lying states will dominate the expressions for t_1 , t_2 and hence for ΔM .^④ The convergence properties of the dispersion integrals are determined by the asymptotic behavior of the absorptive parts of the amplitudes. To lowest order in α , the electromagnetic

Jan 16, 1977

Harari: Mass Differences, 1966

1. Here is the problem: sometimes the naive dM calculation works (pions), and sometimes it fails so badly that it gives the wrong sign. Why? Recall that CFG in 1958 put in the experimental form factors and showed that p - n mass difference comes out with the wrong sign in the pole-approximation to the dM . Thus, CFG disproved the conjecture of FS in 1956 that form factors could explain the anomalous sign.
2. H uses Regge theory to find the answer. First, he classifies mass differences in terms of t -channel isospin amplitudes, something I have not worked out at this time. It turns out that mass differences are either $I_t=1$ or $I_t=2$. H notes that it is only the $I_t=1$ differences that are causing the wrong answers. Basically he will show that the reason is this: for the $I_t=1$ amplitudes (mass differences) like the p - n difference, the pole approximation derives its validity in the first place from a dispersion relation written for the structure functions. The pole is only a good approx if there happens to be no subtraction constant in the dispersion relation. The presence of this constant is determined by the high energy behavior of the integrand, and for $I_t=2$ there are no high Regge trajectories so there is no constant. For the $I_t=1$ however there is the A_2 with intercept .4, so there is a constant. This constant is the reason p - n comes out with wrong sign.
3. Now lets go through details quickly. The mass difference is given by a certain integral of the Compton amplitude for photon-hadron scattering. Happens to be in forward direction, so $t=0$. Variable nu plays role of s , incident energy. This amplitude is T_{uv} . Using uv sym, and conserved current (ie, gauge invariance), you can express T_{uv} in terms of two structure functions called here t_1 and t_2 . Obviously these are related to the now standard W_1 and W_2 functions of deep inelastic. Hence equation (2). There is a d^4q Feynman integral, but angles are done and then the energy or v contour is Wick/Cottingham rotated to yield equation (3). The idea is now to estimate the structure functions and plug in and compute dM . By the way, seems to me that in 1978 these functions are now very well known, so you could just do it. But in 1966 Harari must approximate them by pole term of dispersion relations. So equation (4) shows the disprels for the t_1 , also classified by isospin by superscript. The term out front is not the subtraction constant but is rather the pole contribution to the integrand. Recall that pole contribution can be written in terms of couplings which in turn are given in terms of the various E and M form factors for a given particle, hence (5) and (6). The Regge step is now invoked and you conclude that the $I=1$ disprel for t_1^1 must have a subtraction constant. This is now included in (10). Thus, the naive pole computation is really a computation of the 2nd and 3rd term in (11), involving the form factors. H goes

on to estimate the size of this subtraction constant . You pick out the constant by taking a certain limit, but this limit also picks out the magnetic moment of neutron and proton. The sign of the determined constant now comes out correctly for the $p-n$ difference.

4. What are other tests for this little theory? Regge part of it makes predictions for the photon-proton cross section (now known) . I wonder how this came out? Other mass differences are very hard to measure along with magnetic moments.

Ball & Zachariasen
(1968)

Mass Splittings and Ghost Tadpoles*

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The neutron-proton mass difference is expressed, on the assumption that it is purely electromagnetic in origin, in terms of partial-wave amplitudes for the process $N\bar{N} \rightarrow \gamma\gamma$ at zero total energy, and also in terms of the single $n=0$, $M=0$ $O(4)$ amplitude for the same process. This representation allows quite directly a qualitative understanding of the sign of the n - p mass difference, and also allows, with more drastic assumptions, an approximate numerical evaluation of the mass difference based on experimentally determined parameters from the A_2 Regge trajectory. This numerical estimate is in reasonable agreement with experiment. The approach is easily generalized to other electromagnetic mass differences, and is shown to yield the tadpole model.

I. INTRODUCTION

THE difference in sign between the experimental neutron-proton mass difference and the value calculated on the assumption of point Dirac nucleons demonstrates unequivocally the importance of effects due to the strong interactions of the nucleons, if one is to ascribe the mass splittings solely to electromagnetism. The first attempt to estimate some of these effects was by Feynman and Speisman in 1956.¹ They demonstrated that the inclusion of electromagnetic form factors (presumably present because of the strong interactions) in the coupling of photons to nucleons could lead to the correct sign for the mass difference, depending on the detailed behavior of the form factors. As experimental information on the form factors became available, however, it became clear that in fact the form factors did not behave in such a way as to produce

a sign change.² It therefore became necessary to look at other consequences of the strong interactions.

As a first step in this direction, it was observed by Harari³ that the existence of the Regge trajectory associated with the A_2 meson made it extremely unlikely that the simple Feynman-Speisman-type calculation of the n - p mass difference could have turned out to be correct. This is in marked contrast to $I=2$ mass differences, where such a calculation does work fairly well, and where there is no analog of the A_2 trajectory. It was further noticed that, qualitatively, one might expect the existence of the A_2 to reflect an unusually strong interaction in the $J=0^+$, $I=1$ states of nucleon and antinucleon, and that this strong force could produce a sign change in the mass difference.⁴ The physical processes which seem to be important are illustrated in Fig. 1. The sign change is associated with the existence of an extinct bound state (that is, a bound state with zero residue⁵) of negative mass squared, with $J=0^+$, $I=1$, which is produced by the very strong force. This extinct bound state can be thought of as the "tadpole" which has been invoked as a more or less phenomenological way of obtaining the correct mass shifts.⁶ [Since the A_2 is a member of an $SU(3)$ octet,

"scalar mesons"

?

mixed

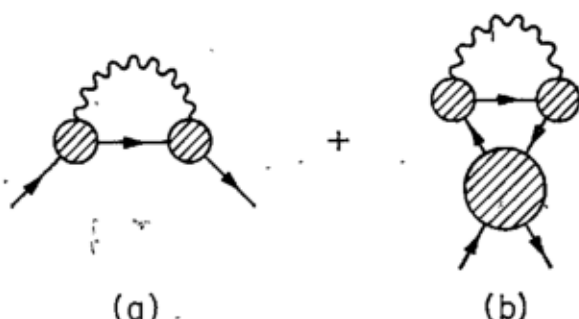


FIG. 1. The classes of diagrams which are of importance for the mass difference. Diagram (a) is the usual Feynman-Speisman diagram, with experimental form factors included at the photon-nucleon vertices. Diagram (b) represents the additional effects due to the strong interaction in the $J^P=0^+$ channel of $N\bar{N}$ scattering.

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¹ R. P. Feynman and G. Speisman, Phys. Rev. 94, 500 (1954).

² M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters 2, 7 (1959).

³ H. Harari, Phys. Rev. Letters 17, 1303 (1966).

⁴ M. Suzuki and F. Zachariassen, Phys. Rev. Letters 17, 1033 (1966); H. Fried and T. Truong, *ibid.* 16, 559 (1966); H. Pagels, Phys. Rev. 144, 1261 (1966); M. Suzuki, *ibid.* 171, 1791 (1968).

⁵ It should be noted that the ghost-elimination mechanism cannot be that suggested by M. Gell-Mann [in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962), p. 539] or the "no-compensation mechanism" [S. Y. Chu, C. B. Chiu, and L. L. Wang, Phys. Rev. 161, 1563 (1967)] but must rather be that of G. F. Chew [Phys. Rev. Letters 16, 60 (1966)]. The reason for this is that we require a zero of the *sense* D function where the A_2 trajectory goes through zero, and the Gell-Mann and no-compensation mechanisms produce a zero only in the *nonsense* D function. The Chew mechanism, on the other hand, has a zero in the *sense* D function. Fortunately, the Chew mechanism in fact seems to be consistent with fits to the A_2 . See Ref. 10.

⁶ S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964). /

the extinct bound state, or tadpole, presumably is also; therefore, all predictions of the tadpole model are obtained.] The "tadpole" is thus a consequence of the strong-interaction dynamics rather than a new elementary particle.

It is our intent here to explore further and to elaborate on this physical picture of what is important for the $I=1$ mass shifts. To this end, we should like to express the mass differences in terms of the physics of the $N\bar{N}$ channel rather than in terms of virtual Compton scattering as is usually done. Our first step, then, will be to derive from the usual Cottingham formula⁷ an expression for δM in terms of the partial-wave amplitudes for $N\bar{N} \rightarrow \gamma\gamma$ for virtual photons of equal mass. This is done in Sec. II. The process evidently is of interest at zero total energy; $O(4)$ symmetry thus applies and we also express δM in terms of the $n=0$, $M=0$, $O(4)$ amplitude for $N\bar{N} \rightarrow \gamma\gamma$.

The formula obtained in Sec. II is specialized in Sec. III to two-particle unitarity, including any number of two-body channels as intermediate states in the $N\bar{N} \rightarrow \gamma\gamma$ amplitudes. With the assumption that these amplitudes satisfy unsubtracted dispersion relations, it is shown, qualitatively, how the sign change produced by the existence of the A_2 can come about. We also outline, in this section, a possible calculation based on $O(4)$ symmetry and the Bethe-Salpeter equation which includes the same basic physics, and which may permit the mass shift to be related more directly to experimentally determined A_2 trajectory parameters.

In Sec. IV, we make some simple numerical estimates, with the intent of showing that the physical ideas we are using not only can provide the correct sign but may even give a result which is quantitatively not too unreasonable, though it is important to emphasize that numerical estimates require far more drastic assumptions than are necessary for understanding the sign.

Finally, in Sec. V we comment on the relation of our results to the tadpole model, and in Sec. VI we give a brief summary of what we believe to be the basic conclusions.

II. RELATION OF δM TO THE PROCESS $N\bar{N} \rightarrow \gamma\gamma$

Our starting point is the usual formula for the electromagnetic mass shift of a nucleon to first order in α , the fine structure constant, which we write as

$$\delta M = \int \frac{d^4 q}{(2\pi)^4} \left(\frac{-i}{q^2} \right) \delta_{\mu\nu} T_{\mu\nu}(q, -q; p, -p), \quad (2.1)$$

where $T_{\mu\nu}(q, \bar{q}; p, \bar{p})$ is the Feynman amplitude for the process $N\bar{N} \rightarrow \gamma\gamma$, for virtual photons of mass q^2 , where p and $\bar{p}$ are the nucleon and antinucleon four-

momenta, q and $\bar{q}$ are the photon four-momenta (we take $q^2 = \bar{q}^2$), and where μ and ν are the photon polarizations.⁸ The nucleon and antinucleon are, of course, on the mass shell.

We may place Eq. (2.1) between the spinors $\bar{u}_{p,-\sigma}$ and $u_{\bar{p},\sigma}$, sum over spins and set $\bar{p}$ equal to $-p$. Note that the relation

$$\begin{aligned} \sum_{\sigma} (\bar{u}_{p,-\sigma} \delta_{\mu\nu} T_{\mu\nu}(q, \bar{q}; p, \bar{p}) u_{\bar{p},\sigma}) \\ = - \sum_{\lambda=-1}^1 \sum_{\mu=-1/2}^{1/2} T_{\lambda\lambda,\mu\mu}(l, \nu; q^2) \end{aligned} \quad (2.2)$$

holds, where $T_{\lambda\lambda,\mu\mu}$ are (essentially) helicity amplitudes for $N\bar{N} \rightarrow \gamma\gamma$, and where we define

$$l = (q + \bar{q})^2 = (p + \bar{p})^2$$

and

$$\nu = q \cdot p / M.$$

As a result, we may rewrite Eq. (2.1) in the form

$$\delta M^2 = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} T(l, \nu; q^2) |_{l=0}, \quad (2.3)$$

where we define

$$T(l, \nu; q^2) = - \frac{|p|}{M} \sum_{\lambda, \mu} T_{\lambda\lambda,\mu\mu}(l, \nu; q^2) \quad (2.4)$$

and, of course, $|p| = (\frac{1}{4}l - M^2)^{1/2}$. [Precisely the same result, Eq. (2.3), obtains for a boson mass shift, where $T(l, \nu; q^2)$ is the amplitude for $BB \rightarrow \gamma\gamma$, summed over (equal) helicities of the two photons.]

We may also apply the reasoning invented by Cottingham⁷ to Eq. (2.3). This yields, in a straightforward manner, the equation

$$\delta M^2 = - \frac{1}{8\pi^3} \int_{-\infty}^0 \frac{dq^2}{q^2} \int_{-(q^2)^{1/2}}^{(-q^2)^{1/2}} d\nu (-q^2 - \nu^2)^{1/2} \times T(0, i\nu; q^2). \quad (2.5)$$

Now the helicity amplitudes $T_{\lambda\lambda,\mu\mu}$ have a very simple partial-wave expansion, namely:

$$T_{\lambda\lambda,\mu\mu}(l, \nu; q^2) = \sum_J (2J+1) P_J(z) T_{\lambda\lambda,\mu\mu}^J(l; q^2), \quad (2.6)$$

where z is the cosine of the center-of-mass scattering angle for $N\bar{N} \rightarrow \gamma\gamma$. Thus we have, in the physical region for the process, the relation

$$\nu = l/4M - [(l - 4M^2)^{1/2}(l - 4q^2)^{1/2}/4M]z. \quad (2.7)$$

The same partial-wave expansion evidently holds

⁸ Our normalization is such that the S matrix is related to $T_{\mu\nu}$ by $S = 1 - i(2\pi)^4 \delta^4(q + \bar{q} - p - \bar{p}) (\theta_p | T_{\mu\nu} | u_p) e_\mu e_\nu / (16E\bar{E}q\bar{q})^{1/2}$; otherwise we use the Feynman conventions throughout.

⁷ W. H. Cottingham, Ann. Phys. (N. Y.) 25, 424 (1963).

Setup of (2.14)

① $T_{\lambda\lambda;\mu\mu}(t, \nu; q^2) = \text{"}\nu\text{"} \rightarrow \text{diagram} = \text{helicity amplitude for } N\bar{N} \rightarrow \gamma\gamma$



$$\delta M_\mu = \text{diagram} = \sum_{\lambda=\pm 1} \int \frac{d^4 q}{(2\pi)^4} \left[\frac{-i}{q^2} \right] T_{\lambda\lambda;\mu\mu}(t, \nu; q^2)$$

= SM for nucleus of helicity μ .

② Seems to me there should be a helicity averaging matrix, but maybe when you average over μ it goes away. So

$$\delta M = \int \frac{d^4 q}{(2\pi)^4} \left[\frac{-i}{q^2} \right] \left[\frac{1}{2} \sum_{\mu=\pm 1/2} \sum_{\lambda=\pm 1} T_{\lambda\lambda;\mu\mu}(t, \nu; q^2) \right]$$

$$= \frac{M}{|\vec{p}|} T(t, \nu; q^2)$$

conventional norm.

where $|\vec{p}|^2 = E^2 - m^2 = \left(\frac{t}{4} - m^2\right) = \left(\frac{t}{4} - m^2\right)$ so $|\vec{p}| = \sqrt{\frac{t}{4} - m^2}$

One end + channel momentum.

③ So $\delta M = \int \frac{d^4 q}{(2\pi)^4} \frac{-i}{q^2} \frac{M}{|\vec{p}|} T(t, \nu; q^2)$

④ Now $\nu = \frac{p \cdot q}{M} = \frac{E_p E_q - \vec{p} \cdot \vec{q}}{M} = \frac{(\frac{t}{2}) - z \cdot \sqrt{\frac{t}{4} - m^2} \cdot \sqrt{\frac{t}{4} - q^2}}{M}$

since $q^2 = \text{photon mass}$.

$$\Rightarrow \nu = \frac{t}{4M} - \frac{z}{4M} \sqrt{t - 4m^2} \sqrt{t - 4q^2} \quad (2.1) \checkmark$$

⑤ So do PWA on $T(t, \cos\theta; q^2)$ as in (2.6). In t-channel, since both have same helicity, $\mu = \mu_1 = \mu_2 = 0$ so P only. Again, ∇ a crossing detail here.

⑥ So, next do Cottingham to get (2.5). This ν integration becomes the z -integration of (2.15). OK, I accept (2.14).

① (2.14) says that all t -channel partial waves contribute to δM , not just $J=0$. Also, $t=0$ only, and $q^2 < 0$ only.