

Notes on Misc Papers & Books

Feynman & Hibbs - Quantum Mechanics and Integrals (1965)

Streater & Wightman - PC, Spin and Statistics and All That (1964)

Chew - S-matrix Theory of Strong Interactions (1961)

Martin & Spearman - Elementary Particle Theory (1970)

Feynman - Photo-hadron Interactions (1972)

Gasiorowicz - Quantum Physics (1974)

Bogoliubov & Shirkov - Intro to the Theory of Quantized Fields (1959)

Phil Lucht notes (1977)

Feynman & Hibbs - Quantum Mechanics and Integrals (1965)

Feynman and Hibbs.

Preface: this alternative to the conventional method of teaching nonrelativistic quantum mechanics was used by Feynman for while at CalTech. Then he evolved into different methods, but student Hibbs has recorded and expounded on the methods in this book. It does not work for QED exactly, but there are some special cases that are clarified by the path integral approach. Also, they claim there is something intuitive to be gained by the method.

Chapter 1. It starts off with Feynman's usual two slit interference experiment. The act of determining which hole the electron goes through destroys the interference. The uncertainty principle is stated in just this way: any determination of the alternative taken by a process capable of following more than one alternative destroys the interference between alternatives. For the screen experiment, this can be reduced to the more usual form $\Delta p \Delta x = h$.

The 90° collision of two particles is given as a second example. If the resulting particles are distinguishable, the two final amplitudes 1,2 and 2,1 do not interfere because the fact that you could say which happens removes the interference. If the resulting particles are the same completely, then amplitudes interfere according as whether they are fermions or bosons.

A third example is the background in crystal scattering of neutrons. If a scattered neutron has no spin flip, then you cannot tell where it hit in the crystal so full interference is present and you get the Bragg points. But, if a neutron spin is flipped, you could in principle tell which nucleus it hit. The wave then is known to emanate outward from that particular nucleus and forms a background. The point is that you don't have to actually do any measurement. It's just that you could in principle do it.

The picture of multiple screens with lots of holes is used to suggest the path integral approach. Every trajectory through empty space is in some sense an alternative and I guess all these alternatives must be added (integrated). Hence you are integrating all the paths.

Finally, there are some comments about how this book is of itself a very incomplete description of quantum mechanics and should really be used only in conjunction with a more systematic and conventional book like Schiff.

Note: I read Ch 1 a long time ago + took a few notes. Can't find them now.

2-1 The Classical Action.

- ① On the one hand, this is the thing you extremize to derive the Euler-Lagrange equations. i.e.

$$S = \int_{t_a}^{t_b} L(x, \dot{x}, t) dt$$

$$\delta S = 0 \Rightarrow \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \left(\frac{\partial L}{\partial x} \right) = 0$$

- ② $S(t_a, t_b)$ is also something you can calculate directly in class mech. For a free particle, if you specify a trajectory $x(t) = x_0 + vt$, then

$$S_{cl} = \frac{m}{2} \frac{(x_b - x_a)^2}{(t_b - t_a)}$$

2-2, 2-3

- ③ Feynman gives rule that $K(b, a) = \sum_{\text{paths } a \rightarrow x(t) \rightarrow b} \exp(i S[x(t)]/\hbar)$

Because $\delta S = 0$ for trajectories near the classical one, these near ones add in phase in this path sum. Far away trajectories tend to cancel one another and don't matter in the classical limit.

2-4 Paths.

- ④ We write the generalized form for a sum over all paths. This:

$$K(b, a) = \int_a^b \mathcal{C}^{(i/\hbar) S[b, a]} \mathcal{D}x(t)$$

like a dx .

Still not very explicit.

- ⑤ Because the action is additive, Feynman derives the idea that amplitudes for successive events should multiply!

over

⑥ Now using this product rule, we can say:

$$K(b,a) = \int dx_c K(b,c) K(c,a)$$

$$= \int dx_1 dx_2 \dots dx_n K(b, x_1) K(x_1, x_2) \dots K(x_n, a).$$

Then it turns out that you should use:

$$K(x+dx, x) = \frac{1}{A} e^{(i/\hbar) \epsilon \cdot L}$$

because $S(t, t+\Delta t) = \int_t^{t+\Delta t} L dt \approx \Delta t \cdot L = \epsilon \cdot L.$

When you sum, the expo is recovered.

- ① They show how you can explicitly do the infinite multiple integral to find the free particle kernel. By examining this kernel + taking classical limits, we learn how to identify W and K .

3-2-33

- ② Next, they set us up to compute the amplitude $K(x,0)$ for particle to go from 0 to x but, at time T it must be between $x_0 - b$ and $x_0 + b$. All other trajectories are omitted. This is a one-dim slit setup. The answer is written explicitly in terms of an integral over free particle propagators, a so-called Fresnel integral. (where will I do optics??)

- ③ Next, an illustration of uncertainty principle with slits.

3-4

- ④ Interpretation of wave function

$$\Psi(x,t) = K(x,t; x_0, t_0) \text{ - whatever you like}$$

Clearly,

$$\Psi(x',t) = \int K(x',t; x,t) \Psi(x,t) dx$$

because: $K(x',t; x_0, t_0) = \int K(x',t; x,t) K(x,t; x_0, t_0) dx$

So we have connected Saxon with path integrals.

3-5

- ⑤ Suppose we try to compute $K(b,a) = \int_2^b Dx(t) \exp\left[\frac{i}{\hbar} \int_{t_a}^{t_b} L dt\right]$. This is a path integral.

We know the classical action. It turns out that the answer is:

$$K(b,a) = e^{(i/\hbar) S_{cl}} F(t_b, t_a) !$$

- ⑥ It is emphasized that the multiplicative factor is very unimportant in most app's. It's the expo that counts. So, the classical action is it! For example, for a free particle

$$K(b,a) \sim e^{i(x_b - x_a)^2 / 2T}$$

For a harmonic osc, see (3-59).

$0 \rightarrow e_n$

⑦. 3-6. Now we have a series of potentials S_n which we are supposed to compute the classical action and therefore the kernel (or propagator). One of these problems is that quoted by Akers. (3-11, page 64) This "external force harmonic oscillator" problem is important for Q.E.D.!!

3-7, ⑧ Extend to more than one coordinate. See (3-69).

3-8 ⑨ The same separation of variables that works so well in classical mech, also works here. Causes a separable kernel $K \equiv K_1 K_2$.

3-9 ⑩ A good explanation of the meaning of a functional $f[x(t)] \neq f(x(t))$.

3-10 ⑪ Here, he treats a two coordinate problem. We let a harmonic oscillator (X) interact with a particle (x). Then, the action S is a functional of paths x and X . In order to compute K , you first need $T[x]$. Then $K = \int T[x] \mathcal{D}x$. The effective action is now very messy.

OK, made enough of this book for a while.

AFRR: Chapter 1 "Strong Interactions"

I am reading this thing fast to see if they have some new ways to look at things. They claim that they will stress aspects relevant to current algebra. It starts off as usual with the Mandelstam variables, though they call them s, t and $\bar{s}$. They remark that the "substitution rule" which I think I would call crossing has not been rigorously proved from even the axioms of field theory.

Next comes unitarity. In particular, the simple elegant form of unitarity in the partial waves in the elastic regime is remarked. Unfortunately, the properties of analyticity are very messy in partial waves. They stress over and over this problem of being unable to state unitarity and analyticity neatly using the same variables. This is supposed to be an "index" of the complication of the relativistic theory.

After a brief example of the removal of kinematical singularities (as opposed to dynamical ones. Analyticity of the first kind states that the only dynamical singularities are those unitarity branch points, I think.), they go right into dispersion relations, which is of course what analyticity is all about. It is assumed that in some sensible region the function $T(s, t)$ is real analytic. They write the disp $\bar{r}$ el first with no subtractions. If the amplitude gets too big at infinity, you have to make N subtractions and introduce N unknown subtraction constants. Or, on the other hand, if you amplitude goes to 0 exceptionally fast at infinity, you end up with "desubtracted" dispersion relations which provide superconvergence relations.

There is a good discussion of the Mandelstam Representation. It is the statement that there are no wierdo singularities. From the MandyRep you can get all other dispersion relations. For a large class of potential nonrel theories the thing is definitely true (with subtractions of course), but in the rel theories there are problems (anomalous thresholds ???)

The rest of the chapter deals with second kind analyticity. First they do the whole thing in terms of the Khuri series expansion (power series). The idea is that you have to assume that inversion formulae interpolate your series coefficients even where the inversion seems to diverge. Carlson's theorem is invoked (and referenced) and a S-W is done and out pop the Khuri poles.

I have not read the rest yet. They go into the Regge theory, daughters, superconvergence, conspiracy, McDowell, trajectory doubling, Toller poles etc etc.

The chapter closes with some appendices.

- 1) projection operator for particle of any spin. (?)
- 2) spin formalism that competes with the helicity formalism (their own baby)
- 3) Lorentz group shortie.

Streater & Wightman - PC, Spin and Statistics and All That (1964)

Chapter One: Relativistic Transformation Laws

1.1 Superselection Rules. I think here they are talking about single particle Hilbert space. You are not allowed to make a state like $0.3 |p\rangle + 0.4 |PI\rangle + \dots$ because such a state (proton mixed with pion) does not have a definite baryon number. No one has ever made a particle beam which was such a mixture of particles of different types. By "types" we refer here to the superselecting quantum numbers such as: B, Q . Similarly, proton and neutron cannot mix, so I and I_3 are superselectors. Also, cannot mix any fermion with a boson, because $R(2PI)$ would yield a different ray if you could mix these (but rotation of $2PI$ should not change ray).

However, this should not be confused with the idea of a direct product of states. You can make a state $|p; PI\rangle = |p\rangle \otimes |PI\rangle$, even though each single state has different baryon number. Here you are not adding things in the same Hilbert space, you are forming a product Hilbert space.

Let Θ be the set of all observables, and Θ' be all operators which commute with these observables. Usually one assumes that operators in Θ' like B, Q commute. This allows the Hilbert Space to be broken into superselecting subspaces, so that a state is always in one of these subspaces and moves around inside a subspace. I gather this is the idea of orthogonal direct sum of Hilbert spaces, as in the right side of a group reduction. These subspaces are called coherent subspaces.

1.2 Symmetry Operations. A symmetry operator $a' = Va$ is a symmetry operator if $|a/b| = |a'/b'|$. Such an operator must, according to theorem 1.1, be either unitary or antiunitary. The idea that $V^{-1} = V^\dagger$ is obvious from the above def. of a symmetry. Wigner shows, however, that the operator must be either linear or antilinear, but not a mixture. Also, one must restrict to the action of V on some coherent (superselecting) subspace. Either V maps this thing onto itself, or onto another coherent subspace, and this map is 1:1.

1.3 Lorentz and Poincare. It is shown that Lorentz group L has four components, in the usual notation. The connection to $SL(2, C)$ is made. Note use of τ^μ for regular Pauli matrices, so that $X = x^\mu \tau^\mu$ = the same thing I use. They also define $\tilde{X} = x_\mu \tau^\mu$ which is the kind of thing you get after parity negates three momentum. The rule is $X' = aXa^\dagger$ as usual. Then comes the complex Lorentz group $L(C)$. Two connected pieces instead of 4, and they use this form: $X' = aXb^T$ whereas I think I prefer the dagger. Finally, the Poincare group P and the complex Poincare group $P(C)$ are mentioned.

Then comes what I think is an error. They speak of the rep $(j/2, k/2)$ as if it were the same as $(\frac{1}{2}, 0)^j \times (0, \frac{1}{2})^k$, when actually the rep $(j/2, k/2)$ is the highest rep in this product. Nevertheless, the $(1, -1)$ rules as in (1.27) are correct, apart from my different (Taylor) convention.

Next, there is some P,C,T discussion. 30 years ago it was noted that the operator T is antiunitary. Therefore the PCT operator Θ is also antiunitary. "The substitution rule for PCT is replace $\phi(x)$ with $\phi(-x)$ and write all operator products in reverse order."

Now on page 17 we have some new (to me) material.

How do you make parity in $SL(2,C)$? I have a separate sheet on this, and the answer is shown as equation (1-34). It follows that T is given by the negative as in (1-35). Obviously you are in trouble, though, if you are trying to construct a linear representation of the full Lorentz group because not all your operators are linear! In particular, parity is antilinear. To fix this up, you have to do a certain doubling of your representation carrier space as in (1-37). I know that this off-diagonal matrix business is the same as the idea of $(\frac{1}{2}, \frac{1}{2}) \oplus (\frac{1}{2}, \frac{1}{2})$ which makes the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ representations of the full Lorentz group out of reps of the proper Lorentz group.

The idea is simpler to understand in the case of Dirac spinor shown in (1-47). There, you know that you have to take the $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ in order to get a representation of the full IG, since parity connects these two pieces. Thus, a Dirac particle needs two spinors, one of each kind, and this is what you see in (1-47). That parity mixes these two is shown in the fact that the matrix for parity is exactly off-diagonal. It is a similar thing happening in the vector case. You have $(\frac{1}{2}, \frac{1}{2}) \oplus (\frac{1}{2}, \frac{1}{2})$. The two pieces look the same, but they get mixed by parity. Hence, parity is off diagonal in (1-37) also.

In contrast, the time reversal operator wants to be antilinear, so it always appears as a diagonal ~~2x2~~ 2x2 matrix sense.

Somehow charge conjugation is conjured into the picture so they can talk full CPT. In the end, they try to make up general rules for the separate P,C,T operations both for wavefunctions and operators. In the general case, you still have this doubling due to parity (and also, apparently, charge conjugation). In the general case, a particle has a "field" with any number of first and second kind labels. This field needs a partner field with all undots dotted etc. As usual, the authors think of higher fields as direct products of lower spinors, rather than things like $(0,s)$.

A possible way to view C is this: since you want C to dagger operators so they make antiparticles instead of particles, you in passing complex conjugate things taking first kind to second kind. Thus, you need the off diagonal matrix shown in (1-52) to get things back so that pieces transform consistently. Just a guess.

Section 1.4: The vacuum state presumably does nothing under $U(a,A)$.

Next, they discuss certain wavefunctions $\chi(p)_m = (p_m | \chi)$. They discuss these rather than the states for the usual normalization reason. The general transformation rule is something like (1-58) which I agree with, and the completeness and orthogonality have the little $D(L^2)$ factors as shown in (1-57), which I have come to understand.

Again, they think of a particle of spin- s as a system $(0, \frac{1}{2})^{2s}$. They always avoid the full D^{os} functions. At least they are consistent in this. Notice that all indices are first kind, as they say, so you don't have any $(\frac{1}{2}, \frac{1}{2})$ or vector particles, per se.

The rest of the section is not too relevant for me now, but I quote some interesting results:

- 1) the phase in the ray representation is discussed.
- 2) a tricky simple reason is given why the $U(a,A)$ are unitary and not antiunitary symmetries. (Many one can be written as a square).
- 3) it is possible to have a zero mass particle of infinite spin, formally.
- 4) the assumption of asymptotic completeness is: $H_{in} = H_{out} = H$.

Chapter Three: Fields and Vacuum Expectation Values.

In classical EM and gravity, hermitian observable fields were introduced to get rid of action at a distance. In RFT one has also unobservable non-observable fields.

3.1 Axioms.

0. State = ray in Hilbert Space. States transform as cu rep of $SL(2, \mathbb{C})$. Vacuum exists.

I. Here, field operators instead of being $\phi(x)$ are more generally taken as $\phi(f(x))$ where F is a test function of some sort. These field operators and their adjoints are defined at least on a part of H which is dense in H , H being the full Hilbert Space. This domain within H is called D , perhaps to remind us it is dense in H . The space of vectors gotten by doing polynomials of smeared fields is called D_0 . Maybe $D = D_0$ but not known at time of writing.

II. The fields transform with certain reps of $SL(2, \mathbb{C})$ or same plus parity. Examples given, all obvious.

III. The usual LC (local commutativity) conditions are stated: The commutator or anticommutator vanishes at spacelike separations. Also called microscopic causality.

Comments: there are problems with the usual equal time commutation relations, eg, when you smear they may still be singular. The equal time commutators are often replaced with another assumption: fields are irreducible (smeared fields are). This then means that any operator can be written in terms of the fields.

Definition: field theory is what you have if application of polynomials in the smeared fields to the vacuum gives a set of vectors D_0 dense in H . (called "cyclic vacuum")

IV. Asymptotic completeness: $H_{in} = H_{out} = H$. This assumption is on a different level, since to frame the assumption you must know about scattering states.

smeared field: instead of using $\phi(x)$ you use $\phi(f) = \int d^4x f(x) \phi(x)$, ie, you integrate your regular field against a test function to smooth it out a little. Perhaps should think of $f(x)$ as a ball around some spacetime point. Then the LC condition in terms of smeared fields says that commutator vanishes if balls are spacelike separated. These balls have compact support, do not extend far.

3.2 Free field theories. These satisfy all the axioms and are known to exist, although their scattering theory is trivial since they don't interact by definition. As a matter of notation, an n -particle state in the Fock Space is called:

$$\psi^{(n)}(p_1, p_2, \dots, p_n)$$

Using free fields, you can make certain curious new fields called Wick polynomials. S-matrix is same as for the free fields.

Perhaps Axiom III on LC is too strong. Theorem 4.1 will show that if you only insist on LC at huge spacelike separations, then you can prove axiom III. Other comments on the time slice axiom and generalized free fields which I skip.

3.3 Properties of Expectation Values. We now begin a set of about 9 theorems of this chapter of the book. First, here are some definitions:

Wightman function: $\mathcal{W}(x_1, x_2, \dots, x_n) \equiv \langle 0 | \phi_1(x_1) \phi_2(x_2) \dots \phi_n(x_n) | 0 \rangle$.

You write this as a script W. Fields can be different, some adjoints etc. Notice that field label goes with its spacetime argument. Here is an example of permuted W function:

$$\pi(1, 2, 3) = (2, 1, 3) \Rightarrow \mathcal{W}(x_1, x_2, x_3) \equiv \mathcal{W}(x_2, x_1, x_3)$$

Mathematically, one first defines the Wightman thing with smeared fields inside. This makes the thing a multilinear functional in the $f_i(x)$. Then using the Nuclear Theorem you can show that the existence of such a thing implies the existence of the Wightman as above in the sense of a tempered distribution. I.e., you smear it to get meaning, like a delta function.

Alternatively, you can deal with states like this:

$$\Psi \equiv \int dx_1 \dots dx_n f(x_1, x_2, \dots, x_n) \phi_1(x_1) \phi_2(x_2) \dots \phi_n(x_n) | 0 \rangle$$

Whereas D_0 was the space whose vectors were smeared field polynomials, the space whose vectors are as shown above is called D_1 . Big deal.

Theorem 3.1 (a) Since the fields transform in a certain way as noted in axiom II, the Wightman functions transform in an obvious way. No need to write this down, page 107.

Theorem 3.2 (b) We know that $\mathcal{W}(x_1+a, x_2+a, \dots) = \mathcal{W}(x_1, x_2, \dots)$ just by translating the fields. Thus, script-W depends only on coordinate differences, so we now use printed W:

$$\begin{aligned} \mathcal{W}(x_1, x_2, \dots, x_n) &\equiv W(x_1 - x_2, x_2 - x_3, \dots, x_{n-1} - x_n) \\ &= \underline{W(\xi_1, \xi_2, \dots, \xi_{n-1})} \end{aligned}$$

$$\text{fourier } \mathcal{W}(x_1, \dots) = \tilde{W}(p_1, p_2, \dots, p_n)$$

$$\text{fourier } W(\xi_1, \dots) = \tilde{W}(q_1, q_2, \dots, q_{n-1})$$

(c) Hermiticity:

$$\langle 0 | \phi_1(x_1) \phi_2(x_2) \phi_3(x_3) | 0 \rangle = \langle 0 | \phi_3^\dagger(x_3) \phi_2^\dagger(x_2) \phi_1^\dagger(x_1) | 0 \rangle^*$$

(d) Local Comm. This says that if all your differences like $x_1 - x_4$ are spacelike, then you can relate different permutations of Wightman functions by the obvious statistics, since fields then commute or anticommute.

Theorem 3.3 (e) Something is positive definite. Dont see relevance.

Fact: The theorems (a) through (e) show that any set of tempered distributions must be the vacuum expectation values of some field theory (reconstruction, see below).

Theorem 3.4: Cluster Decomposition Property.

$$\mathcal{W}(x_1, x_2, x_3 + \lambda a, x_4 + \lambda a) \rightarrow \mathcal{W}(x_1, x_2) \mathcal{W}(x_3, x_4) \quad \text{as } \lambda \rightarrow \infty$$

if $a = \text{spacelike}$. This theorem is proved very simply from previous assumptions. It says that at large spacelike separations things cannot interact. Obviously the basic assumption will be the LC condition. Proof depends on no massless particles, though.

At this point, authors inject a statement of C invariance and CPT invariance. I skip for now because these things seem out of place. The point is that they add further restrictions on the Wightman functions:

Theorem 3.5: The function $W(\xi_1, \dots, \xi_{n-1})$ exists and is holomorphic in "the tube". This analytic function has a boundary value which is the Wightman function mentioned above. There is a unique analytic continuation of this function to the "extended tube".
Note: it is W and not $\mathcal{W}$ about which this theorem is stated.

In particular, you can consider the complex LT $(-1, 1)$. You find that:

$$W(\xi_1, \dots, \xi_{n-1}) = (-1)^J W(-\xi_1, \dots, -\xi_{n-1})$$

where J is the total number of undotted indices, or in my or Taylor's case, this would be a product of $(-1)^{2s_i}$ where each field transforms as (s_j, s_j') or reverse. Converted to $\mathcal{W}$ functions the above says:

$$\mathcal{W}(z_1, z_2, \dots, z_n) = (-1)^J \mathcal{W}(-z_1, -z_2, \dots, -z_n)$$

The order is not reversed. Also, in Taylor's form the phase would be just unity since all reps are really $(0, s_i)$.

Theorem 3.6: This says that two Wightman functions of W type which are related by permutation are connected by analytic continuation. This is obvious at a totally spacelike point (all differences are spacelike), and proof is very short. You get to utilize the "edge of the wedge theorem". Another way to say it: the extended tube and any permuted extended tube have a real environment in common (which contains the totally spacelike points, obviously).

Theorem 3.7:

3.4 The Reconstruction Theorem. If you have a set of tempered distributions $W^{(n)}$ for all $n = 1, 2, \dots$ which satisfy conditions (a) through (e) above as well as have the cluster decomposition, then you can prove that these functions can be interpreted as the vacuum expectation values of some field theory. This sort of reverses all the proofs given above, which showed that in a field theory the Wightman functions have all those properties, hence "reconstruction theorem". This theorem is proved (8 pages) only for the hermitian scalar field case, due to notational simplicity.

3.5 Symmetries.

Theorem 3.8: (for parity) Suppose there exists a unitary operator $U(I_g)$ which transforms a scalar field in the obvious way. Then

- 1) on the vacuum it can give only a phase
- 2) the operator $U(I_g)$ is completely determined on the space D (defined at start of chap)
- 3) the action of $U(I_g)$ on $U(a, L)$ is as you think, 3-59.

The point here is that a symmetry operator like parity, time reversal, or C , is really completely determined in the theory just by its transformation property on the fields.

We now have a long discussion on problems with phases when you do C, P and T on operators which are non-observable.

Theorem 3.9: Suppose your Wightman functions in your field theory have the PCT property as quoted in (3-66) or (3-39) [a property which will be proved in chapter four]. Then your field theory must have a CPT operator Θ which acts on an arbitrary field as shown in (3.67).

This theorem thus "extends" the reconstruction theorem to include the PCT symmetry. Similarly you could show that the parity property ~~is~~ implies a parity operator in the field theory, and so on.

7/10/77

Chapter Four: General Theorems of QFT.

4.1 Theorem 4.1: if a commutator vanishes for x, y in any spacelike separated sets (say far separated), then the same must be true for all x, y which are spacelike separated, ie, LC condition. The proof obviously makes use of transformation properties of fields.

4.2 Polynomial Algebra stuff. Let O be any set in spacetime. (open set of course) Let $P(O)$ be polynomials of smeared fields which are defined on O . This set happens to be a $*$ algebra (see Pozzi).

Theorem 4.2: ~~If~~ Your polynomial operators defined only over O give a set of vectors which are dense in H , provided that this is true for all spacetime. Ie, if your full vectors defined over R^4 give a set of vectors which is dense, then the smaller (in a sense) set of vectors from the operators restricted to piece O of R^4 is also dense. Recall that this means that the vacuum is a cyclic vector, by definition.

Theorem 4.3: if T is an operator in $P(O)$ and T kills the vacuum, then T identically zero. I think they forgot to add that T is supposed to commute with all operators too. I don't see what they are trying to say here.

Theorem 4.4: a lemma for the next theorem.

Theorem 4.5: In a "field theory" (vacuum is cyclic so polys on vac are dense in H), the smeared fields form a set of irreducible operators. Ie, cyclic vacuum implies irreducible field operators implies any operator may be expressed in terms of fields. This shows it is better to assume cyclic vacuum rather than irreducible property.

4.3 The PCT Theorem. First, a definition:

(page 71)

Jost point: the real point $(\xi_1, \dots, \xi_n)$ is called a Jost point if any "positive" linear combination of these coordinates is a spacelike 4-vector. Thus, at a Jost point you know that: $\xi_i = SL$; $\xi_i + \xi_j = SL$ and so on. When we write $\xi_1 = x_1 - x_2$ and $\xi_2 = x_2 - x_3$ this second fact implies that $x_1 - x_3 = SL$. Thus, at a Jost point all the differences $x_i - x_j$ are spacelike.

Theorem 4.6: Let ϕ be a hermitian scalar field doing axioms I, II but not LC.

Define:

PCT condition: $\langle 0 | \phi(x_1) \phi(x_2) \phi(x_3) | 0 \rangle = \langle 0 | \phi(-x_3) \phi(-x_2) \phi(-x_1) | 0 \rangle = \text{reverse + negate}$

Weak LC condition: $\langle 0 | \phi(x_1) \phi(x_2) \phi(x_3) | 0 \rangle = \langle 0 | \phi(x_3) \phi(x_2) \phi(x_1) | 0 \rangle = \text{reverse}$

The theorem then says:

- (a) if PCT holds at all x , the WLC holds at all Jost x .
- (b) if WLC holds near a Jost point (neighborhood), then PCT holds everywhere (for all x).

Now, assume this theorem is true. Then since a field theory has LC, you can certainly reorder the fields at a Jost point, and in particular you can reverse the field order. I.e., usual LC implies WLC at any Jost point. Thus, in any field theory, PCT holds for all x .

Comments: in the proof there is some tubology, but the main ideas are very simple. The complex Lorentz transformation allows you to negate all arguments with no reordering. Then assumption of WLC allows you to reverse the order near a Jost point. This proves PCT near a Jost point, which statement you then analytically continue to all points x . Nothing to it.

Theorem 4.7: PCT Theorem for General Spin. Again, the PCT theorem reverses order and negates the π_i . However, it also contains two phase factors. A factor $(-1)^J$ where again J = number of undotted indices [this phase is ~~xxx~~ unity in Taylor]. Also, there is a factor i^F where F is the number of fermion fields. This same phase i^F also appears in the spin version of WLC condition [order reversal].

To repeat, in a field theory you have LC condition. This means you have WLC or order reversal symmetry (with phase) at Jost points. This in turn implies their version of PCT (which includes order reversal) at Jost points, which you then continue to all points.

I can imagine that someone else might identify CPT simply with the negation of all the arguments and no order reversal. Then CPT is just that complex Lorentz transformation, and CPT theorem just depends on fact that you can uniquely continue a W function to complex arguments (extended tube = Hall Wightman theorem).

4.4 Spin and Statistics.

Theorem 4.8: a field cannot commute with another field, but anticommute^(spacelike) with the adjoint of that other field. And vice versa. In particular, if a field commutes with itself at spacelike separation, then the anticommutator of the field with its adjoint at spacelike cannot vanish!

Theorem 4.9: (Spin Statistics Scalar Field.) if you assume wrong statistics for a scalar field, (ie, scalar field with its own adjoint), then you find that basically $\phi = 0$.

Theorem 4.10: (General Spin-Statistics Theorem.) If you assume wrong statistics for any field with itself, you find that that field must vanish.

Comment: these proofs (to the last two theorems) are quite simple. They depend on the complex Lorentz transformation that one associates also with CPT, therefore, they depend on analyticity. Use smeared fields in a simple way.

Now the subject turns to the statistics between different fields. Some good examples are first given, there is an intermediary theorem 4.11, and then comes:

Theorem 4.12: In any field theory with "abnormal commutation relations" (ie, between different fields) you can always do a "Klein transformation" to get to a field theory with normal commutation relations between different fields.

Comments: this new field theory has the same S-matrix, but it may have different values for some observables. This point is not made clear. Ie, it may be that the two field theories so related are not entirely equivalent. The abnormal field theory will have certain extra symmetries called "even-odd rules".

The proof of 4.12 takes up 6 pages, with theorem 4.13 along the way.

4.5 Haag's Theorem.

The question is this: can you say that at a particular time you can map your interacting fields to the IN fields by some unitary transformation? Like the $U(t)$ of BD? Haag says you cannot.

Theorem 4.14: Consider two fields both of which ^{$T_4 \otimes SU_2$} ~~Poincare~~ transform in the same way. If at a particular time t these fields are related by a unitary transformation V , then V on one fields vacuum gives the other, modulo a phase. In particular, this means that the two fields have the same Wightman functions at equal times.

Theorem 4.15: If the VEV of $\phi \phi$ gives the usual Δ^+ propagator function, then ϕ must be a free field.

Theorem 4.16 (Haag's Theorem) Suppose that one of the two fields of theorem 4.14 is free field. Then the other must be free. The implication here is that you cannot have a free field and an interacting field together in the world. In essence, this is saying that the assumption of V connecting ϕ to ϕ_{in} is wrong.

Theorem 4.17 (Generalized Haag) If two fields transform the same way under $SL(2, C)$ and are related as in theorem 4.14, then all ~~VEV~~ Wightman functions are the same for the two theories up to 4 operators. If one theory is free, this gives the Haag above.

The suggestion here is that the concept of "equal time" may not be meaningful,
and that you must always smear time as well as space.

4.6 Equivalence classes of local fields. Skip.

Chew - S-matrix Theory of Strong Interactions (1961)

Chew: "S-matrix theory of strong interactions"

(1961)

This book contains 100 pages of "lectures" plus 6 historically interesting papers. They are:

Landau	1959	"Analytic singularities of Green's Functions in Field Theory"
Cutkosky	1960	"Singularities and Discontinuities of Feynman Amplitudes"
Mandelstam	1958	"Getting the P-N amplitude from dispersion relations and unitarity."
Kibble	1960	"Kinematics and the M-Representation"
BGKT	1960	"Verify the M-Representation for some potential scattering"
Regge	1960	"Bound States, Shadows, and the M-Representation"

The lectures are not what one would call highly organized, but then this was right in the middle of the operation. The most interesting aspect is that this book contains Chew's strongest statement of his bias, his point of view on the subject. Most of this philosophical jazz is in the first lecture entitled general outlook from which I take several quotes:

"Although he has long since renounced the S-matrix approach, the original work was done by Heisenberg in 1943... he has thrown all his efforts behind the idea of a single underlying field.... He believes that simplicity lies only at the center while on the periphery there is confusion."

"I tend not to believe in the existence of a "center" ... besides, the periphery is subject to experimental tests"

"So that there can be no misunderstanding of the point of view of these lectures, let me say at once that I believe the conventional association of fields with strongly interacting particles to be empty." "I do not have firm convictions about leptons or photons" "Like an old soldier, (field theory) is destined not to die but just to fade away"

"Postulate: the S-matrix is a Lorentz invariant, analytic function of all momentum variables with only those singularities required by unitarity."

"A more concrete way of saying the fundamental postulate, then, is to say that once one is given certain simple singularities (eg, some of the poles), the location and strength of all other singularities are determined by the constraint of unitarity in physical regions. (The solution of such a problem I presume to be consistent with the prescription given by Landau and Cutkosky)

"A fundamental principle therefore might be one of maximum smoothness: The S-matrix has no singularities except where absolutely necessary. There is no "reason" for it to have any others.

"Which of the strongly interacting particles are elementary? ... because of general philosophical convictions I am convinced that there can be only one sensible answer, and that is that none of them is elementary"

"...one may begin anywhere, taking an arbitrary singularity as a starting point and attempting to reach as much of the S matrix from this point as computability allows... couplings assume the maximum value consistent with unitarity.. saturation of unitarity."

"We have absolutely no ideas as to the origin of the strong interaction symmetries."

".. an analytic function is determined through the Cauchy relations by a kind of Coulomb's law for a potential due to point charges (poles) and line charges (branch cuts). The point charge is the residue of the pole, and the line charge density is the discontinuity across the cut. A sort of reciprocal dependence on distance favors nearer singularities"

"The 'nearby singularities' (nearby to the origin) are associated with one and two-particle configurations and are the 'long range forces'. In our incomplete theory, we know only about the long range and medium range forces. The far out multiparticle branch points determine the short range forces, and that is where things get tough" "Thus, we should have a good knowledge of large-L angmom states in scattering."

"My guess is that we will eventually be able to handle 3-body as well as 2-body with fast computers, but nothing more complicated."

"In principle, only one mass is independent and sets the scale for the S matrix"

Here is a quick summary of the contents of those lectures:

1. Introduction, see above.
2. Lorentz invariant amplitude and substitution law. Here, the M variables and kinematics are introduced.
3. The Landau Rules and the m-rep. The idea of the obvious-singularities representation of M is stated. Regions drawn in. Note that Chew uses s_3 for his main, direct channel (we would use s now). There is a good deal of confusion here between the single and double spectral functions.
4. Cutkosky's generalized unitarity relations. Cut showed that you can find the sings of an arbitrary feynman diagram simply by putting all the propagators to delta functions. I.e., put all internal lines "on shell". For diagrams that I usually call "unitarity diagrams", Cuts technique duplicated unitarity. However, his method says more, eg, it gives the contriubtion of a prticular box graph to the double spectral function.
5. Charge and Spin generalization (ie, isospin in spin). A and B etc.
6. Physical interpretation of singularites. He calls the direct thresholds of a channel "physical sings", whereas the invisible cross channel thresholds are called "forces". If you just write a fixed-s disprel, the cut terms look just like what you get in potential theory using a continuum of Yukawas. In fact, the mass at the start of the disprel cut corresponds to the lightest mass in the Yukawa distribution and thus controls the range of the interaction. This is why near sings affect long range forces. I think this is the crux of the lectures so I will write out this disprel:

$$A(q^2, \cos\Theta) = \frac{1}{\pi} \int_0^\infty dq'^2 \frac{\rho_3(q'^2)}{q'^2 - q^2} + \frac{1}{\pi} \int ds'_1 \frac{A_1(s'_1, \dots, s_3)}{s'_1 - s_1} + \frac{1}{\pi} \int ds'_2 \frac{A_2(\dots, s_3)}{s'_2 - s_2}$$

$\sim s_3 \leftarrow \text{fixed } s_3$
 $s_1 = \mu$
 $s_2 = \mu$
 $\approx \frac{g}{s_3 - m^2}$
 $LE = s_1$
 $A_2 = \text{disc across here}$
 $A_1 = \text{disc across here}$

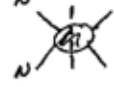
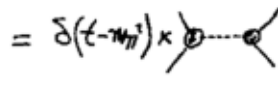
6 con'd. In this dispresl, the pole term contributes only to $L=0$ waves. For higher waves, A is controlled by A_1 and A_2 (the line charges). In particular, A feels the pieces of A_1 and A_2 that are closest in the t -plane. Or, you might say, the pieces that have the longest range, in the Yukawa analogy. If A is np elastic scatter, then the nearest A_1 singularity is the pion pole (which gives a delta $\times g$ to A_1). So for high partial waves, the pion pole by itself should be a good approximation! (Note: high partial waves suggests to me peripherality, large impact parameter, high energy)

"We do not yet have a definite method for calculating multiparticle matrix elements" he says. This was the need that bore the multiperipheral model, that ate up Toller theory, etc

"One channel provides forces for the other two — which in turn generate the first... a self-determining situation. Our task is now to understnad how many arbitrary parameters there are and how to calculate."

7. The 2-body dynamic equations: definition of the "potential". This is where the lectures get a little rough. First, Chew considers the Yukawa deal of BGKT and writes their m -rep for f . The boundary for RHO is easy to see and starts at $4t_0^2$ or higher. By screwing around with various s and/or t discs, and using elastic unitarity (which here is everything), he comes up with equation (7.14) which says: $f_2 =$ function of f_2 . If you know RHO (the Yukawa distribution, the potential), you can compute f_2 and hence RHO_2 (the double spectral) everywhere, by a scheme I think is known as "Mandlestam iteration". Of course, you need that potential to get off the ground. Next, Chew finds the object in the rel theory that corresponds to $RHO(0)$ of the nonrel theory. He has, as advertised, defined the potential. Given this potential, you could solve by iterating as in the nonrel case. But, the potenail is a mess of spectral functions. How do you evaluate it???

8. "Evaluation of Longrange and Medium Range Forces". What that title means is that he will show us how to estimate the poential (= force) for a reaction (3) due to the close-in sings of the cross channel (2). Now, from the deinition in lecture 8 or 7, when s_2 is less than 4 pion masses, the potential for $N-N$ scatter is exactly the cross channel absorptive part. This absorptive part for this range of $s_2 = t$ vanishes except at $t = \text{pionmass}$, so A_2 is a delta function (like single Yukawa RHO would be a delta function. Thus:

When $t < 4m_\pi^2$, "force" or "potential" for  is $t \rightarrow$  = $\delta(t - m_\pi^2) \times$ 
 So, $V = \delta(t - m_\pi^2) \times [\text{diagram}]$.

This potential is called the long-range force (see title of chapter). Now, for the next higher range of t , the potential as defined in Ch.7 is the full absorptive part minus an elastic piece, so:

When $4\mu^2 < t < 9\mu^2$, $V = \text{abs}(\text{diagram}) - \text{abs}(\text{diagram})$ ← elastic in S .

However, in order to know these absorptive parts (in order to know the potential) you need to know a different, contained 2-to-2 amplitude. In the long/medium force regime, then, in order to know NN , you will need to know $PI-N$. And in order to know $PI-N$, you will need $PI-PI$. Somehow, $PI-PI$ is at the bottom of everything (and unfortunately is the one that no "correcting" experiments have been done with.)

" At the base of the theory is the $PI-PI$ problem, which provides all its own long-range forces and an important part of many other 2-body forces..."

stop for now.

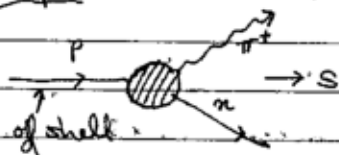
Chapter 2. He defines his "Lorentz invariant amplitude" and the invariants s, s_2 and s_3 . Lorentz transforms cannot connect the "channels" because a.l.t. cannot change $|p_i|$ to $-|p_i|$; no way. Nevertheless, the "substitution rule" postulates that the same amplitude $A(s_1, s_2, s_3)$ describes all 3 channels. (The other 3 reactions are PCT-related, for a total of 6 channels). For the "sub. rule" to have content, you must know how to continue A across nonphysical zones. This is what the Mandelstam is supposed to do for you.

Chapter 3 "Landau Rules and Mandelstam Representation".

First, some references

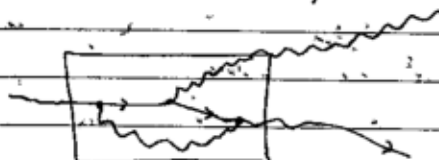
- a) Landau — The singularities of a Feynman diagram occur when s_1, s_2, s_3 are such that all internal momenta are on their mass shells. Furthermore, only poles + branch points occur.

Example: consider the vertex:

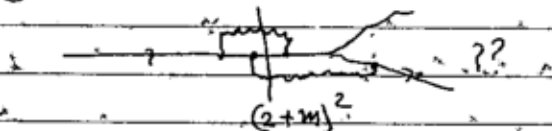


$$g(s, t, m^2) = g(s) \Rightarrow \text{what analytic structure?}$$

Pick some graphs:



Looks like cut in s at $(1+m)^2$. So it is conjectured that the full $g(s)$ has cut at $s = (1+m)^2 +$ then perhaps higher cuts

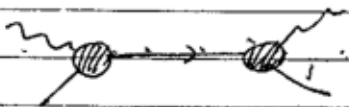


At any rate, if you face all cuts and not real, then no matter how many branch points, you can let some g represent the discant; so:

$$g(s) = \frac{1}{\pi} \int_{(m+1)^2}^{\infty} \frac{ds' \rho(s')}{s' - s}$$

$$\text{Im} g(s) = \rho(s) \quad \Theta[s - (m+1)^2]$$

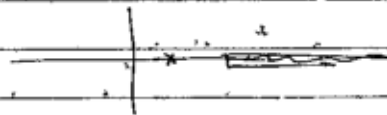
Example 2: Now consider the subset of $\mathcal{A}$, namely (ie, piece of unitarity sum?) No!! This is a Feynman graph not a "pole" unitarity thing. Not Cutovsky yet).



OK, so the subset of all Feynman diagrams is

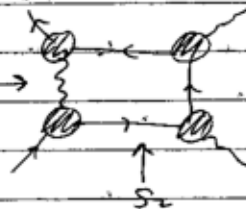
$$\text{Diagram} = g^2(s) S_F(s) g^2(s) = F(s)$$

$$F(s) = \frac{1}{s-m^2} + \int_{(m+1)^2}^{\infty} \frac{g(s') ds'}{s-s'}$$



The point is this: as a Feynman diagram subset, this thing has more singularities than just the usual pole!!

Example: Consider another subset: $s_1 \rightarrow$ etc.



Here, it seems very reasonable that:

$$\text{amp} = \int ds_1' ds_2' \frac{p_{12}(s_1', s_2')}{(s_1' - s_1)(s_2' - s_2)}$$

Here, p_{12} gives the double discontinuity + is real. It is 0 when, according to Landau, the invariants are such that the internal lines shown cannot be on shell (e.g., if either s_1 or s_2 is too small.)

Note: This p_{12} gives nothing in a physical region because the cross variable (say s_1) is not big enough to hit the horizontal mass shells. In fact, s_1 is negative in the s_2 phys. region.

Clearly need s_1 and $s_2 > 0$ to get $p_{12} \neq 0$.

Mandelstam then conjectures that there are no other singularities besides those illustrated by these two subsets. Thus, if you knew the single discontinuities across the cuts, you could compute the doubles, and you would know A everywhere!

Chap 4 "Cutkosky's Generalized Unitarity Relation"

The reference here is

Cutkosky, J. Math. Phys. 1, 429 (1960) — 5 pages!
and, PRL 4, 624 (1960).

The question is "How do you find those double spectral functions?"

Case I: poles and coupling constants



Claim that the amplitude for this blob, $A_{\text{blob}}(s, t)$, does have a pole at $s = m^2$ whose residue is the product of two on-shell vertex functions. I can see this in field theory because you draw a Feynman diagram. Chew says, however, that it is true outside the realm of F.T. I would maybe show this by applying unitarity at $s \approx m^2$. But why am I allowed to use unitarity outside the physical region?

Case II: and what about the discontinuities?

Again, in field theory Cutkosky showed that if you want to know how a particular graph contributes to the disc, the rule is "calc. the graph after replacing all internal propagators with delta functions."

Example: $s \rightarrow \text{graph} \leftarrow \text{graph}$, disc = $v_1 v_2 \delta(s - m^2)$

Why is this a "pole". $A(s+i\epsilon) - A(s-i\epsilon) = 2i \text{Im} A(s+i\epsilon) = iR \delta(s - m^2)$

$$\Rightarrow A(s) \stackrel{i\epsilon}{\approx} \frac{R}{s - m^2 + i\epsilon}$$

$$A(s+i\epsilon) = \frac{R}{s+i\epsilon - m^2} \Rightarrow \text{Im} A(s+i\epsilon) = -\pi R \delta(s - m^2)$$

So you are left with (real) vertices. This then justifies C's rules in this simple case.

Example: Feynman graph

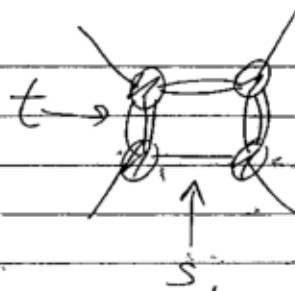
If you put in one delta for each line, that delta kills part of the Feynman loop integral. The rest of the integral (the d^4p) is still there as phase space of on-shell lines.

Now, Cutkosky applied to this class of Feynman diagrams yields the usual unitarity relation

$$2\text{Im} A_{fi} = \sum_n \langle f | T | n \rangle \langle n | T | i \rangle$$

← phase space + sum

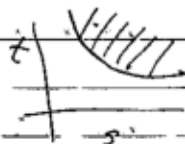
But Cutkosky does still more. He gets you the double disc by putting all lines to δ 's in a box.



typical box fermion diagram. Put all lines as $\delta(\dots)$, leaving only phase space integrals. Then this tells you just!

The lowest one: gives that old ball-busting jacobian $\int d^4k \delta(k) \delta(k) \delta(k) \delta(k)$, which ends up giving a link like $\lambda(t, t, t, t)$ which is $\propto$ except... This then defines your region for p in the "both channels elastic" case.

$$(s - 4m^2)(s - 4\mu^2) = 4\mu^2$$



Chap 5 - generalize to include charge + spin

This means including $150\text{ spin} + \text{spin}$. Essentially, if you do things right, you end up with a set of functions - each one of which satisfies the analyticity of the spinless. Chew does the $\pi - N$ case of A^1, A^2, b^1, b^2 , & gives an interesting reference to Frazer + Fulco on the detail that I once worked on. Hearn has a general method to get "clean" amplitudes, & so does Tannenbaum 2 years in the future, and Fubini in 1972.

Chap 6 - physical interpretation of singularities

OK, start off by writing a fixed $-s$ disp rel (with plane) for $p \cdot n$ scatter:

$A(q, \cos \theta) = \int + \int + \int$

$\int ds_1^2 A_1(\dots)$

$\int ds_2^2 A_2(\dots)$

$\int ds_3^2 A_3(\dots)$

called (III, s_3 by Chew)

These cuts contribute to A in a way that looks like a Born approximation, based on the angle dependence, due to a potential and exchange potential, and these look like continuum of Yukawa potentials.

We leave so far (keep only Ch I. contrib):

$$A_{\pi\pi} \sim \int ds' \frac{A_1(s', s'_{-1})}{s' - s_1}$$

The lowest s'_1 contrib has the largest denom in the $S_3(s)$ phys region, so look at low s'_1 . The absolute nearest is the pole ~~in~~ A_1 delta in A_1 at the pole of A due to pin pole in $\pi\pi$ channel: so

$\pi\pi f_2 \approx \pi\pi$, best for large l since π is "largest size".

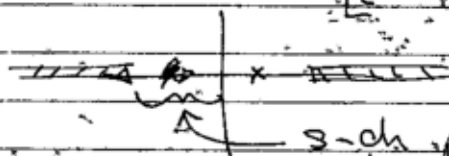
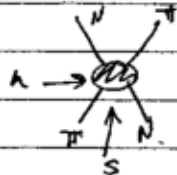
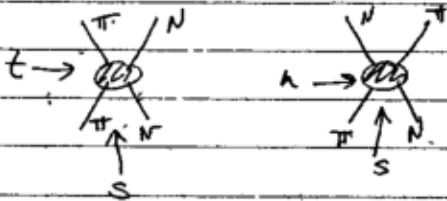
So the big idea here is: "in any channel, the faces are determined by the pole + cuts in the other two channels". This, a "self-determining situation". How many parameters are there?

OK, let's modernize this old language.



fixed u "The physical s -ch amplitude at fixed- u is, via this fixed- u disp rel, influenced by the poles + cuts in the t -channel."

Of course, it also is influenced by the pole + cuts in itself.



s-ch physical region (in variable t). Clearly, you are close to the t -channel pole, especially for $t \approx 0$ (forward).

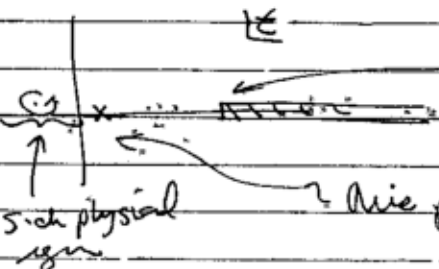
$$s+t+u = \Sigma$$

$$u = \Sigma - s - t$$

$$u = u_0 = \text{pole or cut in the } u \text{ ch.}$$

$$t = \Sigma - s - u$$

But, as $s \rightarrow \infty$, get $t(u_0) = \Sigma - s - u_0$, none way off to left, so



This is second closest contribution.

Thus: "pin pole dominance"

The pole is nearby,

Maybe this pin-pole will cause a pole in the s-channel which will in turn cause that pin pole, i.e., bootstrap.

Chap 7 - The two-body dynamical equations
definition of the "potential"

p 33 of 94.

Now we get down to earth again. Let:

$$V(r) = - \frac{1}{2m^2} \int_{m_0}^{\infty} dm^2 \rho_0(m^2) \frac{e^{-mr}}{r} = \text{Yukawa.}$$

Potential in a non-rel problem. What does this give for a scattering? In the Born approx you get:

$$f \sim - \int V(r) e^{i\vec{q} \cdot \vec{r}} d^3r \quad \text{where } \vec{q} = \vec{m} \cdot \vec{T}.$$

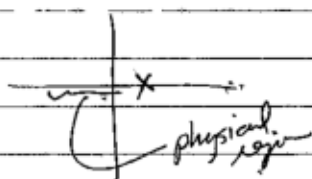
In our case:

$$\begin{aligned} f &\sim + \frac{1}{2m^2} \int_{m_0}^{\infty} dm^2 \rho_0(m^2) \cdot \int d^3r \cdot 2\pi \cdot \frac{e^{-mr}}{r} \cdot \int_{-1}^1 dz e^{igzr} \\ f &\sim \frac{1}{2m^2} \int_{m_0}^{\infty} dm^2 \rho_0(m^2) 2\pi \cdot \frac{1}{q} \int_0^{\infty} dr \cdot e^{-mr} \cdot \sin(qr) \cdot \frac{1}{qr} \sin(qr) \\ &\sim \int_{m_0}^{\infty} dm^2 \rho_0(m^2) \cdot \frac{1}{q^2 + m^2} \end{aligned}$$

And $|\vec{q}|^2$ is called $-t$. $-t = (\vec{p}_1 - \vec{p}_3)^2 \approx (\vec{E}_1 - \vec{E}_3)^2 - (\vec{p}_1 - \vec{p}_3)^2$
not same q as in $\propto -|\vec{q}|^2$
 $2q^2(1 \pm \cos\theta)$

So one Yukawa [$\rho_0(m^2) = \delta(m^2 - M_0^2)$] would give a pole at

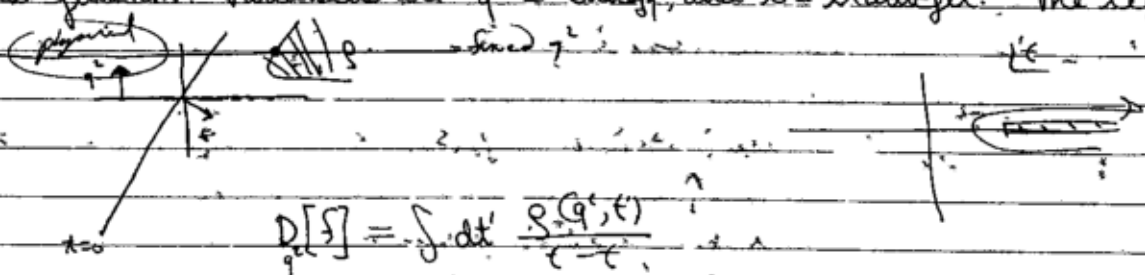
$|\vec{q}|^2 = -m^2$, an "unphysical" momentum transfer.



Well, at $t = +m^2$

Here you see why (p33) $1/m \sim t_0$ is associated with the range of the potential.

So this Yukawa model thing does satisfy Mandelstam's req: There is one double spectral function. Variables are $q^2 = \text{energy}$, and $t = \text{transfer}$. The region is



$$D[S] = \int dt' \frac{S(q^2, t')}{t - t'}$$

So g is the double disc across cut.

Now he is going to derive the M-iteration scheme.

$$\begin{aligned} f_3 &\equiv D_{s_3} F(q^2, t) & s_3 &= \text{non-rel channel} \\ f_2 &\equiv D_t F(q^2, t) & &= \text{P.S.} \end{aligned}$$

(7-9) = rewrite of (7-1) using (7-8).

Nonrelativistic unitarity says:

$$\text{Im} f = \int dt' f^* f \quad \text{in } q^2 > 0 \text{ and } -4q^2 < t < 0.$$

Use the exp (7-9) in the RHS of (7-3), and $f_3 = \text{Im} f$, so

$$f_3 = \iint dt' dt'' f_2^* f_2 \frac{1}{\sqrt{K}} \ln \dots$$

This relates the " s_3 -disc" f_3 to the " t -disc" f_2 .

$$\text{Now do Im m (7-10): } g = \text{Im} f_3 = \int dt' dt'' f_2^* f_2 \sqrt{K}$$

This is the usual result for g in the elastic case.

So we now have a loop:

$$\begin{array}{c} g \xrightarrow{(7.8)} f_2 \\ f_2 \xrightarrow{(7.12)} g \end{array}$$

In fact, put it all together in (7.14):

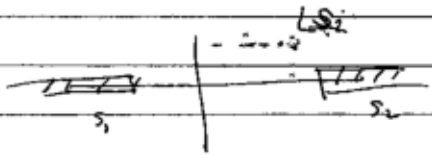
$$f_2 = g(t) + \iint dt' dt'' dq^2 \dots f_2^* f_2$$

Main idea: if you know g , then analyticity + unitarity tells you the full solution.

Now, in the rel theory what do we use for potential?

this stuff is not really clear

① Write $A_3 [= D_3]$ in terms of a dispel for fixed $s_3 [= s]$.



$$A_3(s_1, s_2, s_3) = p_0(s_3) + \int ds'_2 \frac{p_{32}}{s'_2 - s_2} + \int ds'_1$$

↑
write a fixed $|s_3|$ dispersion
relation for $D_3[A] \equiv A_3$.

② Now decompose $A_3 = A_3^{(el)} + A_3^{(in)}$; $p_{23} = p_{23}^{(el)} + p_{23}^{(in)}$ etc

③ Rewrite ① for "elastic only": [elastic in ch III]

$$A_3^{(el)}(s_2, s_3) = p_3(s_3) + \int ds'_2 \frac{p_{32}^{(el) III}(s_3, s'_2)}{s'_2 - s_2} + \int ds'_1 \frac{p_{13}^{(el) III}(s'_1, s_3)}{s'_1 - s_1}$$

④ Now write full unitarity for the elastic piece!

$$A_3^{(el)}(s_3, z) = \int dR' A^*(s_3, z') A(s_3, z)$$

⑤ Now disperse A^* in terms of A_2, A_1, A_3 whatever.
Then compare with ③ to equate

$$p_{32}^{(el) III}(s_3, s_2) \sim \int ds'_2 \int ds'_1 \frac{A_2^* A_3}{K^{1/2}}$$

OK, but this has only got you the III-elastic piece of p_{32} .
Now

⑥ Let $p_{32} = p_{32}^{el III} + p_{32}^{in III}$

⑦ Next, use the M-rep dispel for A_2 :

$$A_2 = p_2 + \int p_{32} + \int p_{21} = \underbrace{\left[p_2 + \int p_{32}^{(in) III} + \int p_{21} \right]}_{\text{call this } \sqrt{V}^{III}} + \int p_{32}^{(el) III}$$

Now this looks like the old elastic problems with a given potential. Looks like (7-8):

$$f_2 = p_0 + \int p_{21}^{\text{elastic}}$$

potential of channel 3
for ch 3

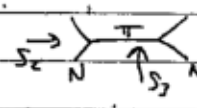
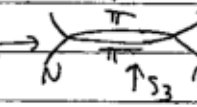
Same now have: $A_2 = V_3^2 + \int g_{32}^{el, III}$

where V_3^2 is the potential seen in channel 3, not channel 2. It is taking the 2 discontinuity that exposes the 3 channel potential as in (7-1) yukawa case.

However, we hypothesize that this 3 ch potential is generated by the channel 2 spectral function.

So we close the chapter asking: how in the hell are you going to provide input to these equations? What starter potential will you use?

Chapter 8: "Evaluation of the Long Range + Medium Range Forces."

Let: $S_2^{(min)} = \text{lowest 2 channel threshold}$ 
 $S_2^{(min)'} = \text{next 2 ch then}$ 

Then, for $S_2^{(min)} < S_2 < S_2^{(min)'}$, you are below the elastic g_{32} function [below the S_2 cut for all S_3], so:

$A_2 = V_3^2$ exactly: [for limited S_2 as shown]

So we have $V_3^2 = g_{\pi N}^2 \delta(S_2 - m_\pi^2)$

Claim: to know V_3^2 for larger S_2 , say $S_2 > S_2^{(min)'}$, you need a Cutkosky analysis.

Claim: the $\pi-\pi$ interaction is at the base of everything, even tho experimentally elusive.

Chap 9. Single Spectral function as Partial Waves.

The SSF occurs only in the S-wave, since no l dependence.

Now, insert the fixed $-s_3$ [s_2 plane] dispersive A into the PW inversion. Like setting up for froissart projection.

$A(s)$

s

This "Picture" draws a blank.

~~LHC~~
LHC

~~RHC~~
RHC.

So now on page 48 + what have I learned?

1. old Kramers + Str.
2. Landau/Cutkosky + double spectral
3. idea of a "potential"
4. idea of pion control, somehow...
5. cuts in $\mathcal{A}(S)$.

So far, not very satisfying. Remember, 1960

Chap 10 — a discussion of the N/D method and partial waves. Some kind of pole exp. Better to read it elsewhere.

Chap 11 — blank. (arb params in $\mathcal{I}^m$ — same as field theory).

Chap 12 — $\pi\pi$ scattering. (p63) A set of coupled equations to be solved by iteration — baloney.

Chap 13: "Principle of Maximum Strength" — nature will saturate the Froissart limit — cross sections are as large as they consistently can be. Here is a classic statement of Chew's philosophy. Copy it later.

Chap 14 "T π dynamics" — by starting with a weak g + adiabatically increasing it, solving all equations + keeping consistent, you might get the whole $\pi\pi$ theory. Then you stop when unitarity is hit, ~~and stop~~ it should pop out the right resonances. All you need is the right equations.

Chap 15 — "EM form factor": — Claims that as of 1960, no evidence of QED failure. So use QED to probe charge "structure" of the pion.

from
p. 48

Chap 18 Summary + conclusion —

Martin & Spearman - Elementary Particle Theory (1970)

The Various and Multicolored Amplitudes of Martin and Spearman.

First, write a general S-matrix element as $\langle f | S | i \rangle$. Then, decompose the i and f states as: $|i\rangle = |P_i^\mu\rangle \otimes |\alpha_i\rangle$ and $\langle f| = \langle P_f^\mu| \otimes \langle \alpha_f|$. Also, write $S = I \otimes S_P$. Here, I indicates that $[S, P_\mu] = 0$, ie, that the total four-momentum is a conserved quantity. Thus, as far as the "orbit" portion of the S-matrix element is concerned, we have $\langle P_f^\mu | S | P_i^\mu \rangle = \langle P_f^\mu | I | P_i^\mu \rangle = \langle P_f^\mu | P_i^\mu \rangle = \delta^4(P_f^\mu - P_i^\mu)$. The operator S_P then acts only within the little Hilbert space for a particular choice P (usually one chooses $P = 0$; then S_P is an operator in the "barycentric subspace".) In other words, write:

$$\langle f | S | i \rangle = \langle P_f^\mu | I | P_i^\mu \rangle \otimes \langle \alpha_f | S_P | \alpha_i \rangle = \delta^4(P_f^\mu - P_i^\mu) \langle \alpha_f | S_P | \alpha_i \rangle$$

Now, for two-particle helicity states in the cms, it turns out that $|\alpha_i\rangle = \sqrt{\frac{4W}{P}} |\theta \phi \mu_1 \mu_2 \gamma\rangle$. This state is carefully defined as by Jacob and Wick, but we do not need to worry much about its origins. The point is that we can now say:

$$\langle f | S | i \rangle = \delta^4(P_f^\mu - P_i^\mu) \frac{4W}{\sqrt{P P'}} \langle \theta' \phi' \mu_3 \mu_4 \gamma' | S_P | \theta \phi \mu_1 \mu_2 \gamma \rangle$$

In order to remove from this amplitude the chance of a non-interaction, define the T operator by $S = I + iT$. In the direct product notation, this means $S = I \otimes S_P$ and $T = I \otimes T_P$ and $I = I \otimes I$ so we can rewrite the T definition as $I \otimes S_P = I \otimes I + i I \otimes T_P$. In the little Hilbert space, then, one has simply $S_P = I + iT_P$. Therefore, in general we can say,

$$\begin{aligned} \langle f | S | i \rangle &= \langle P_f^\mu | P_i^\mu \rangle [\langle \alpha_f | \alpha_i \rangle + i \langle \alpha_f | T_P | \alpha_i \rangle] \\ &= \langle f | i \rangle + i \langle P_f^\mu | P_i^\mu \rangle \langle \alpha_f | T_P | \alpha_i \rangle \end{aligned}$$

In the particular case again of two-particle cms states we have:

$$\langle f | S | i \rangle = \delta^4(P_f - P_i) \frac{4W}{\sqrt{P P'}} \left[\delta(\Omega_f - \Omega_i) \delta_{\mu_3 \mu_1} \delta_{\mu_4 \mu_2} \delta_{\gamma \gamma'} + i \langle \theta' \phi' \mu_3 \mu_4 \gamma' | T_P | \theta \phi \mu_1 \mu_2 \gamma \rangle \right]$$

where the factor $\langle \alpha_f | \alpha_i \rangle$ is written out explicitly.

Now that the elements $\langle \alpha_f | T | \alpha_i \rangle$ are clearly defined, we can show that the invariant cross section is given by,

$$d\sigma = \frac{\pi^2}{\text{flux}} |\langle \alpha_f | T | \alpha_i \rangle|^2 dQ$$

where flux = $\sqrt{(p_1 p_2)^2 - m_1^2 - m_2^2} = p \sqrt{s}$ cms
 $= m_2 p_{LAB}$ lab
 $= s/2$ H.E. limit

and where

$$dQ = \prod_{i=1}^n \left(\frac{d^3 p_i}{2E_i} \right) \delta^4 \left(\sum_{i=1}^n p_i - p_{init} \right)$$

where there are n final particles. Alternatively, $\frac{d^3 p_i}{2E_i} = \delta^4(p_i^2 - m_i^2) d^4 p_i$ so that

$$dQ = \prod_f \delta^4(p_f^2 - m_f^2) d^4 p_f \delta^4(p_f - p_i)$$

Again, specializing to the $2 \rightarrow 2$ case, we have:

$$\begin{aligned} dQ|_{\text{cms}} &= \delta^4(p_1^2 - m_1^2) d^4 p_1 \delta^4(p_2^2 - m_2^2) d^4 p_2 \delta^4(p_1 + p_2 - p_i) \\ &= \delta^4(p_1^2 - m_1^2) \delta^4(p_2^2 - m_2^2) d^4 p_2 \\ &= \left(\frac{q_f}{4s} \right) d\Omega \end{aligned}$$

Then what comes out is:

$$\frac{d\sigma}{d\Omega}|_{\text{cms}} = \frac{\pi^2}{4s} \left(\frac{q_f}{q_i} \right) |\langle \alpha_f | T | \alpha_i \rangle|^2$$

Or, using $dt = 2q_1 q_2 dz$ or $d\Omega = d\phi dz \rightarrow \frac{2\pi dt}{2q_1 q_2}$ we can write,

$$\frac{d\sigma}{dt} = \frac{\pi^2}{4s q_i^2} |\langle \alpha_f | T | \alpha_i \rangle|^2$$

The relation between $\langle \alpha_f | T | \alpha_i \rangle$ and the so-called "invariant amplitude" M_{fi} used by many other authors is $\langle \alpha_f | T | \alpha_i \rangle = -(4\pi^2)^{-1} M_{fi}$. The M_{fi} -formalism has two differences from the MS method. First, the single (and two-) particle states are normalized differently ($\langle p | p' \rangle = (2\pi)^3 \delta^3(p - p')$ for M_{fi} users; this is the non-invariant norm), and secondly, the T matrix is defined with $-i(2\pi)^4$ rather than with $+i$. Thus, the differences end up being $-(2\pi)^4 \cdot (2\pi)^{-6} = -(4\pi^2)^{-1}$.

Using the explicit helicity states, we can rewrite these cross sections as:

$$\frac{d\sigma}{d\Omega} = \frac{\pi^2}{4s} \left(\frac{q_f}{q_i}\right) \frac{16S}{q_i q_f} |\langle \dots | T_P | \dots \rangle|^2 = \frac{4\pi^2}{q_i^2} |\langle \phi \mu_3 \mu_4 | T_P | \phi \mu_1 \mu_2 \rangle|^2$$

$$\frac{d\sigma}{dt} = \frac{\pi}{q_i q_f} \frac{d\sigma}{d\Omega} = \frac{4\pi^3}{(q_i)^3 q_f} |\langle \phi \mu_3 \mu_4 | T_P | \phi \mu_1 \mu_2 \rangle|^2$$

except $\left(\frac{q_f}{q_i}\right)$.

The "f-type" helicity amplitudes are defined so there is no constant in $\frac{d\sigma}{d\Omega}$.

Thus, we define the f amplitudes by:

$$f_{34,12} = \frac{2\pi}{\sqrt{q_i q_f}} \langle \phi \mu_3 \mu_4 | T_P | \phi \mu_1 \mu_2 \rangle = \frac{\pi}{2\sqrt{s}} \langle \chi | T_P | \chi \rangle = -\frac{1}{8\pi\sqrt{s}} m_{fi}$$

and the cross sections become:

$$\frac{d\sigma}{d\Omega} = |f_{34,12}|^2 \times \left(\frac{q_f}{q_i}\right) \quad \frac{d\sigma}{dt} = \frac{\pi}{(q_i)^2} |f_{34,12}|^2 = \frac{1}{(q_i)^2} \frac{1}{64\pi s} |m_{fi}|^2$$

On page 326 M and S give the amplitude $\langle \chi | T_P | \chi \rangle$ the new name $T_{cd,ab}$.

Thus, we can say:

$$\frac{d\sigma}{d\Omega} = \frac{\pi^2}{4s} \left(\frac{q_f}{q_i}\right) |T_{cd,ab}|^2 \quad \frac{d\sigma}{dt} = \frac{\pi^3}{4s q_i^2} |T_{cd,ab}|^2$$

$$f_{34,12} = \frac{\pi}{2\sqrt{s}} T_{cd,ab}$$

While we are discussing all these amplitudes, we might as well record all the corresponding optical theorems. They are:

$$\sigma_{\text{pt}}(s) = \frac{4\pi}{q_i} \text{Im} [f_{12,12}(s, t=0)] = \frac{2\pi^2}{q_i \sqrt{s}} \text{Im} T_{00,00}(s, t=0) = -\frac{1}{2q_i \sqrt{s}} \text{Im} m_{ii}$$

m_{ii}^{LAB}

Finally, we can write the partial wave helicity expansions for these amplitudes. Specifically, (p178)

$$\langle \phi \mu'_1 \mu'_2 s' | T_P | \phi \mu_1 \mu_2 s \rangle = \frac{1}{4\pi} e^{i(\mu'_1 - \mu_1)\phi} \sum_{\lambda} (2\lambda + 1) d_{\mu'_1 \mu_1}^{\lambda}(\theta) \langle \mu'_1 \mu'_2 s' | T_P^{\lambda} | \mu_1 \mu_2 s \rangle$$

$$f_{\mu'_1 \mu'_2, \mu_1 \mu_2} = \frac{1}{2\sqrt{q_i q_f}} e^{i(\mu'_1 - \mu_1)\phi} (\dots) = \frac{\pi}{2\sqrt{s}} T_{cd,ab}$$

Martin & Spearman State Notations

7

A state vector belonging to a representation of the Lorentz group may be written as:

$$|P\rangle \otimes |\alpha\rangle$$

Here the state alpha lies within the "little Hilbert space". The little group is the full rotation group. The state of one particle may be written as:

$$|P\rangle = |\vec{p}, m\rangle \quad \langle \vec{p}, m | \vec{p}', m \rangle = 2E \delta^3(\vec{p} - \vec{p}')$$

$$|\alpha\rangle = |j, \mu\rangle \quad \langle j, \mu | j', \mu' \rangle = \delta_{jj'} \delta_{\mu\mu'}$$

$$|j, m; \vec{p}, \mu\rangle = |\vec{p}, m\rangle \otimes |j, \mu\rangle$$

The action of a Lorentz group element on such a one particle state is given by:

$$U(\Lambda, a) |j, m; \vec{p}, \mu\rangle = e^{-iP \cdot a} \sum_{\mu'} D_{\mu\mu'}^{(j)}(R) |j, m; \vec{p}', \mu'\rangle \quad \rho' = \Lambda \rho$$

Wigner rotation

Obviously the little group here is a subgroup of the Lorentz group.

The general-frame two-particle state is given by,

$$|p_1 p_2, \mu_1 \mu_2, \chi\rangle = |j_1 m_1, \vec{p}_1, \mu_1\rangle \otimes |j_2 m_2, \vec{p}_2, \mu_2\rangle$$

with the normalization,

$$\langle p'_1 p'_2, \mu'_1 \mu'_2, \chi' | p_1 p_2, \mu_1 \mu_2, \chi \rangle = 4p_1^0 p_2^0 \delta^3(\vec{p}_1 - \vec{p}'_1) \delta^3(\vec{p}_2 - \vec{p}'_2) \delta_{\mu_1 \mu'_1} \delta_{\mu_2 \mu'_2} \delta_{\chi \chi'}$$

On the other hand, we can define special cms two particle states, viz,

$$|P \Theta \phi, \mu_1 \mu_2, \chi\rangle$$

These have the normalization,

$$\langle P' \Theta \phi', \mu'_1 \mu'_2, \chi' | P \Theta \phi, \mu_1 \mu_2, \chi \rangle = \frac{4W}{\lambda P P'} \delta^3(\vec{P}' - \vec{P}) \delta(\Omega' - \Omega) \delta_{\chi \chi'} \delta_{\mu_1 \mu'_1} \delta_{\mu_2 \mu'_2}$$

This is also the normalization of those any-frame $p_1 p_2$ states mentioned just above.

The factor out front follows from some jacobians in the change $p_1 p_2$ to P_u .

Next, do a direct product decomposition on these states:

$$|P \Theta \phi, \mu_1 \mu_2, \chi\rangle = |P\rangle \otimes |\alpha\rangle$$

If we demand that $\langle P | P' \rangle = \delta^3(P - P')$, then the $|\alpha\rangle$ must be normalized like so:

$$\langle \alpha | \alpha' \rangle = \frac{4W}{P} \delta(\Omega' - \Omega) \delta_{\mu\mu'}$$

Thus, define some new states with the obvious norm, then alpha in terms of these states.

$$\langle \Theta \phi, \mu_1 \mu_2, \chi | \Theta \phi', \mu'_1 \mu'_2, \chi' \rangle = \delta(\Omega - \Omega') \delta_{\mu_1 \mu'_1} \delta_{\mu_2 \mu'_2} \delta_{\chi \chi'}$$

$$|\alpha\rangle = \sqrt{\frac{4W}{P}} |\Theta \phi, \mu_1 \mu_2, \chi\rangle$$

So,

$$|P \Theta \phi, \mu_1 \mu_2, \chi\rangle = [|\vec{P}, \mu\rangle] \otimes \underbrace{\left[\sqrt{\frac{4W}{P}} |\Theta \phi, \mu_1 \mu_2, \chi\rangle \right]}_{|\alpha\rangle}$$

cms.

$$C_J = \sqrt{\frac{2J+1}{4\pi}}$$

Now we can do the angular momentum states. (helicity states)

We can define a set of states corresponding to either spatial set:

$$\begin{cases} |\rho \theta \phi, \mu_1 \mu_2 \rangle = \sum_{JM} C_J D_{M, \mu_1 - \mu_2}^{(J)}(\phi, \theta, -\phi) |\rho JM; \mu_1 \mu_2 \rangle \\ |\rho JM, \mu_1 \mu_2 \rangle = \frac{2J+1}{4\pi C_J} \int d\Omega D_{M, \mu_1 - \mu_2}^{(J)*}(\phi, \theta, -\phi) |\rho \theta \phi, \mu_1 \mu_2 \rangle \\ \langle \rho JM, \mu_1 \mu_2 | \rho' J' M', \mu_1' \mu_2' \rangle = \frac{4W}{\sqrt{PP'}} \delta^4(P-P') \delta_{JJ'} \delta_{MM'} \delta_{\mu_1 \mu_1'} \delta_{\mu_2 \mu_2'} \delta_{rr'} \end{cases}$$

That was the first set of states. The other set is:

$$\begin{aligned} |\theta \phi, \mu_1 \mu_2 \rangle &= \sum_{JM} C_J D_{M, \mu_1 - \mu_2}^{(J)}(\phi, \theta, -\phi) |JM, \mu_1 \mu_2 \rangle \\ |JM, \mu_1 \mu_2 \rangle &= \frac{2J+1}{4\pi C_J} \int d\Omega D_{M, \mu_1 - \mu_2}^{(J)*}(\phi, \theta, -\phi) |\theta \phi, \mu_1 \mu_2 \rangle \\ \langle \theta \phi, \mu_1' \mu_2' | JM, \mu_1 \mu_2 \rangle &= \frac{2J+1}{4\pi C_J} D_{M, \mu_1 - \mu_2}^{(J)*}(\phi, \theta, -\phi) \delta_{\mu_1 \mu_1'} \delta_{\mu_2 \mu_2'} \delta_{rr'} \end{aligned}$$

In the angular representation we can still do the total decomposition as:

$$\begin{aligned} |\rho; JM, \mu_1 \mu_2 \rangle &= [|\rho \rangle] \otimes [|\alpha \rangle] \\ &= |\rho \rangle \otimes \left\{ \sqrt{\frac{4W}{P}} |JM, \mu_1 \mu_2 \rangle \right\} \end{aligned}$$

The normalization of these states then is:

$$\langle \rho' J' M', \mu_1' \mu_2' | \rho JM, \mu_1 \mu_2 \rangle = \frac{4W}{\sqrt{PP'}} \delta^4(P-P') \delta_{JJ'} \delta_{MM'} \delta_{\mu_1 \mu_1'} \delta_{\mu_2 \mu_2'} \delta_{rr'}$$

The usual partial wave expansion is then:

$$T(s, t) \equiv \langle \alpha_f | T_0 | \alpha_i \rangle = \frac{4W}{\sqrt{PQ}} \langle \theta' \phi', \mu_1' \mu_2' | T_0 | \theta \phi, \mu_1 \mu_2 \rangle$$

$$\langle \theta' \phi', \mu_1' \mu_2' | T_0 | \theta \phi, \mu_1 \mu_2 \rangle = \frac{1}{4\pi} e^{i(\mu_1' - \mu_1)\phi} \sum_J (2J+1) d_{\mu_1' - \mu_1}^J(\theta) \langle \mu_1' \mu_2' | T_0 | \mu_1 \mu_2 \rangle$$

Finally, using the alpha notation and the definition of T, unitarity says:

$$\begin{aligned} \langle \alpha_f | T_0 | \alpha_i \rangle - \langle \alpha_i | T_0^\dagger | \alpha_f \rangle &= i \sum_\lambda \int dQ \langle \alpha_f | T^\dagger | \lambda \rangle \langle \lambda | T | \alpha_i \rangle \\ \langle \alpha_f | T_0 | \alpha_i \rangle &= \langle \alpha_f | \alpha_i \rangle + i \delta^4(P-P_i) \langle \alpha_f | T_0 | \alpha_i \rangle \\ &\quad \uparrow \quad \quad \quad \uparrow \\ &\quad \text{note } n_0(2\pi)^4 \text{ and } +i \end{aligned}$$

Charge Conjugation, Field Theory, and M+S page 269

Suppose that, in field theory, we define:

$$\psi_{\bar{n}} = G \psi_n G^{-1} \quad \text{where} \quad \psi_n \sim b_n + d_n^\dagger$$

Here, ψ_n creates a neutron on the left vacuum:

$$\langle 0 | \psi_n \sim \langle n |$$

Or, ψ_n destroys a neutron coming in:

$$\psi_n |n\rangle \sim |0\rangle$$

The question now is: what does $\psi_{\bar{n}}$ do? Well,

$$\langle 0 | \psi_{\bar{n}} = \langle 0 | G \psi_n G^{-1} = \langle 0 | \psi_n G^{-1} = \sim \langle n | G^{-1} = \pm \langle \bar{n} |$$

Thus, depending on the charge phase chosen for $C|n\rangle = \pm |\bar{n}\rangle$, either $(+\psi_{\bar{n}})$ or $(-\psi_{\bar{n}})$ creates an outgoing anti neutron.

In M+S, the symbol $\bar{n}$ is used for $\psi_{\bar{n}}$. Thus, they say: "Adopting our convention [i.e., $C|n\rangle = -|\bar{n}\rangle$], n destroys neutrons, but $-\bar{n}$ destroys antineutrons."

This seems to me to be a matter of name only. If $\psi_{\bar{n}} \equiv G \psi_n G^{-1}$ and $C|n\rangle = -|\bar{n}\rangle$, then you get $\psi_{\bar{n}} = -\psi_n^\dagger$. But you could have instead defined $\psi_{\bar{n}} \equiv -G \psi_n G^{-1}$, then there would be no problem.

Feynman - Photo-hadron Interactions (1972)

Comments on Chapter One of Photon Hadron Interactions
(ie, the first five lectures)

1. In the first lecture Feynman lays the golden egg: postulates that there exists an entity known as the hadronic electromagnetic current. The idea is that in an electron-hadron interaction, the electron itself never gets it on with the hadron directly, but does so only via a photon. Thus, this is an extension of QED principles. The hadronic em current is the "thing" that the photon propagator latches onto, in the Lorentz sense.

2. In the second lecture the first appropriate question is: why should this "hadronic em current" be a conserved current??? I am happy to simply assume this, but F gives some motivations on page 6,7.

3. Next, he defines $V_{\mu\nu}$ as "the thing that two photons can latch onto".



In QED there is no such thing, its just the current interacting twice. If the electron line were to close on itself, then $V_{\mu\nu}$ would be the infamous vacuum polarization tensor, called $I_{\mu\nu}$ in BD.

In the hadron case it is not completely obvious that this tensor can be written as the time ordered product of two currents, and F spends pages 8 through 13 convincing himself that this is in fact the case. The result is given in (3.3), and the nasty fact is that there very well may be "seagulls". Clearly, F hopes this is not so, but cannot prove it is not so.

So much for lecture 2.

4. In the case of Pion vs. Photon, we know that the "vacuum polarization tensor" has both current-current piece, and a seagull piece. So, why the hell should there be no seagull stuff in general with hadronics!!? It is a very peculiar thing that there are no seagulls in QED. F makes a few comments on this, page 13.

5. From gauge invariance, we know that the vac pol tensor is "conserved", ie obeys the gauge condition as usual. This has the direct implication that the commutator of J_0 with the current J_μ is equal to the divergence of those seagulls, see 3.7. If no seagulls, then we have the very nice equal time commutator condition, at least for the em current. Later we will see that in general the et commutator does have some current riding on the right side.

I spend most of my notes on this chapter 3 trying to remember what a seagull was, Now I will never forget.

6. Chapter 4. Here, observing that from the isospin point of view the em current is a mixture of scalar and vector, F generalizes all our previous results to the scalar and multiplet of isovector currents. In particular, we have some fancy equal time com rels (the non-axial part of "current algebra"), plus from these we have various divergence conditions of the "generalized vac pol tensor".

It is an easy step to further generalize beyond isospin's SU(2) to any Lie gauge group.

7. Suppose your gauge group has mass breaking? How does this affect our basic equations (com rels and divergence conditions.)? For one thing, it is easy to show via translation invariance that even the current itself is no longer conserved! (not to mention the vac pol tensor)

So, to the vac pol divergence you must add some kind of extra term which takes account of this mass breakage, see 4.12. The "glimmer" that I feel here is this: in an ultraviolet problem, this second term in the divergence of $V_{\mu\nu}$ will be relatively unimportant, so this suggests that $SU(3)$ can be OK at high energies and short spacetime intervals. Is this why quark models work so well at high energy scattering?

8. Before leaving this 4th lecture, F points out specifically how the function $v(q^2)$ is relevant to the modification of the photon propagator (which is why we are calling it the vac pol tensor in the first place.) If we can get some information on v , then we will know how hadrons contribute to the Lamb shift, etc etc.

This is the subject of lecture 5. There, we find that data on v lies waiting in e^+e^- goes to hadrons at SLAC and PEP.

9. Lecture 5. First we show directly how $p(m^2)$ is connected to total cross section and hence is measurable with relative ease. (Scott Whittaker has spent his last three years acting out this idea). However, it is clear that $p_{\mu\nu}$ has a simple connection to $V_{\mu\nu}$ since both involve a pair of currents. Thus, p connects to v . p is what we can find in the experiment, v is what we want to know!

10. In fact, the relation between v and p takes the form of a little dispersion relation. F is just waiting for data. No wonder he was surprised to find PSI.

11. The close of this lecture has definitions in F's notation of the various propagators and commutators.

Note: I would imagine that all hadronic effects on such things as Lamb shift would be very small because hadron masses are so large, c although pions are on the same level as muons.

Feynman Lecture 1, "General, I"

① Feynman makes his usual plea: "instead of doing hadron-hadron collisions, use photon as a probe." To this I might say: "The photon has no strong interaction, so you will learn nothing about hadrons." But F. hopes that QED is cause of hadrons, namely, the parton model.

② No coupling: these are just the asymptotic states.

③ Here we have the basic concept: (one photon coupling).

hadronic
em
current

$$\text{amp} = \langle f | J_\mu(q) | i \rangle \cdot \frac{4\pi e^2}{q^2} \cdot \langle \text{had}, f | J_\mu(q) | \text{had}, i \rangle$$

† We assume factorization which is implied by this formula.

④ Thus, we are assuming a Lagrangian:

$$\mathcal{L} = e q_\mu J^\mu$$

em hadronic current operator

$$S = e^{-i \int d^4x \mathcal{L}(x)} \approx 1 - i \int d^4x e q_\mu J^\mu$$

Now, let $S = 1 - iT$. Then $iT = i \int d^4x e q_\mu(x) J^\mu(x)$.

Unitarity says:

$$S S^\dagger = 1 \quad (1 - iT)(1 + iT^\dagger) = 1 \quad -iT + iT^\dagger + TT^\dagger = 0$$

$$i(T - T^\dagger) = TT^\dagger \quad T^\dagger - T = iTT^\dagger$$

$$\text{Thus: } \int d^4x e q_\mu^\dagger(x) J^\mu(x) = \int d^4x e q_\mu J^\mu = i$$

Well, he claims that: [unitarity in lowest order $\Rightarrow J_\mu = J_\mu^\dagger$]. Not clear, but I assume it.

- J_μ conserved
- $V_{\mu\nu}$ introduced.

2

Feynman, Lecture 2 "General, II"

① "Why is the hadronic em current conserved?" My feeling is that J_μ conserved $\Rightarrow$ Q conserved, as observed. F. gives some other arg.

② Later, we shall prove that $[J_\mu(x_1), J_\nu(x_2)] = 0$ for $X \neq 0$. ✓

③ Two photons:



$$\text{Now } \int g_\mu J^\mu d^4x \rightarrow ?$$

$$\langle \text{ph}_\mu, \text{had}_f | \text{ph}_\nu, \text{had}_i \rangle = \int d^4x, d^4y \langle \text{ph}_\mu | \text{had}_f | T [a_\mu(x), a_\nu(y)] | \text{had}_i \rangle$$

Well, I think: $\mu \nu$ blob = $V_{\mu\nu}(x_1, x_2)$ = Bose symmetric

Whatever $V_{\mu\nu}$ is, we can probe it by shooting on "independently" polarized photons."

$$\text{blob} = -e \int g_\mu(x) J^\mu(x) d^4x$$

$$\text{blob} = -e \int g_\mu(x) a_\nu(y) V_{\mu\nu}(x, y) d^4x d^4y$$

blob presents its face to a pair of photons.

$$\text{So, } T \equiv 1 - i \int J_\mu a_\mu - \iint V_{\mu\nu} a_\mu a_\nu + \dots$$

④ F. spends the rest of this lecture discussing the relation between $V_{\mu\nu}$ and $J_\mu J_\nu$. First, he proves a weaker theorem:

$$V_{\mu\nu}(1, 2) = J_\mu(1) J_\nu(2) \text{ if } X \neq 0$$

I have not gone thru the proofs in detail. Maybe later.

Then, after much detail, the stronger theorem is shown:

$$[V_{\mu\nu}(1, 2) = J_\mu(1) J_\nu(2) \text{ if } \frac{1}{2} \text{ ie, } t_1 > t_2]$$

The basis of these proofs is a) unitarity b) causality.

conserved current

causality

$V_{\mu\nu}$ as second order blob

the " $V_{\mu\nu}$ Theorem" proved.

Feynman, Lecture 3 "GENERAL, III"

① What is a "seagull"? Let's try some Photon-Photon Field Theory:

$$\mathcal{L} = e A_\mu J^\mu \quad \text{where} \quad J^\mu = \bar{\psi} \gamma^\mu \psi$$

Check out this current: $\bar{\psi}(x) \gamma^\mu \psi(x) = ? J^\mu$

Then: $J^{\mu\dagger} = \bar{\psi}^\dagger(x) \gamma^\mu \psi^\dagger(x) \quad ?$

Well, what is the flux for a KG field? It is:

$$J^\mu = \partial_\mu \phi^\dagger \cdot \phi - \partial_\mu \phi \cdot \phi^\dagger \quad \text{or}$$

② Here is this old problem of "the meaning of $+$ ". First off, I assume that $(AB)^\dagger = B^\dagger A^\dagger$ for field operators. Now, suppose

$$A_\mu = (\partial_\mu B) \cdot C \quad \text{then} \quad A_\mu^\dagger = C^\dagger (\partial_\mu B)^\dagger = C^\dagger \partial_\mu B^\dagger$$

Of course ∂_μ is a c-number operator, so:

So, consider

$$j^\mu = i(\phi^\dagger \partial^\mu \phi - \phi \partial^\mu \phi^\dagger)$$

Then $j^{\mu\dagger} = -i(\partial^\mu \phi^\dagger \cdot \phi - \partial^\mu \phi \cdot \phi^\dagger) = i(\partial^\mu \phi \cdot \phi^\dagger - \partial^\mu \phi^\dagger \cdot \phi)$

Now we could do a "parts" to get $= i(-\phi \partial^\mu \phi^\dagger + \phi^\dagger \partial^\mu \phi)$
 $= i(\phi^\dagger \partial^\mu \phi - \phi \partial^\mu \phi^\dagger)$

So it is only after thinking "parts" that you achieve hermiticity.

③ [What does "parts" mean here?] Usually we have:

$$\int d^4x j^\mu(x) A_\mu(x)$$

But $\int d^4x \partial^\mu f(x) A_\mu(x) = \int d^3x f(x) A_\mu(x) - \int d^4x f(x) \partial^\mu A_\mu(x)$
 $= ?$

I thought this followed from current conservation?

$$\partial_\mu j^\mu = i(\partial_\mu \phi^\dagger \partial^\mu \phi + \phi^\dagger \partial_\mu \partial^\mu \phi - \partial_\mu \phi \partial^\mu \phi^\dagger - \phi \partial_\mu \partial^\mu \phi^\dagger)$$

review of meaning of "seagull"

to show that scalar current is hermitian

Well, suppose we have a wavepacket for a π^+ . X

$$\phi(p) = \int d^4p \delta^4(p - m) \phi(p) |p\rangle = |\phi\rangle \quad c_2 = 1$$

Then $\phi(x) = \int d^4p \delta^4(p - m) e^{-ipx} \phi(p)$

Well: $\langle \phi | \mathcal{J}_\mu(x) | \phi \rangle = ?$

$$|\phi\rangle = \int d^4p \delta^4(p - m) \phi(p) |p\rangle = \int d^4p |p\rangle \cdot \int \frac{d^4x}{(2\pi)^4} \phi(x) e^{ipx}$$

Well, $\langle \phi | \mathcal{J}_\mu | \phi \rangle = \langle \phi | x | \mathcal{J}_\mu | x \rangle \langle x | \phi \rangle = \phi^*(x) \mathcal{J}_\mu \phi(x) \phi(x) ?$

Keep x^μ as a parameter!

$$\begin{aligned} \langle \phi | \mathcal{J}_\mu(x) | \phi \rangle &= \int d^4p \delta^4(p - m) \phi^*(p) \langle p | \mathcal{J}_\mu(x) | \int d^4p' \delta^4(p' - m) \phi(p') | p' \rangle \\ &= \int d^4p \delta^4(p - m) \phi^*(p) \phi(p) \langle p | \mathcal{J}_\mu(x) | p \rangle \end{aligned}$$

Or, with plane waves:

$$\langle p_2 | \mathcal{J}_\mu(x) | p_1 \rangle = \langle p_2 | e^{-i\psi(x)} \mathcal{J}_\mu \psi(x) | p_1 \rangle$$

There are no derivatives. But:

$$\langle p_2 | \mathcal{J}_\mu(x) | p_1 \rangle = \langle p_2 | ie (\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger) | p_1 \rangle$$

$$= \langle 0 | a_+(p_2) ie (\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger) | a_+(p_1) | 0 \rangle$$

$$\phi \sim a_+^\dagger + a_-$$

$$= ie c_1 c_1 \left[e^{+ip_2 \cdot x} \partial_\mu e^{-ip_1 \cdot x} - e^{-ip_1 \cdot x} \partial_\mu e^{+ip_2 \cdot x} \right]$$

Well:

$$[a_+(p_2), \phi^\dagger(x)] = e^{+ip_2 \cdot x} c_1$$

$$= ie c_1 c_1 \left[-ip_{1\mu} e^{i(p_2 - p_1) \cdot x} - ip_{2\mu} e^{i(p_2 - p_1) \cdot x} \right]$$

$$= ie c_1 c_1 (p_1 + p_2)_\mu e^{i(p_2 - p_1) \cdot x}$$

So here is a brand new result:

$$[\langle p_2 | j_\mu(x) | p_1 \rangle = e \cdot C_1 \cdot C_2 \cdot (p_1 + p_2)_\mu \cdot e^{i(p_2 - p_1) \cdot x}] \leftarrow \text{KG fields.}$$

Matrix element
for a
Klein-Gordon
field

and
current

normal order
details

Rule:

→

where $C_2 = X$. Used: $j_\mu = ei(\phi^\dagger \partial_\mu \phi - \phi \partial_\mu \phi^\dagger)$

① If we did parts on both terms, result would be the same.

② How do we show $\partial_\mu j^\mu = 0$? What about normal orders!

$(:AB:) = :BA:)$ of course. [for bose fields]

$$So : \phi^\dagger \partial^\mu \phi : = : \partial^\mu \phi \cdot \phi^\dagger :$$

$$Thus: j^\mu = i(: \phi^\dagger \partial^\mu \phi : - : \phi \partial^\mu \phi^\dagger :)$$

$$What is (:AB:)^\dagger = :B^\dagger A^\dagger: = :A^\dagger B^\dagger:$$

If these are fields, then:

$$\phi = \phi^{(+)} + \phi^{(-)}$$

$$[\phi^{(+)}]^\dagger = \phi^{(-)}?$$

Only for scalar field:

Each: BD define $\phi^{(+)}$ as the $a(k)$ piece!

minus sign if fermion

$$Theorem: : \phi_i(x) \phi_j(y) : = \phi_{i(+)}^{(+)} \phi_{j(+)}^{(+)} + \phi_{i(+)}^{(-)} \phi_{j(+)}^{(+)} + \phi_{i(-)}^{(+)} \phi_{j(-)}^{(+)} + \phi_{i(-)}^{(-)} \phi_{j(-)}^{(-)}$$

$$OK: (:AB:) = :BA: + \text{bose, - fermion.})$$

③ I guess then that the normal ordered K-G current is clearly conserved. But even in QM, NR.

$$j^\mu(x) = i(\phi^\dagger(x) \partial^\mu \phi(x) - \phi(x) \partial^\mu \phi^\dagger(x))$$

$$\partial_\mu j^\mu(x) = i(\partial_\mu \phi^\dagger(x) \partial^\mu \phi(x) - \dots) = 0$$

This works only because $[\phi, \phi^\dagger] = 0$.

④ Now we can get the "rules"



$$-i \int d^4x A_\mu j^\mu \approx S, so$$

$$-ie \epsilon_\mu (p+p')^\mu = M_f$$

③ So back to where we started, what are "seagulls"? In BD they seem to just add $e^2 A^2 \phi^\dagger \phi$ to $\mathcal{L}$ to get them. Ah yes! With no interaction,

$$\mathcal{L}_f = \frac{1}{2} (\partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi)$$

Now to "get em", replace $\partial_\mu \rightarrow \partial_\mu - ie A_\mu$. We pick up

$$\begin{aligned} & (\partial_\mu \phi^\dagger - ie A_\mu \phi^\dagger) (\partial^\mu \phi + ie A^\mu \phi) \\ &= \partial_\mu \phi^\dagger \partial^\mu \phi - ie A_\mu (\phi^\dagger \partial^\mu \phi - \partial^\mu \phi^\dagger \phi) + e^2 A_\mu A^\mu \phi^\dagger \phi \end{aligned}$$

Repeat this:

$$\partial_\mu \phi^\dagger \partial^\mu \phi$$

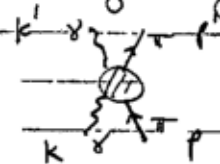
$$\boxed{\partial_\mu \rightarrow \partial_\mu + ie A_\mu}$$

$$\rightarrow (\partial_\mu$$

$$\begin{aligned} i\partial \rightarrow i\partial - eA & \Rightarrow i\partial_\mu \rightarrow i\partial_\mu - eA_\mu \\ & \Rightarrow \partial_\mu \rightarrow \partial_\mu + ieA_\mu \end{aligned}$$

$$\text{So: } \mathcal{L}_f \rightarrow \mathcal{L}_f - ie A_\mu [\phi^\dagger \partial^\mu \phi - \partial^\mu \phi^\dagger \phi] + e^2 A_\mu A^\mu \phi^\dagger \phi$$

④ Seagull Feynman rule:



$$= \langle k', p' | e^2 A_\mu A^\mu \phi^\dagger \phi | k, p \rangle$$

↑ 2 ways to contract photons.

$$= g_{\mu\nu} e^2 \epsilon^\mu \epsilon^\nu \cdot 2$$

this is a seagull

Why the i? Well, $S = 1 - i \int d^4x A_\mu A^\mu \phi^\dagger \phi$ (to $\pm$ sign).

⑤ Back to Feynman. To add seagulls there only because we have not investigated the $t_1 = t_2$ case. There could be terms involving $\delta^4(t_1 - t_2)$ or its gradients.

This means you may have to include seagulls at your "local" level, but second order.

$$\mathcal{L} = \mathcal{L}_f(i) a^\dagger(i) + \mathcal{L}_{\text{seagull}}(i) a^\dagger(i) a(i)$$

↑ seagullness.

⑩ Mention of "Schwinger terms".

⑪ Gauge: we know that $q_\mu \rightarrow q_\mu + \partial_\mu \chi$ cannot change physics. This is the classical reason why you can choose an $q_\mu \rightarrow \partial_\mu a^\mu = 0$. Then this $\Rightarrow \partial_\mu T^\mu = 0$ by parts. Argument goes either way.

$$(\partial_\mu T^\mu = 0) \Rightarrow (\partial_\mu a^\mu = 0)$$

Well, $A_\mu \rightarrow A_\mu + \partial_\mu \chi$ = an invariance because $F_{\mu\nu}$ only is observable + $F_{\mu\nu} \rightarrow F_{\mu\nu}$ under gauge.

⑫ For $T \rightarrow T$ under $q_\mu \rightarrow q_\mu + \partial_\mu \chi$, we need:

$$\partial_\mu T^\mu = 0 \quad \text{and} \quad [\partial_\mu T^\mu, V^{\mu\nu}(1,2)] = 0 \leftarrow \text{new!}$$

From this second condition we can show:

$$[T_0(1), T_0(2)]_{\text{et}} = - \partial_\mu \overset{\text{seagulls}}{\tilde{S}^\mu} \delta^3(\vec{x}_1 - \vec{x}_2)$$

Hopefully, "no seagulls" ~ then $[T_0(1), T_0(2)]_{\text{et}} = 0$. Notice that this is a part of "extended current algebra hypothesis", see Adler Dashen.

This seems to be an uncertain matter. F. presents the whole business as "a research problem. Also related to Schwinger terms."

- isospin + weak. 4
- general gauge group
- mass breakage and $\delta g^{\mu\nu}$
- photon prop. mod. by V .

Feynman Lecture 4: "GENERAL, II"

① From the N. rule we know that J_μ is not pure isoscalar or isovector. If it were isoscalar, then all members of an isoplet would have the same charge, $Q \sim Y$. If isovector, then p and n would have opposite charge, $Q \sim I_3$. Everyone knows that $J = J^S + J^V$.

② Now assume existence of J^V and J^S . Generalize all rules such as:

$$[J_\mu^a(1), J_\nu^b(2)] = 0 \quad \text{X} \quad a = \text{vector index or scalar}$$

$$T = -i \int J_\mu^a(1) q_\mu^a(1) d\tau_1 - \frac{1}{2} \iint V_{\mu\nu}^{ab}(1,2) T(q_\mu^a(1), q_\nu^b(2)) d\tau_1 d\tau_2 + \dots$$

Thus, we have 4 "photons", somehow.

③ Some current algebra: We know that $J^V \sim I_+$, which raises I_3 , so which raises Q . I.e., $[Q, I_+] |I, m\rangle = Q I_+ |I, m\rangle - I_+ Q |I, m\rangle = (Q \cdot (I, m+1) - I_+ q |I, m\rangle) = (q+1) c |I, m+1\rangle - q \cdot c |I, m+1\rangle = c |I, m+1\rangle = I_+ |I, m\rangle$. So:

$$[Q, I_+] = I_+ \rightarrow [Q, J_\mu^V] = J_\mu^V$$

$$[Q, J_\mu^V(1)] = J_\mu^V(1) = \int d^3x_2 \{ [J_0^V(2), J_\mu^V(1)] + [J_0^S(2), J_\mu^V(1)] \}$$

Now use $Q = \int d^3x_2 (J_0^V(2) + J_0^S(2))$. Then set $t_2 = t_1$. Then

$$[J_0^a(2), J_\mu^V(1)]_{et} = \delta(x_1 - x_2) \delta^a_V + \text{deriv.} \quad [\text{Schwingers}]$$

I guess $[Y, I] = 0$, so drop that term. Conclusion: if no Schwinger terms, then:

$$[[J_0^V(2), J_\mu^V(1)]_{et} = \delta^3(x_1 - x_2) J_\mu^V(1)] \leftarrow \text{typical current algebra statement.}$$

④ Consider $V_{\mu\nu}$ again:

$$V_{\mu\nu}^{ab}(1,2) = T(J_\mu^a(1) J_\nu^b(2)) + \text{seagulls}_{\mu\nu}$$

$$\begin{aligned} [J_0^a(1) V_{\mu\nu}^{ab}(1,2)] &= 0 + \delta(t_1 - t_2) [J_0^a(1) J_\nu^b(2)] + \text{sea} \\ &= \delta^4(1-2) J_\nu^b \end{aligned}$$

← current algebra used here.

EM current is + isoscalar + isovector 3

assume other vector components exist

extend all commutator + divergence results.

⑤ General gauge group: $T(a^\mu) = T(a^\mu + \partial^\mu \chi + \chi \times a^\mu)$ i.e., there are n photons, n gauge functions, etc.

$$T(a) \Rightarrow T(a + \partial \chi + \chi \times a) \quad [\text{functional}]$$

$$T[a + \partial \chi + \chi \times a] = T[a] + \frac{\delta T}{\delta a} \times (\partial \chi + \chi \times a)$$

Invariance means $\int \frac{\delta T}{\delta a} (\partial \chi + \chi \times a) = 0$.

Example: $T[a] = -i \int \bar{\psi}_\mu(1) q_\mu(1) d\tau_1 + \frac{1}{2} \int \int V_{\mu\nu}(1,2) \{q_\mu(1) q_\nu(2)\} \dots$

$$\frac{\delta T}{\delta a(1)} = -i \bar{\psi}_\mu(1) + \frac{1}{2} \int d\tau_2 V_{\mu\nu}(1,2) q_\nu(2)$$

OK, no big thing. Results will be:

$$\left[\begin{aligned} \partial_\mu V_{\nu\lambda}^{ab}(1,2) &= \delta(1-2) f_{ab}^c \bar{\psi}_\nu(1) \\ \partial_\mu \bar{\psi}_\mu^a &= 0 \end{aligned} \right]$$

results for any gauge group.

⑥ We have started with group Q , and have extended to $\mathbb{R} \times Q$. Can we go to $SU(3)$? Mass breakage $\Rightarrow \partial_\mu \bar{\psi}_\mu^a$ not conserved. Why?

$$\langle 1 | \partial_\mu \bar{\psi}_\mu^a | N \rangle = \langle 1 | \partial_\mu \bar{\psi}_\mu^a | N \rangle = \langle 1 | \partial_\mu \bar{\psi}_\mu^a | N \rangle$$

Stop. $\langle 1 | A(x) | N \rangle = \int \frac{d^4 q}{(2\pi)^4} e^{i q \cdot x} \langle 1 | A(q) | N \rangle$

So: $\langle 1 | \partial_\mu \bar{\psi}_\mu^a(x) | N \rangle = \int \frac{d^4 q}{(2\pi)^4} e^{i q \cdot x} \langle 1 | \partial_\mu \bar{\psi}_\mu^a(q) | N \rangle$

Stop: $\langle 1 | A(x) | N \rangle = e^{i(p_1 - p_2) \cdot x} \langle 1 | A(0) | N \rangle$

So $\langle 1 | \partial_\mu \bar{\psi}_\mu^a(x) | N \rangle = i(p_1 - p_2)_\mu \langle 1 | \bar{\psi}_\mu^a(0) | N \rangle$

Special case. $N \rightarrow \Delta$ "at rest." $\langle 1 | \bar{\psi}_\mu^a(0) | N \rangle = i(m_N - m_\Delta) \langle 1 | \bar{\psi}_\mu^a(0) | N \rangle \neq 0!$

This means maybe:

$$\partial_\mu V_{\nu\lambda}^{ab} = \delta_{\mu\nu} \bar{\psi}_\lambda^b + \{ \partial_\mu \bar{\psi}_\nu^a, \bar{\psi}_\lambda^b \}_T$$

↑
new piece. ✓

extend to general gauge group.

for $SU(3)$, mass breakage $\Rightarrow \partial_\mu \bar{\psi}_\mu^a \neq 0$

The point here is that this second piece could still be smooth at $l=2$, hence negligible there. So at least we have a glimmer of "no SL₂ breakage for $l=2$ = small." ✓

② How do you F.T. $V_{\mu\nu}(x_1, x_2)$?

$$V_{\mu\nu}(q_1, q_2) \equiv \int d^4x_1 d^4x_2 e^{+iq_1 \cdot x_1} e^{+iq_2 \cdot x_2} V_{\mu\nu}(x_1, x_2).$$

Now, shove on a vacuum on each side:

$$\langle 0 | V_{\mu\nu}(x_1, x_2) | 0 \rangle = 0. \quad \langle 0 | V_{\mu\nu}(0, x_2 - x_1) | 0 \rangle$$

So: F.T. of $V_{\mu\nu}(0, x_2 - x_1)$ is:

$$\langle 0 | V_{\mu\nu}(q_1, q_2) | 0 \rangle = \int d^4x_1 d^4x_2 e^{iq_1 \cdot x_1} e^{iq_2 \cdot x_2} \langle 0 | V_{\mu\nu}(0, x_2 - x_1) | 0 \rangle.$$

Let $x^\pm = x_2 \pm x_1$. Then $d^4x_1 d^4x_2 = ?$ $\frac{x^+ + x^-}{2} = x_2$

$$dx_1 dx_2 = \frac{\partial(x_1, x_2)}{\partial(x^+, x^-)} dx^+ dx^- = \begin{vmatrix} \partial x_1 / \partial x^+ & \partial x_1 / \partial x^- \\ \partial x_2 / \partial x^+ & \partial x_2 / \partial x^- \end{vmatrix} dx^+ dx^- \quad \frac{x^+ - x^-}{2} = x_1$$

$$J = 1 = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = \frac{1}{2}. \quad \text{Perhaps compensated.}$$

Well:

$$\langle 0 | V_{\mu\nu}(q_1, q_2) | 0 \rangle = \int d^4x^+ d^4x^- e^{iq_1 \cdot x^+} e^{iq_2 \cdot x^-} \langle 0 | V_{\mu\nu}(0, x^-) | 0 \rangle.$$

$$\text{Well: } \int d^4x^+ e^{iq_1 \cdot x^+/2} e^{iq_2 \cdot x^+/2} = \int d^4x^+ e^{ix^+ \cdot (q_1 + q_2)}$$

$$= (2\pi)^4 \delta^4(q_1 + q_2). \quad \text{Let with } e^{iq_1 \cdot (-x^+/2)} e^{iq_2 \cdot (+x^+/2)} = e^{ix^+ \cdot (q_2 - q_1)} = e^{ix^+ \cdot q_2}$$

So:

$$\langle 0 | V_{\mu\nu}(q_1, q_2) | 0 \rangle = \int d^4x^- e^{ix^- \cdot q_2} \langle 0 | V_{\mu\nu}(0, x^-) | 0 \rangle \cdot (2\pi)^4 \delta^4(q_1 + q_2)$$

$$\text{Thus: } \langle 0 | V_{\mu\nu}(q_1, q_2) | 0 \rangle = (2\pi)^4 \delta^4(q_1 + q_2) \cdot c \cdot \int d^4x^- e^{iq_2 \cdot x^-} \langle 0 | V_{\mu\nu}(0, x^-) | 0 \rangle. \\ = 0 \text{ unless } q_1 = -q_2.$$

③ Also: $\left[\text{diagram of a loop with two external lines} \right] = V_{\mu\nu}(-q, q)$

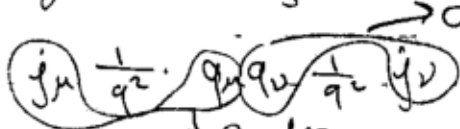
So:  = $\langle 0 | V_{\mu\nu}(-q, q) | 0 \rangle = \dots$
 $= -q_\mu q_\nu V(q^2) + q_{\mu\nu} b(q^2)$ by L.I.

But, $q^\mu V_{\mu\nu} = 0 \Rightarrow 0 = -q^2 V(q^2) q_\nu + q_\nu b(q^2)$

$\Rightarrow b = q^2 V$, so $= (g_{\mu\nu} q^2 - q_\mu q_\nu) V(q^2)$

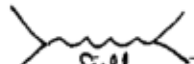
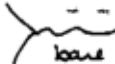
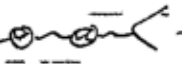
⑨ So:  = $\langle 0 | V_{\mu\nu}(-q, q) | 0 \rangle = (\delta_{\mu\nu} q^2 - q_\mu q_\nu) V(q^2)$

In application, second term gives nothing:

 = 
 $\downarrow 0 \text{ also}$

So effectively,  = $g_{\mu\nu} q^2 V(q^2)$

Then:

 =  +  +  + ...
 $= \frac{1}{q^2 (1 - 4\pi e^2 V(q^2))}$

So "hadronic" renormalizes the charge at a vertex, not the photon mass: (just as vac. pol. in QED)

$e_e^2 = \frac{e_0^2}{1 - 4\pi e_0^2 V(q^2)}$

But "charge" is pole residue, so set $q^2 = 0$. Nevertheless, the full photon propagator should be:

$V(q^2) \approx a + bq^2 + \dots$

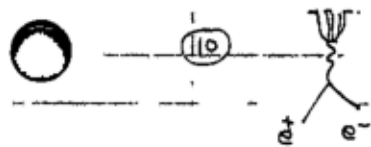
prop = $\frac{1}{q^2 - 4\pi e^2 a b q^4} = \frac{1}{q^2 (1 - 4\pi e^2 a b q^2)}$

= $\frac{1}{q^2 (1 - q^2/\Lambda^2)} = \frac{1}{q^2} \cdot (1 - q^2/\Lambda^2)^{-1}$

$\approx \frac{1}{q^2} [1 + q^2/\Lambda^2] \approx \frac{1}{q^2} + \frac{1}{\Lambda^2}$
 $\sim$ Veltman correction, Λ large. "vacuum pole. correction"

gauge condition
 $\Rightarrow$ general
 from on

hadronic
 renormalize
 charge
 and
 propagator.



$$\text{disc } \textcircled{0} = \text{Im } \textcircled{0} \sim \text{Im } V.$$

Imaginary part of $\textcircled{0}$

It would seem that

$$\left| \begin{array}{c} \text{photon} \\ \swarrow \searrow \\ e^+ e^- \end{array} \right|^2 = \left(\begin{array}{c} \text{photon} \\ \swarrow \searrow \\ e^+ e^- \end{array} \right) \propto \text{Im } V.$$

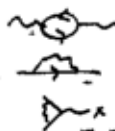
So $\text{Im } V \neq 0 \Rightarrow$ photon can decay into hadrons.

$$\text{decay rate} \sim \frac{1}{\text{lifetime}} \sim \text{Im } V.$$

Of course, if $q^2 = 0$, you are kinematically blocked, as usual.

⑪ So, "polarization of the vacuum" is an observable. The fact that a photon can "split apart" an e^+e^- pair from the vacuum changes something somewhere.

NOTE: at this point I went to BD Vol. 1 and read completely ch. 8. Yes, the vac. pol. really does cause observable effects; so do the other radiative corrections:



- vac. pol $\rightarrow$ Uehling $\sim 2\%$
- elect. self-energy + vertex $\rightarrow$ Lamb.
- vertex $\rightarrow$ anomalous moment.

elec.

Feynman Lecture 5: "GENERAL, II"

① Consider $e^+e^- \rightarrow \text{hadrons}$. Claim:

the em.
hadronic
current

$$\begin{array}{c} \text{em. current} \\ \text{hadronic} \\ \text{current} \end{array} = \bar{u}(1) [-ie\gamma_{\mu}] u(1) \cdot \left[\frac{-i}{q^2} \right] \cdot \langle \text{hadron set} | J^{\mu} | 0 \rangle$$

So we are saying: $J^{\mu} = \langle \text{had} | J^{\mu}(q) | 0 \rangle = \text{current to create hadrons from vac.}$

Well clearly $\sigma_{\text{tot}}(l \rightarrow \text{had}) \propto \sum_n (\text{phase space}) |\langle \text{had} | J^{\mu}(q) | 0 \rangle|^2$

Each possible final state should contribute positively, because you could do each experiment, + get a $\sigma > 0$. So

$$\sigma_{\text{tot}}(l \rightarrow \text{had}) = \sum_m |\langle m | J^{\mu}(q) | 0 \rangle|^2 = \sum_m \langle 0 | J^{\mu}(-q) J^{\nu}(q) | 0 \rangle$$

Of course this lacks onto some trace ρ . So: "you can actually measure this object !!!" say, at SLAC

② So, define:

the em.
hadronic
current
tensor
 $P_{\mu\nu}$

$$[\langle 0 | J_{\mu}(-q) J_{\nu}(q) | 0 \rangle = P_{\mu\nu}(q) = (q_{\mu}q_{\nu} - q^2 g_{\mu\nu}) \Theta(q_0) p(q^2)]$$

If you are below threshold, $\sigma_{\text{tot}} = 0$, so all pieces must be 0; $\sigma_{\text{tot}} = 0 \Rightarrow P_{\mu\nu}(q) = 0$, that is why $\Theta(q_0)$, a $\Theta(q^2 - s_0)$ say.

③ After doing trace etc, you can show that $p(q^2) \geq 0$. (yes, see p.4) $\rightarrow$

④ Note: $\Theta(q_0) - \Theta(-q_0) = \text{sgn}(q_0) = q_0/|q_0|$, thus:

$$\langle 0 | [J_{\nu}(1), J_{\mu}(2)] | 0 \rangle \stackrel{\text{F.T.}}{=} (q_{\mu}q_{\nu} - g_{\mu\nu}q^2) \text{sgn}(q_0) p(q^2)$$

Do this F.T.:

$$\begin{aligned} \langle 0 | [J_{\nu}(x_1), J_{\mu}(x_2)] | 0 \rangle &= f(x_1 - x_2) \text{ from translation} \\ &= \int \frac{d^4 q}{(2\pi)^4} e^{-iq \cdot (x_1 - x_2)} \langle 0 | [J_{\nu}(-q), J_{\mu}(q)] | 0 \rangle \end{aligned}$$

AND
 $p(q^2)$

Now use $\langle 0 | [\bar{\psi}_\nu(-q), \bar{\psi}_\mu(q)] | 0 \rangle = (q_\nu q_\mu - g_{\mu\nu} q^2) \text{sgn}(q_0) p(q^2)$

$$= \int \frac{d^4 q}{(2\pi)^4} (q_\nu q_\mu - g_{\mu\nu} q^2) \text{sgn}(q_0) \int \frac{dm^2}{(2\pi)^2} \delta(q^2 - m^2) p(m^2) e^{-iq \cdot (x_1 - x_2)}$$

Now replace $q_\mu = i\partial_\mu^{(1)}$ $q_\mu q^\mu = q^2 = +i\partial_\mu^{(1)} i\partial^{\mu(1)}$

Then $(q_\nu q_\mu - g_{\mu\nu} q^2) \rightarrow -(\partial_\nu \partial_\mu - g_{\mu\nu} \square)$, so

$$= (-\partial_\nu^{(1)} \partial_\mu^{(1)} - g_{\mu\nu} \square^{(1)}) \int \frac{dm^2}{(2\pi)^2} p(m^2) \cdot \int \frac{d^4 q}{(2\pi)^4} \text{sgn}(q_0) \delta(q^2 - m^2) e^{-iq \cdot x}$$

⑤ What is this integral?

$$\int \frac{d^4 q}{(2\pi)^4} \text{sgn}(q_0) \delta(q^2 - m^2) e^{-iq \cdot (x_1 - x_2)}$$

Well, $e^{-iq \cdot (x_1 - x_2)} \text{sgn}(q_0) = e^{-iq \cdot (x_1 - x_2)} \text{sgn}(t_1 - t_2)$?

$$\text{Check this: } \int_{-\infty}^{\infty} dq_0 [q \cdot x] \text{sgn}(q_0) = \int_0^{\infty} dq_0 (q \cdot x) - \int_{-\infty}^0 dq_0 (q \cdot x)$$

So we really have:

$$= - \left[\int_0^{\infty} \frac{d^4 q}{(2\pi)^4} \delta(q^2 - m^2) e^{-iq \cdot (x_1 - x_2)} + \int_0^{\infty} \frac{d^4 q}{(2\pi)^4} \delta(q^2 - m^2) e^{-iq \cdot (x_1 - x_2)} \right]$$

$$= - \left[\int_0^{\infty} d^4 \dots \right] = - \left[\int_0^{\infty} \frac{d^4 q}{(2\pi)^4} \delta(q^2 - m^2) \left(e^{-iq \cdot (x_1 - x_2)} - e^{+iq \cdot (x_1 - x_2)} \right) \right]$$

↑ hits only at $q_0 = + \dots$

$$= - \left[\int_0^{\infty} \frac{d^3 q}{(2\pi)^3 2E} (e^{-i\tau} - e^{+i\tau}) \cdot \frac{1}{2\pi} \right] = - \frac{1}{2\pi} i \Delta(x_1, x_2)$$

So, Feynman seems to be using:

$$\bar{C}^m(1,2) = -\frac{i}{2\pi} \Delta(1,2; m)$$

$$p(q^2) = \int dm^2 \delta(q^2 - m^2) p(m^2)$$

special analysis?
no, same p.

⑥ Conclusion:

$$\langle 0 | [\bar{\psi}_\nu(x_1), \bar{\psi}_\mu(x_2)] | 0 \rangle = (\bar{\nabla}_\mu \bar{\nabla}_\nu - \bar{\nabla}^2 g_{\mu\nu}) \int \frac{dm^2}{(2\pi)^2} p(m^2) \cdot \left(-\frac{i}{2\pi} \right) \Delta(x_1 - x_2; m^2)$$

↑ One positive sign.

① Next, we want to compute $\langle 0 | \{ \bar{\psi}_\mu(1) \bar{\psi}_\nu(2) \} | 0 \rangle$ in terms of this same function $p(q^2)$ which characterizes $\langle 0 | \bar{\psi}_\mu(-q) \bar{\psi}_\nu(q) | 0 \rangle$. Here, aside from the seagulls which may appear in (4.3), we are computing $\langle 0 | \bar{\psi}_{\mu\nu}(q) | 0 \rangle$ where this means F.T. $\langle 0 | \bar{\psi}_{\mu\nu}(-q, q) | 0 \rangle$.

relation
between
 $V(q^2)$
and
 $p(q^2)$

But $\bar{\psi}_{\mu\nu}$ is characterized by $V(q^2)$, so we shall obtain a relation between p and V .

$$V(q^2) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \frac{p(m^2) dm^2}{q^2 - m^2 + i\epsilon} - i(\text{Seagull})$$

Of course, if S is real as claimed, then $S + \frac{i}{2}p = \text{Im } V$

$$\text{Im } V = -S + \frac{i}{2}p = \dots \text{ Well:}$$

$$iV(q^2) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{p(m^2) dm^2}{q^2 - m^2 + i\epsilon} + S \Rightarrow \text{Im}(iV) = -\frac{1}{2} p$$

② Conclusion: from this disp. for $V(q^2)$, we can get $V(q^2)$ everywhere from $\text{Im}(iV(q^2))$ above threshold which we can get experimentally; i.e., at least, we just got done showing that $p(q^2)$ is obtainable from $\sigma_{\text{tot}}(e^+e^- \rightarrow \text{hadrons})$.

Specifically: [am. now computing $e^+e^- \rightarrow \text{had}$ via $\bar{\psi}^{\mu}$.]

compute
 $\sigma(s)$
for e^+e^-
into
hadrons

$$\text{amp} = (\bar{v}_2 \gamma_\mu u_1) \cdot \frac{-i}{q^2} \cdot e^2 \cdot \langle f | \bar{\psi}^\mu(q) | 0 \rangle \checkmark$$

$$|\text{amp}|^2 = \frac{e^4}{q^4} \cdot \frac{1}{4} \text{trace}([p_1 - m] \gamma_\mu [p_2 + m] \gamma_\nu) \cdot \langle 0 | \bar{\psi}^\mu(-q) | f \rangle \langle f | \bar{\psi}^\nu(q) | 0 \rangle \checkmark$$

average initial.

Then:

$$\text{trace} \approx \text{tr}(\not{p}_1 \not{p}_2) = 4[p_1^\mu p_{2\mu}] = 4[p_1^\mu p_{2\mu} + p_{3\mu} p_{4\mu} - g^{\mu\nu} p_1^\mu p_{2\nu}] \checkmark$$

$$|\langle f | T | i \rangle|^2 = \frac{(4\pi\alpha)^2}{s^2} \cdot [2p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1^\mu p_{2\nu}] \cdot \langle 0 | \bar{\psi}^\mu(-q) | f \rangle \langle f | \bar{\psi}^\nu(q) | 0 \rangle \checkmark$$

Now,

$$\sigma_{\text{tot}} = \sum_n \int dQ_n \frac{|T_n|^2}{4\sqrt{s} p^*} = \frac{(4\pi\alpha)^2}{s^2 \cdot 4\sqrt{s} \cdot \sqrt{s}/2} [p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1^\mu p_{2\nu}] \cdot \langle 0 | \bar{\psi}^\mu(-q) | f \rangle \langle f | \bar{\psi}^\nu(q) | 0 \rangle$$

← completeness used.

Now:

$$[p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1^\mu p_{2\nu}] [q_\mu q_\nu - q^2 g_{\mu\nu}] p(s) \checkmark$$

$$= p(s) \cdot [2p_1^\mu p_2^\nu - g^{\mu\nu} p_1^\mu p_{2\nu}] [q_\mu q_\nu - q^2 g_{\mu\nu}] \checkmark$$

Four terms First ~ well, write small:

$$2(p_1 \cdot q)(p_2 \cdot q) - 2q^2(p_1 \cdot p_2) - q^2(p_1 \cdot p_2) + q^2 \cdot 4(p_1 \cdot p_2)$$

$$= 4s_A - 3s \cdot (p_1 \cdot p_2) + 2(p_1 \cdot q)(p_2 \cdot q)$$

But, $s \approx 2p_1 \cdot p_2$ ✓ $p_1 \cdot q = p_1 \cdot p_1 + p_1 \cdot p_2 \approx p_1 \cdot p_2 \approx s/2$ ✓
 $p_2 \cdot q \approx p_2 \cdot p_1 \approx s/2$ ✓

$$S_0 = 4s_A - 3s \cdot \frac{s}{2} + 2 \cdot \frac{s}{2} \cdot \frac{s}{2} = 2s^2 - \frac{3}{2}s^2 + \frac{1}{2}s^2 = s^2 \left(\frac{4-3+1}{2} \right) = s^2$$

⑨ So, $\sigma_{tot} = \frac{(4\pi\alpha)^2}{2s^2} \cdot p(s) \cdot s^2 = \frac{16\pi^2\alpha^2 p(s)}{2s}$

Result: $\Rightarrow \sigma_{tot}(e^+e^- \rightarrow had; s) = 8\pi^2\alpha^2 p(s) \cdot s^{-1} = \boxed{(4\pi\alpha)^2 p(s) / 2s}$

Now, $p(s) = \frac{2s\sigma(s)}{(4\pi\alpha)^2}$, so

$$V(q^2) = \frac{i}{2\pi} \cdot \frac{1}{(4\pi\alpha)^2} \cdot 2 \cdot \int \frac{s\sigma(s)ds}{q^2 - s + i\epsilon} = iS(q^2)$$

$$V(0) = \frac{i}{2\pi} \cdot \frac{1}{(4\pi\alpha)^2} \cdot 2 \cdot \int \frac{s\sigma(s)ds}{-s + i\epsilon} = iS(0)$$

$$\Rightarrow V(q^2) - V(0) = \frac{i}{2\pi} \cdot \frac{1}{(4\pi\alpha)^2} \cdot 2 \cdot \int \sigma(s)ds \left[\frac{s}{(q^2 - s) + i\epsilon} + \frac{s}{(+s)} \right]$$

$$= i[S(q^2) - S(0)]$$

$$\frac{\pm s + sq^2 - s^2}{+s(q^2 - s)} = \frac{q^2}{q^2 - s}$$

subtracted
dispersion
relation

$$\Rightarrow i[V(q^2) - V(0)] = \frac{i^2}{\pi} \cdot \frac{1}{(4\pi\alpha)^2} \cdot q^2 \int \frac{\sigma(s)ds}{q^2 - s + i\epsilon} = i^2[S(q^2) - S(0)]$$

on
 $V(q^2)$

$$[i(4\pi\alpha) [V(q^2) - V(0)]] = \frac{q^2}{\pi} \cdot \frac{1}{(4\pi\alpha)^2} \cdot \int \frac{\sigma(s)ds}{-s - q^2} + (S(q^2) - S(0))$$

⑩ We know that $\frac{dV(q^2)}{dq^2}$ will be relevant for "hadrons vac pol" etc ie, in general for the photon propagator correction.

So: $4\pi\alpha i V'(0) = \frac{1}{4\pi^2\alpha} \int_{-\infty}^{\infty} \frac{\sigma(s)ds}{s}$

Can get this from data...?

(11) Consider:



Claim: $p(s) = \frac{\text{const}}{(m^2 - m_\phi^2)^2 + \Gamma^2/4}$

$$m^2 = s$$

example:

I could show this specifically by looking up my "resonance formula for $\alpha(s)$ ", then converting to $p(s)$ via:

$$\sigma(s) = (4\pi\alpha)^2 p(s) / 2s.$$

Of course $|\text{amp}|^2$ will look just like this ~ no question on that.

(12) So, dump this $p(s)$ into (5.14) and compute $v(q^2) - v(0)$!
Turns out that

$$i v(q^2) = \frac{\text{const}/\Gamma}{(q^2 - m_\phi^2) - i\Gamma/2}$$

So, I presume you compute from this the vac. pol. correction due to the ϕ -hadron state. Why doesn't he do this? Well, this is the vac. polarization, dummy. The corrected propagator is:

$$\frac{1}{q^2} \frac{1}{[1 - 4\pi\alpha i v(q^2)]} = \frac{1}{q^2} \frac{1}{[1 - \frac{4\pi\alpha \cdot \text{const}/\Gamma}{(q^2 - m_\phi^2) - i\Gamma/2}]}$$

(13) In order to derive our little vac. pol. disp. rel., we used the $\mu\nu = tt$ version of the equation (5.8) which relates $V_{\mu\nu}$ to $\{T_\mu, T_\nu\}$. When you look at an xx case, there appears a divergent $\int g(m^2) dm^2 = C$, see (5.23). For tt this happens not to appear.

Also, (5.23) does not seem to be covariant! (Schwinger term problem).

(14) F. closes out this chapter with some evaluations of various "commutator + propagator functions". I will look at this later when BP is at hand.

propagator from data.

Lecture 6: Low Energy Photon Reactions

1. The basic topic here is the photoproduction of pions off nucleons, in particular at resonance energies ("low energy"). The main object of theoretical interest will be the "current" of: nucleon turns into resonance. That is why we use photoproduction. Thus, we are interested in $(res/J^u/nucleon)$. The idea that such an "amplitude" exists is argued through factorization (sort of a macro-causality). The idea of this lecture and number 7 is to briefly go over the procedure whereby Walker obtained numbers for these current matrix elements from the experiments. Then in lectures on quarks F will make some theoretical predictions for these matrix elements, then we can compare. The Breir form is briefly reviewed.

Lecture 7: more on Walkers work.

1. Briefly, the notions of "background" of a resonance, and the duality-Veneziano idea are presented, along with the threshold behavior form for a resonance width. (Check this later).
2. Then we get right to Walkers stuff. He used Resonances+Background+Pion exchange as his model for what is going on in photoproduction. Resonance is clear, the background was some real, smooth function formed by getting best fit to resonances.

Lecture 8: more on Walkers work.

1. Here, the pion exchange pole is used to account for diffraction which I guess is present a little at low energy (especially at small t). I.e., this shows up in higher partial waves only. Feynman takes the opportunity to show that when you use such a pion pole OPE thing, you have to add certain other terms in order to achieve a gauge invariant interaction.

Aside: gauge invariance means $A^u = A^u + d^u f$ should make no difference. Thus, since $L = A_u J^u$, we must have $d_u J^u = 0$. If q = photon momentum, then in momentum space, this says: $q_u J^u = 0$. Thus, in the OPE term, you replace A^u (or e^u of free photon) with q^u and test to see whether function vanishes. This tests gauge invariance. In general, when a charged particle is produced, you will need "other terms" to get gauge invariance.

2. F makes some remarks on the insignificance of magnetic moment terms in the gauge correction terms M_{II} and M_{III} . The point is that they are smooth and can be absorbed into the "background" constant. Thus, Walker speaks of "electric Born term" with no magnetic moment stuff present.

Lecture 10: "work of Feynman, Kislinger, Ravndal" Up to now F has been discussing some kind of non-rel three-quark harmonic oscillator model for baryons. In this section they try to make things a little more relativistic. The momenta and coordinates appearing in the new hamiltonian are all four-vectors. The only constant appearing in the hamiltonian is Ω , the spacing of resonances, set at 1 GeV. Now, once you have this hamiltonian for the 3-quark system, you can add photons as using via the minimal coupling. This yields an electric current J_μ as shown in 10.5. Then you go ahead as in the original non-rel model and calculate the various couplings ($\text{res}/J_\mu/\text{initial}$) by putting in the appropriate quarks for each baryon (remember, we are here talking about a piece of the photoproduction matrix element). The results of this relativistic modification are given in the table on page 46 along with the non-rel results and the experimental results.

By looking at diagonal matrix elements ($\text{proton}/J_\mu/\text{proton}$) as function of q , you are talking about electromagnetic form factors. A modification of F's work by someone else (FKN) shows that by playing with the bad neg norm states etc, you can generate reasonable exponentially decaying em form factors as desired. Certain modifications and unanswered questions are mentioned.

Lecture 11: "photoproduction of mesons" We are at high energy above the resonances and want to know what happens in the titled exclusive channel (which is only a small piece of the total cross section of γp .) Some experimental results are mentioned: the DCB seems to be roughly independent of s and have exponential t -dependence at small t . Actual data is shown on page 70. Rule is e^{3t} at $t = .8 \text{ GeV}$. Then F goes into some comments about regge theory and helicity flip which I do not understand. Related to rules of Jackson and Hite I think.

Lecture 12: "more on photoproduction" Somehow by using selection ~~xxx~~ rules, F can state that "there must be helicity flip at this vertex" and so on. This enables him to claim that the amplitude (one pion exchange) must look like $t^a/(t-m^2)$, where a is some exponent you figure out from which helicities flip etc. If $a = \frac{1}{2}$ or 1 then you expect to see a dip in the forward direction in the DCS. As page 70 figure shows, you do get this dip in some reactions, but not in others. For example, if you make a Δ^{++} , you predict a dip and you get a dip. But conversion of p to n or vice versa shows a forward spike, though you would predict a dip.

How do we justify this spike? F says: absorption. Ie, you go to impact parameter space and state your OPE amplitude $a(b)$ which happens to be a curve which indicates nothing in the forward direction (ie, area under this $a(b) = 0$). Then you modify this $a(b)$ by multiplication by an "absorption function" which causes the area not to be zero and then gives you something at $t=0$. Hence, the spike.

~~(more lecture 12).~~ Several other reactions are discussed in this very detailed section. The difference between p to n and n to p is given by ρ -exchange in regge theory, and you get the subsequent dip when $d \approx 0$, ie, at $t \approx -0.5$.

The asymmetry between photocross sections for different polarizations is discussed. Your photon can have its polarization either in the reaction plane (parallel) or perpendicular to the reaction plane. Certain exchanges only contribute to one or the other because of certain selection rules based on parity, so you can try to make some kind of theory to explain your asymmetry experiments.

In the photoproduction of a neutral pion, you can exchange a photon. For obvious reasons, this causes a large forward spike at $t=0$. (The Primakov effect). I have rather ignored the last page of this phenomenological lecture for lack of interest.

Lecture 11. This lecture and #12 hit me as rather unfamiliar + foreign: Much work needs be done here by me.

① The 90° Business. He claims that $\langle p \rightarrow \pi^+ n$ and $\langle n \rightarrow \pi^+ p$ would have same $d\sigma/dt$ if "photon coupling of ~~isovector~~ scalar vs. isovector differed by 90°".

WHY? Idea: since $Q = t_3 + \frac{1}{2}Y$, perhaps $J_\mu = J_\mu^{(3)} + J_\mu^{(scalar)}$

By J_μ means:

$$\begin{array}{c} \pi^+ \\ \nearrow \\ \text{---} \circ \text{---} \\ \nwarrow \\ p \end{array} = A_\mu J^\mu \quad J^\mu \dots \langle \pi^+ n | J^\mu | p \rangle$$

Now if $J^\mu =$ "electromagnetic current of the hadrons", then

$$\langle \pi^+ n | J^\mu | p \rangle = \langle \pi^+ n | J_3^\mu | p \rangle + \langle \pi^+ n | J_8^\mu | p \rangle$$

So there should be only two numbers floating around, the rest all Clebsch related. I.e., we know how J^μ breaks isospin.

Now, lets get out these two #'s. Treat current as "spurion"

so:

$$J_8^\mu = f_8 \cdot |0,0\rangle$$

$$J_3^\mu = f_3 \cdot |1,0\rangle$$

$$\text{then: } \langle \pi^+ n | = \langle 1,1; \frac{1}{2}, -\frac{1}{2} | = \sqrt{\frac{1}{3}} | \frac{3}{2}, \frac{1}{2} \rangle + \sqrt{\frac{2}{3}} | \frac{1}{2}, \frac{1}{2} \rangle \quad \checkmark$$

$$\begin{aligned} \text{then } \langle \pi^+ n | J_3^\mu | p \rangle &= \left(\sqrt{\frac{1}{3}} \langle \frac{3}{2}, \frac{1}{2} | + \sqrt{\frac{2}{3}} \langle \frac{1}{2}, \frac{1}{2} | \right) \cdot f_3 \cdot | \frac{1}{2}, \frac{1}{2} \rangle \\ &= \sqrt{\frac{2}{3}} \cdot f_3 \end{aligned}$$

$$\text{but, } J_8^\mu | p \rangle = f_8 | 1,0; \frac{1}{2}, \frac{1}{2} \rangle = f_8 \cdot \sqrt{\frac{2}{3}} | \frac{3}{2}, \frac{1}{2} \rangle + f_8 \cdot (-\sqrt{\frac{1}{3}}) | \frac{1}{2}, \frac{1}{2} \rangle \quad \checkmark$$

$$\begin{aligned} \text{So, } \langle \pi^+ n | J_8^\mu | p \rangle &= \left(\sqrt{\frac{1}{3}} \langle \frac{3}{2}, \frac{1}{2} | + \sqrt{\frac{2}{3}} \langle \frac{1}{2}, \frac{1}{2} | \right) \cdot f_8 \cdot \left(\sqrt{\frac{2}{3}} | \frac{3}{2}, \frac{1}{2} \rangle - \sqrt{\frac{1}{3}} | \frac{1}{2}, \frac{1}{2} \rangle \right) \\ &= f_8 \cdot \left[\frac{\sqrt{2}}{3} - \frac{\sqrt{2}}{3} \right] = 0 \quad \checkmark \text{ by coincidence} \end{aligned}$$

Let's do: $\delta n \rightarrow \pi p$ so $\langle \pi p | J^y | n \rangle$ ✓

Then $\langle \pi p | = \sqrt{\frac{2}{3}} \langle \frac{3}{2}, -\frac{1}{2} | - \sqrt{\frac{1}{3}} \langle \frac{1}{2}, -\frac{1}{2} |$ ✓

$$J^y | n \rangle = f_v \cdot | 1, 0; \frac{1}{2}, -\frac{1}{2} \rangle = f_v \cdot \left[\sqrt{\frac{2}{3}} | \frac{3}{2}, -\frac{1}{2} \rangle + \sqrt{\frac{1}{3}} | \frac{1}{2}, -\frac{1}{2} \rangle \right] \checkmark$$

So, $\langle \pi p | J^y | n \rangle = f_v \cdot [\sqrt{\frac{2}{3}} - \sqrt{\frac{1}{3}}] = 0$ ✓

Why are these things vanishing?? How about

$$\delta n \rightarrow \pi^0 n$$

$$\langle \pi^0 n | J^y | n \rangle = \langle ? \rangle$$

$$J^y | n \rangle = f_v \left[\sqrt{\frac{2}{3}} | \frac{3}{2}, -\frac{1}{2} \rangle + \sqrt{\frac{1}{3}} | \frac{1}{2}, -\frac{1}{2} \rangle \right]$$

$$\langle \pi^0 n | = \langle 1, 0; \frac{1}{2}, -\frac{1}{2} | = \sqrt{\frac{2}{3}} \langle \frac{3}{2}, -\frac{1}{2} | + \sqrt{\frac{1}{3}} \langle \frac{1}{2}, \frac{1}{2} |$$

So now, $\langle \pi^0 n | J^y | n \rangle = \left(\frac{2}{3} + \frac{1}{3} \right) f_v$

And $\langle \pi^0 n | J^z | n \rangle = \left(\sqrt{\frac{2}{3}} \langle \frac{3}{2}, -\frac{1}{2} | + \sqrt{\frac{1}{3}} \langle \frac{1}{2}, \frac{1}{2} | \right) f_s \cdot | \frac{1}{2}, -\frac{1}{2} \rangle$
 $= \sqrt{\frac{1}{3}} f_s$

Also, $\langle \pi p | J^z | n \rangle = -\sqrt{\frac{2}{3}} \langle \frac{1}{2}, -\frac{1}{2} | f_s | \frac{1}{2}, -\frac{1}{2} \rangle = -\sqrt{\frac{2}{3}} f_s$

Summary:

$$\langle \pi^+ n | J_x | p \rangle = \sqrt{\frac{2}{3}} f_x^{(s)}$$

$$\langle \pi^+ p | J_x | n \rangle = -\sqrt{\frac{2}{3}} f_x^{(s)}$$

$$\langle \pi^0 n | J_x | n \rangle = \sqrt{\frac{1}{3}} f_x^{(s)} + f_x^{(v)}$$

These happen to have no isovector component.

Contradiction: according to my isospin calculation, I would predict that

$$(\delta p \rightarrow \pi^+ n) = (\delta n \rightarrow \pi^- p)$$

because amplitudes are same up to sign, namely, $f_{\pi}^{(s)}$ only seems to enter. Data shows these DCS's are not equal. Something is wrong!

Next, we look into comment that

$$\left(\begin{array}{c} \text{vector} \\ \text{scalar} \end{array} \right) = \left(\begin{array}{c} + \text{vector} \\ - \text{scalar} \end{array} \right) \left(\begin{array}{c} \text{vector} \\ \text{scalar} \end{array} \right) \leftarrow \text{I think he is claiming}$$

Well, $\rightarrow$ do scalar first: $\left(\begin{array}{c} \text{vector} \\ \text{scalar} \end{array} \right) = \langle p | \delta p \rangle = \langle p | p \rangle = 1$

$$\left(\begin{array}{c} \text{vector} \\ \text{scalar} \end{array} \right) = \langle \bar{p} | \delta \bar{p} \rangle = \langle \bar{p} | \bar{p} \rangle = 1 \text{ also; but}$$

from another view we get: $\langle \delta | T | p \bar{p} \rangle = \langle \delta | G^T C | p \bar{p} \rangle$

$$= (-1) \langle \delta | T | C(p \bar{p}) \rangle = (-1) \langle \delta | T | \bar{p} p \rangle$$

$\uparrow$ is this the -1 he speaks of??

What is state of $p \bar{p}$?

$$|p \bar{p}\rangle = \left| \frac{1}{2}, \frac{1}{2}; \frac{1}{2}, -\frac{1}{2} \right\rangle = \sqrt{\frac{1}{2}} |1, 0\rangle + \sqrt{\frac{1}{2}} |0, 0\rangle$$

$\uparrow$ is vector $\uparrow$ is scalar

Now, in reverse,

$$|1, 0\rangle = \text{vector} = \sqrt{\frac{1}{2}} \left[\left| \frac{1}{2}, -\frac{1}{2} \right\rangle + \left| -\frac{1}{2}, \frac{1}{2} \right\rangle \right]$$

$-|p \bar{p}\rangle + |\bar{p} p\rangle$ sym

Perhaps: $\langle \delta | p \bar{p}; 1, 0 \text{ state} \rangle = +1 \dots$
 $\langle \delta | p \bar{p}; 0, 0 \text{ state} \rangle = -1 \dots$

$$\text{Then } \langle \delta | T | p \bar{p}^V \rangle = \mp \langle \delta | T | p \bar{p}^V \rangle$$

exactly wrong.

Perhaps I am looking in wrong place.

$$\left. \begin{array}{l} \delta p \rightarrow \pi^+ n \\ \delta n \rightarrow \pi^- p \end{array} \right\} \text{crosses into} \quad \left. \begin{array}{l} \delta \bar{n} \rightarrow \pi^+ \bar{p} \\ \delta \bar{p} \rightarrow \pi^- \bar{n} \end{array} \right\} \text{compare these!}$$

$$\langle \pi^+ \bar{p} | J_+ | \bar{n} \rangle \quad \text{versus} \quad \langle \pi^- p | J_- | n \rangle \quad ?$$

Well, certainly

$$\begin{aligned} C | \pi p \rangle &= | \pi^+ \bar{p} \rangle \\ C | n \rangle &= | \bar{n} \rangle \end{aligned}$$

I assume.

$$\text{Now } C^\dagger J_+ C = C^\dagger J_3 C + C^\dagger J_3 C$$

$$= J_3 - J_3, \text{ must be } 0$$

So, again:

$$\langle \pi^+ \bar{p} | J_+ | \bar{n} \rangle = \langle \pi^- p | C^\dagger J_+ C | n \rangle = \langle \pi^- p | J_3 - J_3 | n \rangle$$

$$\langle \pi^- p | J_+ | n \rangle = \langle \pi^- p | J_3 + J_3 | n \rangle$$

But, now include photon:

$$\langle \pi^+ \bar{p} | J_+ | \bar{n} \gamma \rangle = \langle \pi^- p | -J_3 + J_3 | n \gamma \rangle = \langle \pi^+ h | J_- | \gamma p \rangle$$

$$\langle \pi^- p | J_- | n \gamma \rangle = \langle \pi^- p | J_3 + J_3 | n \gamma \rangle$$

here is a way to compare the two processes. If 90° , then

$$|J_+ + J_-|^2 = |J_3|^2 + |J_3|^2$$

and amps would be same. But I have argued that J_3 term $\equiv 0$. But that was based on isospin conservation...

Lecture 13: "t-channel exchange phenomena" In this brief lecture, F discusses three "theories" which purport to explain peripheral reaction phenomena. First, there is the geometrical viewpoint where you go to impact parameter space and visualize the amplitude $a(b)$ in some intuitive way. For example, in regular diffraction you imagine that $a(b) = 1$ except for small b where there is absorption. This is a grey or black disc situation. For a charge exchange, you might imagine a ring of activity in the b -space where, say, PI^+ is converted to PI^0 or something. The weakness of this geometrical view is that you get no information on energy (s) dependence of things, only the t dependence.

The second theory is regge theory. It has the advantage of correlating different sectors of data. The use of regge cuts can simulate geometric absorption as in the first theory above. F has very little to say about regge theory.

The third idea presented is the so-called "van-Hove series". You can simulate a regge amplitude by summing the Born terms (OPE) for an infinite family of particles, say scalar, vector, tensor. Of course the coupling has to be just right to make this work. As an example, you could sum over the spectrum of the harmonic osc quark model; F claims you do get a regge behavior. I note in passing how F accents things he has worked on.

Lecture 14: "s-channel resonances" This is another theory to explain peripherality. You can imagine resonances of larger and larger J getting active as s increases. In order that these explain dips which remain fixed int, you need very special couplings. F is not at all convinced of duality. He still thinks you might have to add both s and t channel stuff. He mentions the dual model, though, of Veneziano, but does not know how to correct it (analog of absorption). In closing, he mentions that the quark model does lead to peripherality in this way: the amplitude for 3 quarks as a proton to remain a proton requires low relative momentum, ie, small momentum transfer. The amplitude then is only large if a photon is absorbed and a pion emitted by the same quark, so the net transfer is very small. Small p_t requires the pion go off forward, ie, peripherality. A nice intuitive picture, but hard to support since quarks never appear. The dual diagrams are mentioned. In them, interaction is always explained by one quark interacting with one other quark (say between two protons). At least that accounts for the bulk of the amplitude.

Gasiorowicz - Quantum Physics (1974)

Gerasimovic, Ch 4³ on Vector Mesons

(14 pages)

dec. 18, 1974.

① The condition $\partial_\mu \phi^\mu(x)$ is arbitrarily imposed, or at least suggested.

② We hunt for a Lagrangian. He tries out:

$$\mathcal{L} = -\frac{1}{2} f^{\mu\nu\dagger} (\partial_\mu \phi_\nu - \partial_\nu \phi_\mu) + \text{b.c.} + \frac{1}{2} f^{\mu\nu\dagger} f_{\mu\nu} + m^2 \phi_\mu \phi^{\mu\dagger}$$

where $f^{\mu\nu}$ is some kind of "auxiliary field". Then Euler Lagrange says:

$$(a) \frac{\partial \mathcal{L}}{\partial f^{\mu\nu\dagger}} = 0 \text{ since no } \partial^\mu f^{\mu\nu\dagger} \text{ appearing}$$

$$\Rightarrow f_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu$$

$$(b) \frac{\partial \mathcal{L}}{\partial (\partial_\alpha \phi_\mu^\dagger)} = \frac{\partial \mathcal{L}}{\partial \phi_\mu^\dagger} \Rightarrow \partial_\alpha f^{\alpha\mu} + m^2 \phi^\mu = 0$$

By using "extra" fields, he ends up with extra equations. This last one is interesting because it says:

$$\partial_\nu \phi^\nu = 0 \quad \text{as he wanted.}$$

If we use this condition in (a) result, we get:

$$\partial_\mu f^{\mu\nu} = -m^2 \phi^\nu = + \partial_\mu \partial^\mu \phi^\nu \Rightarrow (\square + m^2) \phi^\nu = 0.$$

So this clever Lagrangian gives both equations:

③ The next move is to simply get rid of ϕ^0 and Π^0 altogether. So we are going to drop Lorentz invariance & trick. It is just an exercise to write out:

$$a) \mathcal{L}(\phi^i, \partial_\mu \phi^i)$$

By the way:

$$b) \mathcal{H}(\phi^i, \Pi^i)$$

$$\Pi^\mu = \frac{\partial \mathcal{L}}{\partial (\partial_0 \phi_\mu)} = f^{\mu 0\dagger}$$

$$c) H = \int d^3x \mathcal{H}$$

$$\text{So } \Pi^0 \equiv 0.$$

④ Gas. is careful to use a hermitean lagrangian constructed from fields that are not nec. hermitean themselves.

⑤ For comrels, we exactly duplicate the scalar field.

$$[\pi^i(x), \phi^j(x')]_{et} = -i \delta_{ij} \delta^3(x-x').$$

Then you can compute other components. Notice that

$$\pi^\mu(x) = \dot{\phi}^{\mu+}(x) = 2^\mu \phi^{0+} - 2_0 \phi^{\mu+}$$

So if you wanted $[\pi^\mu, \phi^\nu]$, you would need many results. Gas. then claims that:

$$[\phi^{\mu+}(x), \phi^{\nu+}(y)] = -i \left(g^{\mu\nu} + \frac{1}{m^2} \partial^\mu \partial^\nu \right) \Delta(x-y; m)$$

I will have to do this as an exercise to really believe it.

⑥ Next, he does expansions of $\phi^k(x)$ only, using A and $B^\dagger$ operators, and using

$\mathcal{C}^k(q, \lambda)$ as wavefunctions.

Here, $\sum_{\lambda=1}^3$ are the sum over spin states.

⑦ To get these comrels: $[A(q, \lambda), A^\dagger(q', \lambda')] = 2\omega \delta(q-q') \delta_{\lambda\lambda'}$,
you are forced
then to choose:

$$\sum_{\lambda=1}^3 \mathcal{C}^k(q, \lambda) \mathcal{C}^l(q, \lambda) = \delta_{kl} + \frac{q^k q^l}{m^2}$$

This is the completeness relation.

⑧ Something to keep in mind is that these unusual forms are not resulting from masslessness. Try to obtain for massive need!

⑨ Using the rule ⑦, he tries to find explicit forms for the $\mathcal{C}^k(q, \lambda)$.

What he concludes is:

$$e^k(q,1) = (1, 0, 0)$$

$$e^k(q,2) = (0, 1, 0)$$

$$e^k(q,3) = (0, 0, \frac{\omega_q}{m})$$

These are true spatial vectors,
where we choose

$$q^k = (0, 0, q_z)$$

Thus, there is a "longitudinal"
polarization state, but
no timelike one.

⑩ Next, he repeats all using ϕ^μ Results.

$$e(q,\lambda) = q^\lambda/m \quad \text{where } \lambda = 1, 2, 3.$$

But, $q^1 = q^2 = 0$ in our choice.

Then:

$$\sum_{\lambda=1}^3 e^\mu(q,\lambda) e^\nu(q,\lambda) = - \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{m^2} \right).$$

More exercises.

⑪ Why is spin = 1? i.e., you do know $[J^i, J^j] = \dots$, but why spin 1? i.e., you know you have a $\mathbb{Z}_{\pm 2}$ of $\mathcal{L}$, but why do you have a $D^{(1)}$ of R ?

Well, look at $[J^i]_{jk}$ only. This is the $D^{(1)}$ matrix of $R^{(1)}$ i.e.,

$$[J_3]_{jk} = [M^{12}]_{jk} = \text{stop!}$$

In the standard $\mathbb{Z}_{\pm 2}$ basis, you are not in the standard $D^{(1)}$ basis at the same time. But you could disagree.

More significantly: $J^2 = J_1^2 + J_2^2 + J_3^2 = 2 \cdot I$ in any basis.

But $J^2 |jm\rangle = j(j+1) |jm\rangle$ Thus, $j=1$ ✓

⑫ Now I will do a rough reading of the rest of the chapter + will do carefully tomorrow.

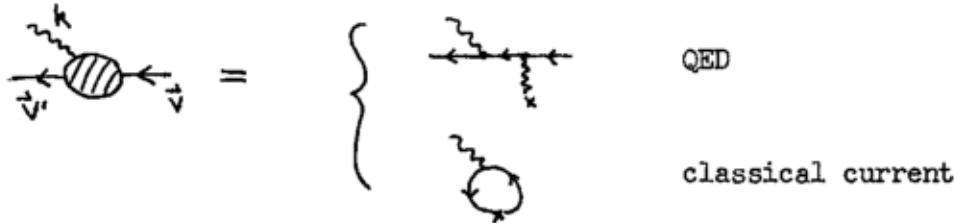
First, we write the S-matrix and M-matrix element for Brem. using the ---x trick. We are thus assuming that the nucleon mass energy is much larger than anything else around.

Next, we look at the soft photon limit of the S matrix. It looks just like the elastic or Coulomb S matrix times a characteristic Brem. factor (a bunch of dot products which are the remains of the propagators). Now, when you write out the computed cross section in the soft limit, you get:

$$d\sigma_{\text{brem}} = (\text{factor}) \times \frac{dk}{k} d\Omega_k d\sigma_{\text{elastic}}$$

The cross section clearly blows up in the usual infrared way as $k \rightarrow 0$. Also, the "factor" has two strong directional peaks in it: these peaks are in the direction of the before and after electron momentum. These peaks were not discussed by Halpern.

These peaks do not require a quantized matter field. If you treat only the photon field as quantized, and let the electron current be classical, the same peaks show up. They come from the time integral of an exponential (see (12.9)). In other words, we use $L = J_\mu A^\mu$ except the current J is classical. Compare:



The next move in the chapter is to rewrite the Brem cross section by assuming a small photon mass μ . The result, after summing over photon polarization states, is given in (12.14). In this expression (12.14) there are three terms.

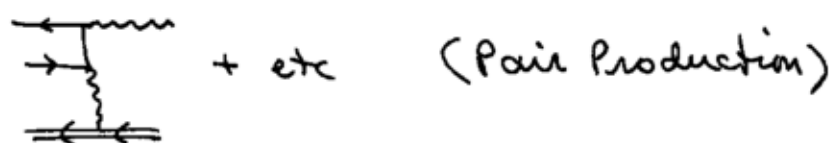
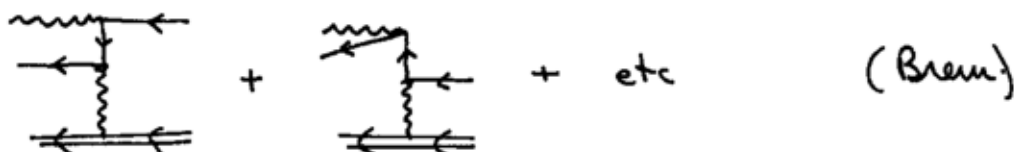
Question: what is the cross section for Brem. where the emitted photon has energy between μ (the small mass) and something call it ϵ ? To answer this question we have to integrate (12.14) over all angles of the photon, and also over the energy range just mentioned. When the momentum transfer $p - p'$ is large, only the third term in (12.14) is significant. This result is given in (12.19) and is clearly log divergent in the photon mass μ . It is claimed that this divergence will be cancelled by another divergence yet to come so let us wait.

The next topic is high energy Brem. Here we use the Wiegacker-Williams ideas discussed in Jackson. In the rest frame of the electron, the em field of the relativistically approaching nucleus looks like a packet of photons. These photons do Compton scattering from the electron and the scattered photons are the Brem photons when you go back to the lab frame. In this method, the relativistic cross section for Brem is obtained by integrating the Klein-Nishina formula with a special weighting for the nucleon photon packet. Gas uses $*$'s to denote variables in the electron rest frame. After doing the integration and transforming back to the lab frame, the result is (12.27). From this cross section, the energy loss and radiation length are easily calculated. In air, a "hot" electron loses $3/4$ of its energy in 330 meters of travel due to Brem in this relativistic limit.

(chapter 12, con'd)

Along the same lines as that rel Brem, Gas discussed pair production in the field of a nucleus. First you go into a frame in which the nucleus approaches the incident photon XXXX. You first transform to a reference frame in which the nucleus is travelling near the velocity of light. Then its field may be treated as the aforementioned photon packet. In this frame then we have photon-quasiphoton scattering or pair production. We know the cross section for this pair production from our lowest order calculation. Stick this in and integrate over the photon packet weighting function to get the pair production cross section. Then from this cross section we can obtain a characteristic length. This is the length a hi energy photon can travel through some material before it produces a pair. Strangely, this length is almost exactly the same as the electron radiation length. See (12.36).

These last two processes can of course be done exactly in perturbation theory. It is the high energy case that prevents us from using the simple -----x technique for the external potential. The relevant diagrams are:



These calculations are all done by Bethe and Heitler. Wonder what they use for the large momentum transfer form factors.

The final remarks of the chapter are that a polarized hard photon transfers all of its helicity to the faster of the pair it produces. Also, when a hot electron brems, the created or emitted photon gets the helicity that the electron had. Several references are given for these ideas and calculations.

Finally, a comment about the WW technique. It uses the on-shell Compton formula, even though one of the photons is really off shell. As CM energy increases this approximation must fail. That one photon is really a propagator and is not transversely polarized etc.

The one-particle-to-vacuum matrix element of the current which drives the Dirac equation is zero. This is simply a result of the fact that the one particle state is stable. We can write this zero matrix element in various ways using the reduction formalism. One example of this is 13.8 .

As an aside, we must keep in mind where the unrenormalized and renormalized operators occur. The reduction formulas in Gasirowics always use the renormed, unhatted operators. All masses in the reduction formulae are experimental, and no Z factors appear. In particular, the external leg factors contain physical masses. On the other hand, the field operators which occur in the lagrangian and the com rels are unrenormalized, hatted operaotrs.

When we write the lagrangian with the usual unrenormed operators, we want the physical mass to appear in the usual place. This of course requires a mass counterterm; this counterterm therefore appears in the Dirac drivigg current which Gas. denoted by the letter f .

So, from the reduction formalism we have an equation relating Z_2 , dm , and a reduction type integral of the VEVTOP of this driving current and its bar. This reduction may be evaluated to order e^2 in the usual way, the Z_2 factors cancel, and we are left with 13.13 which expresses dm as a divergent integral. This integral 13.13 is none other than the usual lowest order correction to the electron propagator. However, it is in terms of the physical mass! And, it involves the bare electron charge.

Evaluation of Z_2 . Again he makes use of the one-particle stability to obtain a reduction type expression for $Z_2 u(p)$. To order e^2 we end up with expression 13.18 for Z_2 . It contains an integral that looks much like what we called dm , but we find that really dm is only one piece of three pieces which comprise the total integral (labelled capital sigma as in B+D). The fact that $u(p)$ sits on the right end does therefore make a difference. It is the factor A that we identify with dm . Then it turns out that the factor B is exactly $z_2 - 1$. Thus, at this point we know exactly how to calculate dm and z_2 to second order in terms of infinite integrals.

Why does Gas. have no external leg factors. It all stems from his strange definition of his one particle states. The creation operator a^+ which creates an electron is defined differently from everyone else. The norm of a OPS in this book is not 1; instead it is E/M for electrons and $2w$ for others. Thus, the leg factors do not appear explicitly in the S-matrix. However, now the density of probability is no longer $1/2 \cdot \pi$ cubed but is say $2w$ times same. In the end, then, these leg factors reenter.

"Classical" Beta Decay

Gasiorowics

Chapter 29

Weak

1. In 1934 Fermi suggested a vector-vector coupling to explain beta decay.
2. There are 5 possible current-current couplings that conserve P.
3. In beta decay the nuclei are nonrelativistic so the reduction is appropriate. When this is done, the P term disappears so there are only 4 terms left in the interaction. The S and V go with the [1], Fermi, and the T and A go with the $[\sigma_k]$, Gamow-Teller.
4. Using these terms, one can calculate the rate. There will of course be AT and SV interference terms. These terms are not observed. (Fierz)
5. Notice that our coupling does not connect nuclear states of differing parity. This is because we only considered the first and largest term in the nuclear matrix element expansion.
6. Pure Fermi transitions are: $(0, \pm)$ to $(0, \pm)$ for (J, P) .
7. Pure Gamow-Teller transitions are: $(J, +/ -)$ to $(J \pm 1, +/ -)$. ie, $dJ=1$
8. All other transitions are mixed. (eg, neutron decay is mixed)
9. Consideration of the π_{12} decays:
 - a. modelled with a π -np vertex.
 - b. parity conservation and Lorentz invariance restrict us to A and P terms
 - c. the electron/muon ratio shows that the P term is zero.
 - d.

In light of the fact that parity is broken in weak interactions, why should any of the above stuff be valid. The parity violating terms are always left out, even in the π decay.

The form of the beta interaction.

Gasiorowics

Chapter 31

Weak

1. On Lee and Yang's suggestion, the Co^{60} decay was investigated for parity violating terms in the year 1957 by Wu et al at NBS. The decay is pure Gamow-Teller so the four terms to start with are A, A', T, T' . In chapter 29 we ignored the A' and T' terms.
~~2. The Co^{60} is a beta decay, by the way.~~
2. The Co^{60} is a beta decay, by the way.
3. An asymmetry is observed and at once parity is dead.
4. The data shows that either T and T' are present, or A and $-A'$; not both.
5. In either case, the electron should have helicity $= -\beta$. This was also checked in 1957 by Frauenfelder.
6. The two component neutrino theory emerges.
7. $\text{Ga}(66)$ is a pure Fermi decay. The neutrino-2 theory says that either S or V terms are present but not both.
8. Question: does the neutrino involved in the fermi decays have the same helicity as the neutrino in the gamow-teller decays? If so, then in a mixed decay there should be interference between the two terms. Experiment did show interference and the conclusion was (S,T) or (V,A) .
9. The question was finally resolved in a correlation experiment. An example is discussed involving K-shell electron capture followed by neutrino and then photon emission. The neutrino that was produced (not an anti- ν) was shown to have negative helicity. This selected out the $V-A$ terms only.
* The S and T terms had the wrong correlations.
10. Longitudinal polarization in pion decay was further evidence. Then the muon decays and the resultant electron again has longitudinal polarization.
11. Another lepton conservation test is μ yields $e + \nu + \bar{\nu}$. Must be different kinds of helicity on those neutrinos, says the theory. So says experiment!
12. The muon-electron- ν - $\bar{\nu}$ hamiltonian is written down with the root 2.
13. From the muon lifetime the value of G_μ is found to be 10^{-5} inverse proton masses.
14. From the pure Fermi O^{14} decay the value of G_V can be measured. It is exactly the same as G_μ . To me this is an amazing result. The muon decay and beta decay must be the same interaction.
15. From the neutron decay (mixed) we find $C_A/C_V = 1.18$.
16. Finally, it is suggested that this 1.18 is the form factor and does not really belong in the interaction current.
17. Note that this entire chapter is about $S=0$ currents.

Strange reactions I

Gasiorowics

Chapter 32

Weak

1. So far we have considered only $\Delta S = 0$ currents.
2. These currents have always carried charge. No neutral currents.
3. For these currents, exper shows that T is OK, hence CP is OK.
4. When strange particles decay weakly, the rule $\Delta S = 1$ always works.
5. The rule $\Delta Q = \Delta S$ also applies.
6. The rule $\Delta T = \frac{1}{2}$ also works.
 - a. evidence 1 is decay of K into two pions. 500X rate difference.
 - b. evidence 2 is lambda zero branching ratio into Pi-Nucleon.
 - c. evidence 3 is sigma-into Pi-N decays(3); graphic amplitude sum prediction.
 - d. cascade into lambda pi branching ratio.
 - e. evidence 5 is K_S and K_L branching ratios (=2) into two pions.
7. The strong weak current now has four obvious terms.
8. There is some evidence for $\Delta T = 3/2$ processes. (K^+ decays).
9. The $\Delta T = \frac{1}{2}$ rule is thought to be a "dynamical enhancement."
10. The $K^0-\bar{K}^0$ system.
 - a. construct the non-diagonal Hamiltonian 2x2 matrix.
 - b. find the linear combinations of K and K-bar which diagonalize H.
 - c. use special K_1 and K_2 notation for these lincoms when CP holds. (32.26)
 - d. using approx CP conservation, show whay one is longer than the other.
 - e. using $\Delta T = \frac{1}{2}$ rule, find some branching ratios = 2.
 - f. in a pure K beam, K-bar appears downstream because of mass difference.
 - g. this allows measurement of the mass difference.
 - h. if a K_L beam hits something, K_S can be regenerated.
 - i. there is recent evidence for CP violation
 - j. this may lead to the death of the current-current simple picture.
11. By the way, the $\Delta S = 0$ currents have $\Delta T = 1, 0$.

Strange reactions II

Gasirowics

Chapter 33

Weak

1. We write the M.G.F. for the lambda decay amplitude and identify the S and P wave pieces.
2. From polarization experiments one finds the parameters alpha, beta, and gamma. From these one can compute a_s and a_p at low q^2 .
3. A large alpha indicates both channels Λ and P are active in the decay. A small alpha shows that only one channel is going.
4. By assuming CP invariance, we can write down the possible SU_3 invariant couplings for the S and P waves separately. These lead to various triangle relations which are borne out experimentally. It is assumed that H_W is transforming as a λ_6 member of an octet.
5. The other non-leptonic decays involve the sigma and cascade.
6. In the leptonic decays ($\lambda = \text{pev}$; eg), the experimental rates are always slower than the theory predicts by factor of 10. Maybe the strangeness-changing current should not be weighted the same.
7. A comparison of the form factors in K_{e2} and π_{e2} decays again suggests that the two kinds of currents are not on the same footing. The K is a $S=1$ current and the π is $S=0$.
8. In the K_{e3} decays there are two form factors possible (maybe more).
9. A simple OPE model for K_3 decay using the K^* resonance predicts an f_+/f_- ratio of ~ 3 .
10. This ratio can be found experimentally in two ways. (1) look carefully at the shape of the lepton energy spectrum. (2) look at the e/u rate ratio however, this gives two possible values for the ratio.
11. In all these rates, the f_- form factor is always multiplied by a few lepton masses so it really doesn't count for much. It is "suppressed".
12. A comparison of K_3 to π_{e3} decay again shows a factor of ten difference in the principle form factors.

Conserved Currents.

Gasiorowics

Chapter 34

Weak

1. The CVC hypothesis is made. The em current has a piece that transforms under SU_3 and T like the π^0 . It is postulated that the weak vector currents transform like the π^+ and the π^- . Isotriplet hypothesis.
 - a. evidence: in π beta decay, CVC predicts $f_+ = \sqrt{2}$. Works.
 - b. evidence: in beta decay CVC predicts that F_2 is same as for em. This is the "weak magnetism" prediction. Works.
 - c. evidence: in e^+/e^- decays of T = 1 nuclei, CVC predicts details of the lepton energy spectrum. These shape details depend on the magnetic form factor F_2 and verification has been shown. This test is like b.
 - d. evidence: in neutrino-proton reactions, again CVC says that the em form factors apply for F_1 and F_2 . When these cross sections are measured experimentally, the theory can be checked again.
2. The above statements apply only to the vector current, not the axial.
3. In the above, the conserved vector current is the isotopic spin current!
4. The fact that J_V is conserved makes the induced pseudoscalar form factor (P) vanish. Gas forgot to mention this. Thus, in weak vector decays there are only two form factors to worry about.
5. Axial current. If it were also conserved, then π_{u2} could not go. It does go.
6. Several theories suggest that the divergence of the axial current is proportional to the pion mass and the pion field (sigma model, eg). In the limit of vanishing pion mass, the J_A is conserved. This is called PCAC.
7. Using PCAC, and considering $(n/J_A/p)$, we come up with a relationship between the F_A Form factor, g (the strong π -N coupling constant) and the NNPI vertex form factor. We also produce a relation between F_A and F_P ; the induced pseudoscalar form factor which is present because the axial current is not conserved. Finally, we can find the constant that appears in the PCAC equation for the axial current divergence. It is simply $F_A(0)/g$.
8. Knowing this constant, we now apply the PCAC equation to π_{u2} decay. The result is a relation between F_A (of beta decay) and f_{PT} of π_2 decay. (and g). Thus, knowing g and F_A , we can predict F_{pi} . The prediction works and PCAC is thus vindicated. This relation of F_A , F_{pi} , g is the original Goldberger-Trieman relation.
9. The above statements apply only to $S=0$ currents.
10. The $S=1$ vector current cannot be conserved because that would block a certain decay which is observed to go.
11. The $S=1$ weak currents are associated with J^4 and J^5 of the octet of conserved currents developed from SU_3 symmetry. The $S=0$ currents of course are associated with J^1 and J^2 . This hypothesis, made on sound observations, at once predicts the $dS \rightarrow dQ$ rule and the $dT = \frac{1}{2}$ rule.
12. The vector currents are directly associated with the SU_3 generators and thus contain only F-type SU_3 coupling. The axial currents however can be a mixture of F and D couplings.
13. Cabibbo's sin and cos are thrown in and universality is saved by writing the hadronic currents J_A and J_V in terms of the angles and the octet currents.
14. It is shown that this result comes from a simple F_7 rotation of the plain isotopic spin current. This leads to the statement that $H_w = F_6$ xform.
15. Consideration of certain K and π decay ratios shows that the angles which appear in the J_V and J_A are the same.
16. Analysis of the semileptonic baryon decays gives the same angle once again.
17. Gell-Mann's extension to $SU_3 \times SU_3$ is mentioned and there is a 5 page derivation of the Adler-Weisberger-Fubini sum rule which relates F_A to pion cross sections.

Bogoliubov & Shirkov - Intro to the Theory of Quantized Fields
(1959)

Bogoliubov + Shirkov [Steklov Math. Institute, Academy of Sc., Moscow]
 "Introduction to the Theory of Quantized Fields"
 (1959)

This is one gargantuan book. Some 700 pages. Here is the contents:

1. Classical Theory of Free Fields (80)
2. Quantum Theory of Free Fields (100)
3. The scattering matrix (90)
4. Removal of Divergences from the S-matrix (70)
5. Application of "divergence removal" to special cases (80)
6. Schrödinger equation + Dynamical Variables (50)
7. Functional Averaging (25)
8. The Renormalization Group (40)
9. Dispersion Relations (100)
- Appendices (50)

Introduction: Bethe's 1947 is credited as the starting point for the "rapid devel. of QFT". The original equations were known in the 1920's, but even then the perturbation theory divergences had been run up against. "Form factors" were tried to cutoff these divergent integrals. Did not get far. The problem was resolved by the "subtraction formalism" which involves renormalizing mass and charge. Bethe used this to compute the Lamb Shift. In 1948, Tomonaga and Schwinger found a covariant (formal) way to present the theory, not very practical. In 1949, Feynman recast the theory in terms of path integrals + Feynman diagrams made their entry [people have been obsessed with "diagrams" ever since]. Dyson then set straight the prescription for unloading the divergences.

Example of theory's success: $\pi \rightarrow \mu \nu$ vs. $\pi \rightarrow \mu \nu$

So QED was beautiful, but application to meson theories was a bomb. So several branches of the theory occurred then. One is that of Tamm-Landau, a second that of using Green's functions.

Derivation of the Dirac Spinor Completeness Relation

Here is what we want to show: $\sum_{\alpha=1}^4 \epsilon_{\alpha} \omega_{\alpha}^{\dagger}(\vec{p}) \bar{\omega}_{\beta}^{\dagger}(\vec{p}) = \delta_{\alpha\beta}$

[A] first show true for $\vec{p}=0$:

$$\left. \begin{aligned} \omega_{\alpha}^{\dagger}(0) &= \delta_{\alpha\epsilon} \\ \bar{\omega}_{\beta}^{\dagger}(0) &= \epsilon_{\beta} \delta_{\beta\epsilon} \end{aligned} \right\} \Rightarrow \sum_{\epsilon=1}^4 \epsilon_{\epsilon} [\delta_{\alpha\epsilon}] [\epsilon_{\epsilon} \delta_{\epsilon\beta}] = \delta_{\alpha\beta} \quad \checkmark$$

[B] second, do Lorentz transform to show true for general $\vec{p}$:

$$\omega_{\alpha}^{\dagger}(\vec{p}) = S_{\alpha\alpha'}(a) \omega_{\alpha'}^{\dagger}(0) \quad \text{where} \quad p = a \check{p}$$

Also, since we know that $\bar{S}^{\dagger} = \gamma_0 S^{\dagger} \gamma_0$ (2.26) we have

$$\bar{\omega}_{\beta}^{\dagger}(\vec{p}) = \bar{\omega}_{\beta'}^{\dagger}(0) \bar{S}_{\beta\beta'}^{\dagger}(a)$$

So start with

$$S_{\alpha\alpha'} \rightarrow \left[\sum_{\epsilon} \epsilon_{\epsilon} \omega_{\alpha}^{\dagger}(0) \bar{\omega}_{\beta}^{\dagger}(0) \right] \leftarrow \bar{S}_{\beta\beta'}^{\dagger} = S_{\alpha\alpha'} \rightarrow [\delta_{\alpha'\beta'}] \leftarrow \bar{S}_{\beta'\beta}^{\dagger}$$

$$\Rightarrow \sum_{\epsilon} \epsilon_{\epsilon} \omega_{\alpha}^{\dagger}(\vec{p}) \bar{\omega}_{\beta}^{\dagger}(\vec{p}) = \delta_{\alpha\beta} \quad \checkmark$$

The Klein Paradox

This is a strange situation that arises in relativistic quantum mechanics. I read about it on p 40 of B+D VI where they give the original reference (~~O.~~ Klein, ~~Z.~~ Physik, 53, 157 (1929)).

In normal quantum mechanics, if you shoot an electron at a potential wall, it bounces off if the height of the wall is more than the energy of the electron. If the wall is of finite height, there is an exponentially decaying leak through probability so that if the wall is of finite thickness, there is a chance that the electron will tunnel through the wall. This is a standard quantum mechanics illustration of "strange things."

Now, if the height of the wall is greater than the electron energy by an amount more than mc^2 , and if we use the appropriate equation to analyze this situation (the Dirac Equation), then we find two very strange results. Inside the wall, which we take as infinitely thick, the wave function does not die off exponentially. Instead, it propagates. However, it propagates in such a way that the "current" (probability current) inside the wall is negative! Secondly, the reflected current is larger than the incident current! This is the Klein paradox.

I have not yet read the solution to this paradox, but it is clear what is happening. The electron hits the wall and reflects. In addition, since the potential is so large, an electron-positron pair is created at the wall. This second electron comes off as a reflection and explains why the reflected current is paradoxically large. The positron propagates into the wall and explains the negative transmission current. The positron is able to survive and propagate inside the wall where an electron would die.

THE SPIN-PROJECTION OPERATOR

$$\mathcal{Z}(\vec{s}) \equiv \frac{1}{2}(1 + \gamma_5 \vec{\sigma} \cdot \vec{s})$$

In rest frame where $\vec{S} = (0, \vec{s})$,

$$\begin{aligned} \mathcal{Z}(\vec{s}) &= \frac{1}{2}(1 - \gamma_5 \vec{\sigma} \cdot \vec{s}) \\ &= \begin{bmatrix} \frac{1}{2}(1 + \vec{\sigma} \cdot \vec{s}) & 0 \\ 0 & \frac{1}{2}(1 - \vec{\sigma} \cdot \vec{s}) \end{bmatrix} \leftarrow \end{aligned}$$

Let $\psi(\vec{s})$ be a 2-spinor such that

$$\vec{\sigma} \cdot \vec{s} \psi(\vec{s}) = \psi(\vec{s}) ; \quad \vec{\sigma} \cdot \vec{s} \psi(-\vec{s}) = -\psi(-\vec{s})$$

Then we have:

$$\omega^1(0) = \begin{bmatrix} \psi(\vec{s}) \\ 0 \end{bmatrix} \quad \omega^2(0) = \begin{bmatrix} \psi(-\vec{s}) \\ 0 \end{bmatrix}$$

$$\omega^3(0) = \begin{bmatrix} 0 \\ \psi(\vec{s}) \end{bmatrix} \quad \omega^4(0) = \begin{bmatrix} 0 \\ \psi(-\vec{s}) \end{bmatrix}$$

Thus we can show that:

$$\begin{aligned} \mathcal{Z}(\vec{s}) \omega^1(0) &= \omega^1(0) & \mathcal{Z}(-\vec{s}) \omega^1(0) &= 0 \\ \mathcal{Z}(\vec{s}) \omega^2(0) &= 0 & \mathcal{Z}(-\vec{s}) \omega^2(0) &= \omega^2(0) \\ \mathcal{Z}(\vec{s}) \omega^3(0) &= 0 & \mathcal{Z}(-\vec{s}) \omega^3(0) &= \omega^3(0) \\ \mathcal{Z}(\vec{s}) \omega^4(0) &= \omega^4(0) & \mathcal{Z}(-\vec{s}) \omega^4(0) &= 0 \end{aligned}$$

Thus, it is $\omega^1(0)$ and $\omega^4(0)$ that are "spin up".

To generalize to $\vec{p} \neq 0$;

$$\begin{aligned} \mathcal{Z}(\vec{s}) \omega^1(0) &= \omega^1(0) \\ \omega^1(\vec{p}) &= \vec{S}^1(a) \omega^1(0) \quad \text{where} \quad p = a \vec{p} \\ \vec{S}^1 \mathcal{Z}(\vec{s}) \vec{S} &= \mathcal{Z}(s) \quad \text{where} \quad s = a \vec{s} \\ [\text{since } \vec{S}^1 \not{p} S &= \not{p}' \text{ and } \vec{S}^1 \gamma_5 S = \gamma_5] \end{aligned}$$

Thus has been shown:

$$\mathcal{Z}(s) \omega^1(\vec{p}) = \omega^1(\vec{p}) \quad \text{etc.}$$

Standard spinors are

$$\begin{aligned} u(p, s) &\equiv \omega^1(\vec{p}) \\ u(p, -s) &= \omega^2(\vec{p}) \\ v(p, s) &= \omega^4(\vec{p}) \leftarrow \text{NOTE!} \\ v(p, -s) &= \omega^3(\vec{p}) \end{aligned}$$

Thus, spin projection says:

$$\begin{aligned} \mathcal{Z}(s) u(p, s) &= u(p, s) \\ \mathcal{Z}(s) v(p, s) &= v(p, s) \end{aligned}$$

One cannot help but wonder why it is that the spin-projection operator has this special form. It would seem more "reasonable" to have $\mathcal{Z}(s)$ be a straight 4-dim version of the 2-dim projector. Instead, we find a minus sign in lower right 2x2 corner (see arrow). Suppose we try to set up the naive 4-dim projector as $\tilde{\mathcal{Z}}(\vec{s}) = \frac{1}{2}(1 + \vec{\sigma} \cdot \vec{s})$. Here, $\vec{\sigma}$ is just a 4-dim version of the Pauli $\vec{\sigma}$. This candidate projection operator is fine in that it now makes $\omega^1(0)$ have "spin up" the way we want it to be intuitively. However, then the equation $\tilde{\mathcal{Z}}(\vec{s}) \omega^1(0) = \omega^1(0)$ is not Lorentz covariant. In order for the equation to be covariant, it must have the same form in any frame (above we have only the rest frame). This means that we need $\tilde{\mathcal{Z}}$ to do this: $\vec{S}^1(a) \tilde{\mathcal{Z}}(\vec{s}) \vec{S}(a) = \tilde{\mathcal{Z}}(s)$. However, the only type of object which transforms in that way is a slash 4-vector, $\vec{S}^1(a) \not{p} \vec{S}(a) = \not{p}'$ where $p' = a p$. The matrices 1 and γ_5 are unaffected by the transformation S . In our candidate projector, the object $\vec{\sigma}$ simply does not transform in the Dirac space as the spatial part of a "4-vector". It cannot be written as terms of the form $\not{p}$ or $\gamma_5 \not{p}$, and these are the only terms that will work. Thus we are stuck with the other form. Note that the γ_5 is necessary to pull the projector into a diagonal form. Lorentz covariance says there is no other way.

Neutrino Helicity in the B+D Gamma Representation

$$\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

The helicity projection operator for dirac 4-spinors u :

$$\hat{h}_4 = \begin{pmatrix} \hat{h} & 0 \\ 0 & \hat{h} \end{pmatrix} \quad \text{where} \quad \hat{h} = \vec{\sigma} \cdot \hat{p}$$

There are several other ways to write this: $\hat{h}_4 = 1 \otimes \hat{h}$

$$\hat{h}_4 = \vec{\sigma}_4 \cdot \hat{p} \quad \hat{h}_4 = \gamma_0 \gamma_5 (-\hat{p} \cdot \vec{\gamma}) \quad \hat{h}_4 = \gamma_0 \gamma_5 \not{S} \quad S = (0, \hat{p})$$

Meanwhile, the neutrino dirac equation says:

$$[\gamma^0 - \vec{\gamma} \cdot \hat{p}] \psi = 0 \quad \begin{bmatrix} 1 & -\hat{h} \\ \hat{h} & -1 \end{bmatrix} \psi = 0 \quad \begin{bmatrix} 1 & -\hat{h} \\ \hat{h} & -1 \end{bmatrix} \begin{bmatrix} \phi \\ \chi \end{bmatrix} = 0 \Rightarrow (\gamma^0 + \not{S}) \psi = 0$$

To verify that this is the dirac $\not{p} \psi = 0$, by the B+D rep solutions:

$$\begin{bmatrix} 1 & -\hat{h} \\ \hat{h} & -1 \end{bmatrix} \begin{bmatrix} u \\ \hat{h} u \end{bmatrix} = 0 \quad \begin{bmatrix} 1 & -\hat{h} \\ \hat{h} & -1 \end{bmatrix} \begin{bmatrix} \hat{h} u \\ u \end{bmatrix} = 0$$

So let ψ be any of the 4 solutions to this dirac equation. What then is the meaning of $\psi_L = (1 - \gamma_5) \psi$?

$$\begin{aligned} \hat{h}_4 \psi_L &= \gamma_0 \gamma_5 \not{S} (1 - \gamma_5) \psi = \gamma_0 \gamma_5 (1 + \gamma_5) \not{S} \psi = -\gamma_0 \gamma_5 (1 + \gamma_5) \gamma_0 \psi \\ &= -\gamma_0 (1 + \gamma_5) \gamma_0 \psi = -(1 - \gamma_5) \psi \\ &= -\psi_L \end{aligned}$$

Thus, ψ_L is a solution of negative (left) helicity. Similarly:

$$\begin{aligned} \hat{h}_4 \psi_R &= \gamma_0 \gamma_5 \not{S} (1 + \gamma_5) \psi = \gamma_0 \gamma_5 (1 - \gamma_5) \not{S} \psi = -\gamma_0 \gamma_5 (1 - \gamma_5) \gamma_0 \psi \\ &= -\gamma_0 (\gamma_5 + 1) \gamma_0 \psi = \gamma_0 (1 - \gamma_5) \gamma_0 \psi = (1 + \gamma_5) \psi \\ &= +\psi_R \end{aligned}$$

So ψ_R has positive helicity. Specifically, in this rep we have:

$$\psi_{L\nu} = (1 - \gamma_5) \psi = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_\nu \\ \hat{h} u_\nu \end{bmatrix} = \begin{pmatrix} [1 - \hat{h}] u_\nu \\ -[1 - \hat{h}] u_\nu \end{pmatrix}$$

Equal but opp upper+lower. If u_ν is spin along the $-\hat{p}$ (left), then

$$\psi_{L\nu} = \begin{pmatrix} 2 u_\nu \\ -2 u_\nu \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} \otimes (u_\nu)$$

In the Weyl rep, things are a little simpler