

The Multiperipheral Model
and its
Application
to the
Rising P-P Total Cross Section

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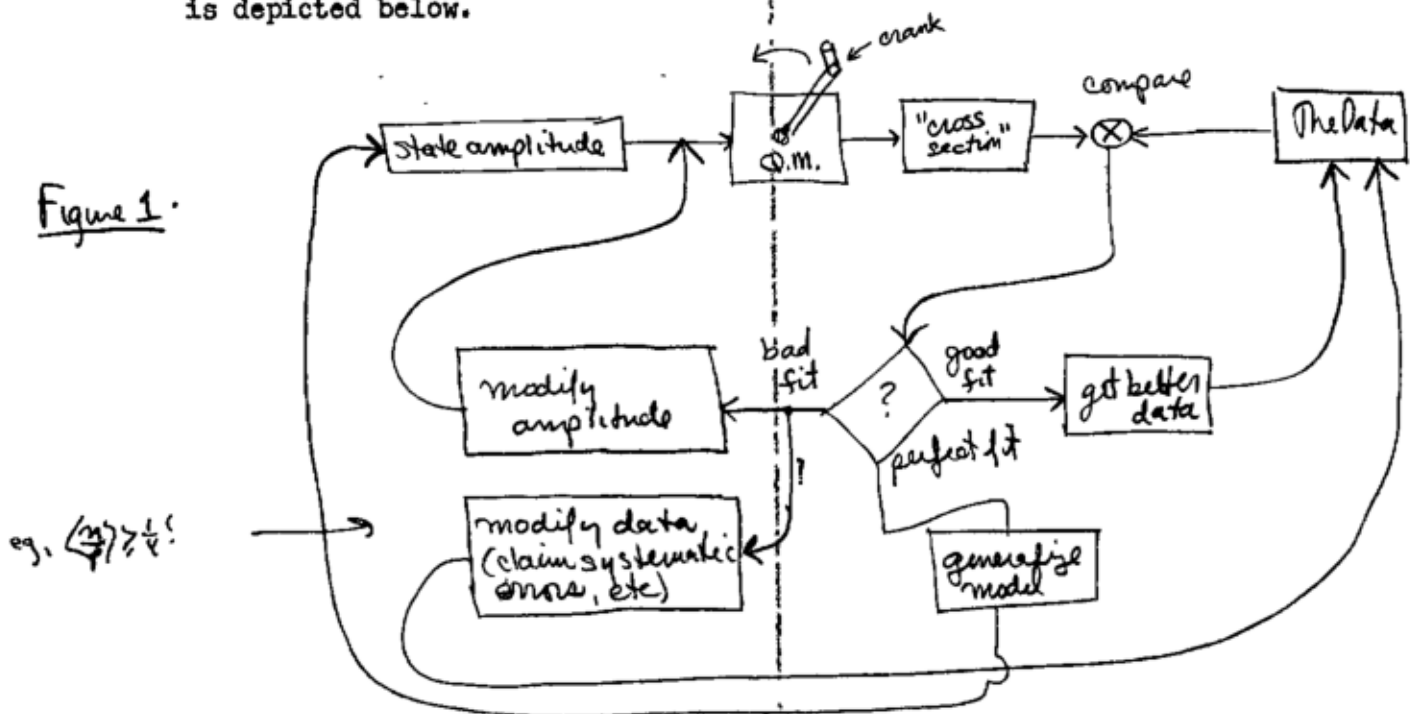
Dec. 7, 1973

1. The basic question and the program; doubts.

The basic question of elementary particle physics seems to be this: what is the most elegant and satisfying way to explain The Data. If this question is too much to chew all at once, we could ask instead: what is the cleanest way to explain a particular chunk of The Data. How might we explain, ^{for example,} the grab-bag of measurements which have been made on $s = 30 - 3000 \text{ GeV}^2$ proton-proton collisions? The multiperipheral model is an attempt at such an explanation.

Like all quantum mechanical models, the multiperipheral model assumes the existence of an amplitude A which one squares, A^*A , to get some kind of indirectly measureable probability for a particular event to occur. The model is an amplitude and as such is already part of another model called quantum mechanics. It would be cruel indeed if "amplitude" were to hold its conventional meaning only in the non-relativistic limit of the true physics. Barring any such conspiracy, the statement of the multiperipheral amplitude clearly defines the model. The old machinery for cranking out cross sections, multiplicities, and so on stands ready to grind through any theorist's amplitude. The program of the theorist is depicted below.

Figure 1.



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It should be noted when we speak of proton-proton collisions that such collisions may represent a poor choice of experiment. I am reminded of Feynman's comment of a few years ago about the Swiss watchmaker's apprentice who spent his evenings smashing together pairs of expensive wristwatches. He was hopeful that, by looking carefully at the various jewels, stems and screws that flew out and by doing appropriate ensemble averages, he could learn how works a watch. Along these same lines, one must wonder how many "events" would be required to lay bare the Schrodinger equation in a systematic study of (krypton-krypton, electron) inclusive spectra. I must admit to having my share of doubts.

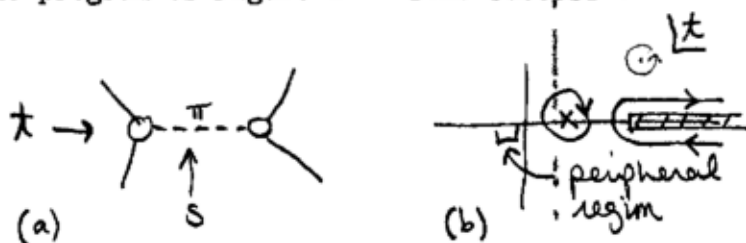
2. Development of the Multiperipheral Model.

The multi-peripheral model was begotten of the peripheral model which in turn was begotten - in a sense - of relativistic quantum field theory. Field theory clarified the covariant propagator associated with an internal line, although such propagators were already known in old fashioned perturbation theory as second-order energy denominators. The covariant propagator of field theory exposes the pole terms of an amplitude. Such pole terms must be present independent of field theory and Feynman diagrams, however, since a fixed mass, one particle intermediate state is by definition such a pole term. There is no reason for such intermediate states to be absent.

The peripheral model¹ exploited the pion pole contribution to the nucleon-nucleon scattering amplitude, but did so in the crossed channel. The idea was simply that a pion pole in the t-channel at $t = m_\pi^2 = .02 \text{ GeV}^2$ lies very close to the region of physical t for near-forward scattering in the s-channel*. If the scattering amplitude were dispersed in the t-plane,

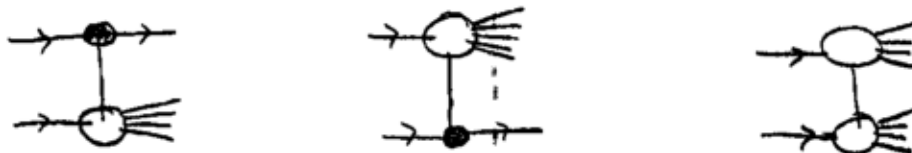
*Near forward suggests scattering at an impact parameter near the periphery of the interaction region.

the activity nearest to $t = 0$ other than the above mentioned pole would arise from the cut contribution starting at $t = 4 m_\pi^2 = .08 \text{ GeV}^2$. Unless the pole residue happened to be anomalously small*, or the cut contribution anomalously large**, for scattering close to $t = 0$ the pole is four times closer than the branch point so the pole should dominate. Thus, this one particle exchange model ought to "explain the data" of the peripheral scattering of nucleons into two body final states. In applications to reactions like $\pi^+ p \rightarrow \pi^+ p$ the peripheral model has had -- after a few cycles through the program of Figure 1 -- some success¹.



The one pion exchange model. Note: sometimes the picture in (a) is taken to imply a factorizable residue.

As the data on high energy multiparticle reactions grew, it was discovered that there was a steep (exponential) cutoff in the momentum transfer variables for such reactions as well as for the elastic reactions. Particles ejected in hot p-p collisions stream out along the line of fire, that is, with small $p_\perp$. The following peripheral mechanisms were suggested:[†]



*Unfortunately, the pion pole residue for NN scattering is anomalously small near $t=0$. In fact, it vanishes at $t=0$!

**The cut is infinitely long, after all.

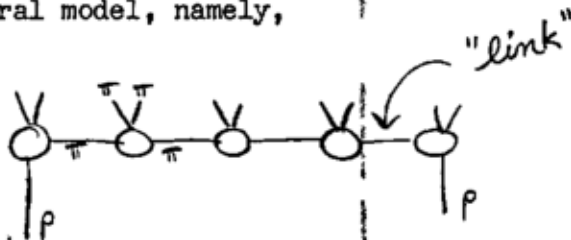
[†] These pictures are, I believe, described as "high mass diffractive dissociation," especially if the pion is replaced by a modern and stylish pomeron.

As before, the rung in these diagrams represents a (possibly factorized) pion pole. Support for such a scattering mechanism existed at least fifteen years ago in the cosmic ray data. Giuseppe Cocconi was analyzing this data back in 1958 and had these conclusions to make:

As a final comment, the most important feature of the model is the possible presence of the two fireballs detached from the original nucleons and from which the secondaries, all with about the same momentum, emerge. It suggests that the final products of NN interactions are rather insensitive to the primary energy, once the ultrarelativistic region is reached, as the high frequency components of the interaction never succeed in emerging from the region of interaction.*

This two-fireball picture was, however, unable to explain in a contortion-free manner 1) the lack of strong correlations between reaction products in the central rapidity region, and 2) the $\log s$ multiplicity growth at high energy. If a fireball, or nova, were to decay as a blob of resonance jelly, the multiplicity of the resultant pions ought to be proportional to $\sqrt{s}$, not $\log s$.

These problems were resolved by the multi-peripheral extension of the peripheral model, namely,



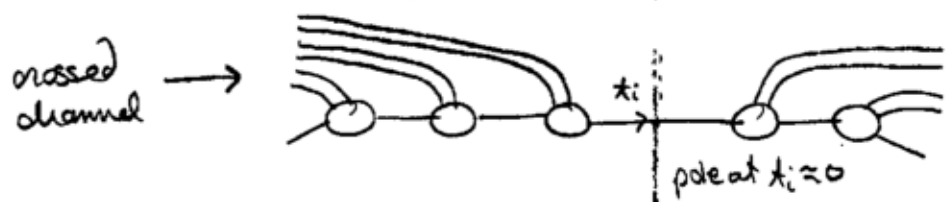
In drawing this picture, we are making several implicit assumptions:

*I cannot resist finishing Cocconi's quotation:

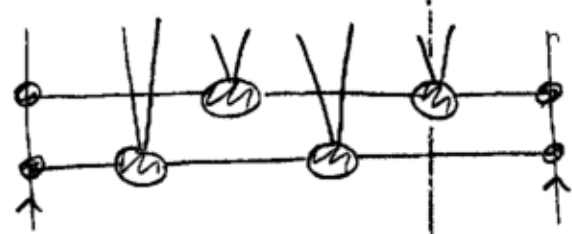
A conclusion of this kind could have the potential consequence of removing some urgency to the building of accelerators of always higher energies. It would in fact make it plausible that the present panorama of the elementary particles is already rather complete, if by particles we mean a property of matter that lasts long enough to be singled out.

1) the pion poles have residues which factorize so that each blob is a function only of its own local momentum variables, and 2) the picture is meant to depict an "average" event in which the blobs are evenly distributed in rapidity. These assumptions correct for the two defects of the two-fireball model listed above: factorization kills off most correlations, and the even rapidity distribution leads to a $\log s$ multiplicity. The main idea here is that the large incident energy is shared democratically by many equivalent blobs. Any blob has a relatively low subenergy and, if the links are in fact pions, the blob is a known quantity, namely the $\pi\pi$ elastic amplitude. Here the word "known" is employed in a loose sense, since off-shell $\pi\pi$ amplitudes are not really known without many extra assumptions of factorization and continuity. (eg, Chew Low)

As interblob links, the pions are justified in the same way as in the peripheral model. If the external lines are properly crossed, the pion is exposed as a pole in some crossed amplitude which is essentially the same as the uncrossed amplitude. Again, the pole is near $t_1 = 0$, where t_1 is the momentum transfer of the pion link,



Before investigating the multiperipheral amplitude, we must wonder why certain other "pictures" are left out, with the assumption that they contribute only slightly to the complete multiparticle amplitude. For example, what happens to the diagram shown below?



This diagram is analogous to the two pion exchange diagram in the peripheral model, and it is not clear why it should be small. Possibly the picture makes no sense at all since the pion lines cannot be isolated as in the preceeding sketch. It is certainly a violation of principle to regard the picture as a higher order graph, as if it were a perturbative Feynman diagram.

Let me quietly brush aside these confusions and use the accepted ASF diagram (the multiperipheral model) since it is topologically tractable and has the pion pole justification just mentioned.

3. What can be done with the Multiperipheral Model?

The multiperipheral amplitude, in its simplest form, is an estimate of the true amplitude to produce n pions (let us neglect all chain end effects). Thus,

$$T_{2 \rightarrow n} \equiv \text{diagram} = \text{diagram} \quad \text{where } \chi = \text{graph}$$

This amplitude, $T_{2 \rightarrow n}$ can be fed into the program of Figure 1 and a fit to the topological or partial cross sections can be carried out.

$$\sigma_n = \frac{1}{4 \cdot \text{flux}} \int d\Omega_n |T_{2 \rightarrow n}|^2.$$

These partial cross sections can then be summed to yield a prediction for the total cross section and the forward elastic absorptive amplitude,

$$\sigma_A = \sum_n \sigma_n = \frac{1}{2 \cdot \text{flux}} (-A(s, 0)).$$

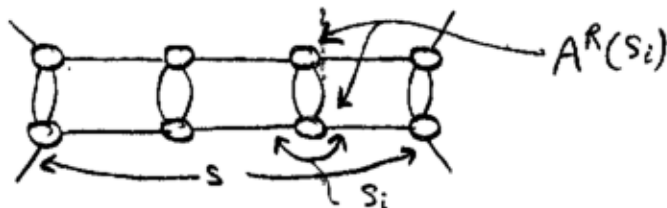
Alternatively, direct channel unitarity can be plied to obtain the non-forward, but small t , elastic absorptive amplitude,

$$s \rightarrow \text{blob} = \text{blob with wavy line} = \sum_n \left[\text{blob with } n \text{ internal lines} \right]$$

$$A_{2 \rightarrow 2}(s, t) = -\frac{1}{2} \sum_n \int d\phi_n T_{2 \rightarrow n} T_{n \rightarrow 2}'.$$

4. Outline of the ASF program.³

The ASF version of the multiperipheral model is the original version. The links are pions, and two pions are produced at each blob. The $n = 3$ contribution to the absorptive elastic amplitude is shown below. The same picture, of course, models the $n = 3$ partial cross section, according to the optical theorem.



The picture suggests the recursion relation,

$$\text{blob with } A_{n-1} \text{ inside} = \text{blob with } A_n \text{ inside}$$

In the ASF notation (their equation 3.4),

$$A_n(p, p') = \int ds_0 \frac{A^R(s_0)}{8\pi^4} \int d^4 p'' A_{n-1}(p, p'') \delta[(p' - p'')^2 - s_0] / (p'^2 - \mu^2)^2,$$

where $A^R(s_0)$ is the resonance-dominated, absorptive $\pi\pi$ amplitude $\sim \sigma_{\pi\pi}$.

By a reasonably straightforward procedure, ASF show that, in the large s limit, each iteration of the recursion relation 3.4 picks up another power of $\log s$, so that,

$$A_1 \sim \frac{\lambda^2}{s}$$

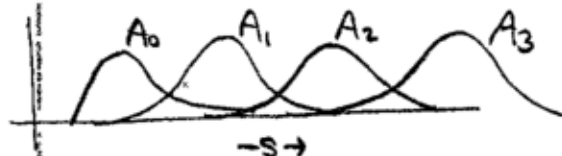
$$A_2 \sim \frac{\lambda^2}{s} (\lambda \log s)$$

⋮

$$A_n \sim \frac{\lambda^2}{s} \cdot \frac{(\lambda \log s)^{n-1}}{(n-1)!} \sim \sigma_n$$

Here, λ represents the size of the $\pi\pi$ integrated cross section.

These partial cross sections are in qualitative agreement with the prong data:



The function $f(s) = \frac{(\log s)^n}{s}$ starts out small, grows to a maximum value of $e^{n(\log n - 1)}$ at $s = e^n$, then dies out. For a specific s , the total cross section is dominated by the partial cross section which peaks at $n = \ln(s)$, reminding us of one of the multiperipheral model's major assets.

The total cross section as we said can be obtained by formally adding the partial cross sections, although ASF do not do this. Formally we have,

$$\sigma_{\text{tot}} = \sum_n \sigma_n \approx \frac{\lambda^2}{s} \sum_n \frac{(\lambda \log s)^{n-1}}{(n-1)!} = \frac{\lambda^2}{s} e^{\lambda \log s} = \lambda^2 s^{\lambda-1}$$

This summation is done in the "leading ln approximation" discussed in Horn⁴p 154. As emphasized there, it is not at all obvious that the series sums in this simple way, since in each order lower terms have been omitted. As ASF show, however, the net effect of these neglected terms is a renormalization of λ to λ' .

ASF show this in the following way. The recursion relation 3.4 is transformed into an exact Bethe-Salpeter-like integral equation for the full absorptive amplitude $A = \sum_{n=1}^{\infty} A_n$.

$$s \rightarrow \text{diagram with } A \text{ in a circle} = \text{diagram with two vertices} + \text{diagram with } A \text{ in a circle and two vertices}$$

By examining the symmetry of the kernel of this equation, ASF conclude that a factored solution is admitted, viz,

$$A(s, \text{other variables}) = s^{\alpha} \phi_{\alpha}(\text{other variables})$$

where other variables are the off-shell squared momenta of the external lines (we still have a forward amplitude here, so $t = 0$). This factorization is the crucial step in the ASF analysis. It says that the multiperipheral model yields the same power law behavior manifested in two-body Regge theory. Power law behavior is obtained by ASF without any mention of the j -plane.

The integral equation for A becomes, via the factorization, an eigenvalue equation; $\{\phi_{\alpha_i}\}$ are some eigenfunctions, and $\{\alpha_i\}$ are the corresponding eigenvalues. The multiperipheral model therefore provides a scheme for computing the exponent of the power behavior of the total cross section for large s ! All one need do is find the leading eigenvalue α_0 .³ In Regge language, the multiperipheral model yields the $t = 0$ intercepts of the appropriate Regge trajectories.

Unfortunately, the eigenvalue equation is a mess on account of the Volterra aspect resulting from the oddly shaped elastic phase space domain,

$$\phi_1(u) = \frac{1}{16\pi^3} \int ds_0 A^R(s_0) \int_0^1 dx x^{\alpha} \int_{x[u + \frac{s_0}{1-x}]}^{\infty} du' \frac{\phi_1(u')}{(u' + \mu^2)^2}.$$

If the lower u' endpoint is approximated by $u' = 0$, then $\phi_\alpha(u)$ is independent of u , the equation is easily soluble, and the eigenvalue obtained for α_0 is the same as that obtained by the naive leading ln approximation, namely, $\alpha_0 = -1 + \text{integrated } \sqrt{s} \pi \text{ resonance cross section.}$

In a model similar to that of ASF, Goldberger⁵ touches on the same problem of calculating the leading eigenvalue α_0 . In his model, which is slightly simpler than that of ASF, the $2\pi \rightarrow 2\pi$ blobs are torn apart and the remaining vertex functions are replaced by constants. Thus, Goldberger's amplitude is this,

$$T_{2 \rightarrow 2} = \text{diagram with 4 external lines and a central blob with 4 internal lines.}$$

His integral equation is given by,

$$\text{diagram with blob A and 4 external lines} = \text{diagram with blob A and 4 external lines, with internal lines} + \text{diagram with 4 external lines and no blob.}$$

Actually, he defines $\tilde{A} = A - (\text{pole term})$ and works with this equation instead,

$$\text{diagram with blob } \tilde{A} \text{ and 4 external lines} = \text{diagram with blob } \tilde{A} \text{ and 4 external lines, with internal lines} + \text{diagram with 4 external lines and no blob.}$$

The question of the $u' = 0$ endpoint of course comes up again. The problem of handling this endpoint was investigated by Goldberger's student Saxton who did the following manipulation that J.D. Jackson would have certainly enjoyed. Saxton kept the exact phase space endpoint $u' = u(x - \frac{m_0^2}{1-x})$ and transformed the $A(u,s)$ equation to the l -representation via a Mellin transform (all at large s). Next, he applied an obscure differential operator to his $a(u,l)$ integral equation, converting it

into a hypergeometric differential equation and taking care to match the right boundary conditions. He then identified the leading pole of the hypergeometric solution as $\alpha_0 = -\frac{3}{2} + \sqrt{\frac{1}{4} + \frac{g^2}{16\pi^2}}$ and ^{concluded} that setting the u^* endpoint equal to zero has the same effect as making the "weak coupling approximation", ie, $g \ll 1$, so that $\alpha_0 \approx -1 + \frac{g^2}{16\pi^2}$. This g^2 term corresponds to the ASF integrated $\pi\pi$ resonance cross section.

Getting back to the ASF program, they proceed to calculate the multiplicities (ln s), inclusive spectra (central plateau), and non-forward elastic amplitude that follow from their model. Their calculation of $A(s,t) = \text{Im } T(s,t)$ follows the same lines as their procedure for getting $A(s,0)$.^{*} The full complex elastic amplitude is obtained from the usual fixed-t dispersion relation and the result coincides with the standard Regge form,

$$A(s,t_1,\dots) = C(t) S^{\alpha(t)} \phi_\alpha(t_1,\dots)$$

$$T(s,t) = C(t) S^{\alpha(t)} \left[i - \frac{\cot}{\tan} \left(\frac{\pi \alpha(t)}{2} \right) \right]$$

Finally, at the end of their paper, ASF observe that their amplitude A is not self-consistent with respect to direct channel unitarity, which states that,

$$\text{Diagram with circle A} = \text{Diagram with circles T and } \pi^0 \text{ in series} + \text{other stuff.}$$

The ^{unitarity} elastic term should exist in . But, if  goes as s^α , then the elastic unitarity term goes as

$$\text{Diagram with three circles in series} = \int dg S^g \rho(t,g) \equiv A_1(s,t)$$

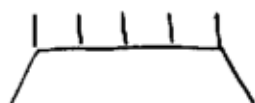
A continuous superposition of Regge poles $l = \xi$ is a cut. It is this

^{*}The function C(t) is positive definite in the ASF model, therefore non-vanishing. It leads to the l=0 ghost difficulty.

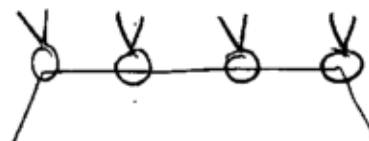
cut in angular momentum that is perhaps considered the most important element of the ASF paper and is always referred to in the literature as "the well-known ASF cut." The cut is really independent of the multiperipheral model and merely states that the iteration of two regge poles is a cut. ASF themselves observed that the elastic unitarity term has, in this model, four particles in the t-channel intermediate states. This situation could never arise in non-relativistic potential scattering and that is why no cuts appear in potential analysis.*

5. Reggeization of the links.

So far we have considered two versions of the multiperipheral model,



and



where the links are always pions. Several authors have suggested replacing the fixed spin pions with variable spin reggeons. In the peripheral model, this replacement becomes two-body Regge theory,

$$\begin{array}{c}
 \text{Diagram: A vertex with two incoming lines and one outgoing line labeled } \leftarrow t, l \\
 \uparrow \\
 A_l \sim \left[\delta_{l,0} \cdot \frac{1}{t - m_\pi^2} \right]
 \end{array}$$

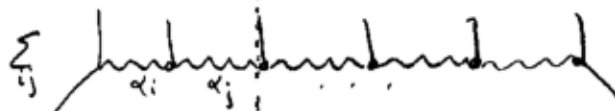


$$\begin{array}{c}
 \text{Diagram: A vertex with two incoming lines and one outgoing line labeled } \leftarrow t, l \\
 A_l \sim \left[\frac{e^{-[l - \alpha(t)] \xi(t)}}{\alpha(t) - l} \right] **
 \end{array}$$

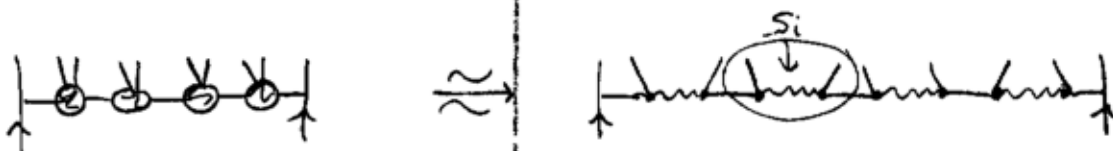
*There is some discussion in Collins and Squires⁶ page 131 that the ASF cut is actually cancelled by some higher unitarity graphs, but then reappears under consideration of so-called non-planar diagrams. This work of Mandelstam leaves me mystified.

**Here I have for fun written the Khuri-Jones form of the partial wave amplitude as described in C&S.

The Reggeists, then, would like to see the following multiperipheral amplitude,

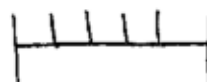


There is, however, a slight problem! The whole point of the multiperipheral ~~model~~ decomposition, as expounded by ASF, is to have low subenergies and to make use of the direct resonance, low energy $\pi\pi$ cross section as a kernel. But here is what the Reggeists are suggesting:



The direct channel $\pi\pi$ low energy absorptive parts are being replaced by the leading cross-subchannel Regge poles as if the subenergies were very large. This clearcut travesty is justified on the basis of Dolen, Horn, Schmidt duality which says (I think) that the leading cross channel Regge poles do describe the low direct resonance amplitudes in an average way and are therefore an acceptable substitute for the direct channel resonances. ⁸,*

What, then, are the implications of these reggeon links? First, they allow an estimate of the short range correlations. The straight chain appears to have no correlations,



but in the Chew-Pignotti model (reggeon link model), a correlation develops (see Chew Goldberger and Low⁹),

* What happened to the background integral? I thought it contained the resonance information.



$$G(p_{i-1}, p_i, p_{i+1}) = \beta(p_i^2, p_{i+1}^2) [(p_{i-1} - p_{i+1})^2]^{\alpha_{in}(p_i^2)}$$

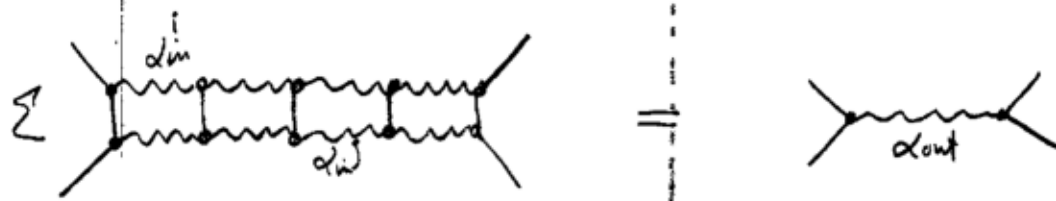
This correlation extends only one link in either direction along the chain. The full amplitude for the chain is then

$$f(\text{end}) G(p_0, p_1, p_2) G(p_1, p_2, p_3) \dots G(p_n, p_{n+1}, p_{n+2})$$

Of course the ASF model also had one-link correlations arising from the $\pi\pi\pi$ blobs. The slight difference is that in the reggeon case, there are two ways to view the chain, so the correlations go both ways,

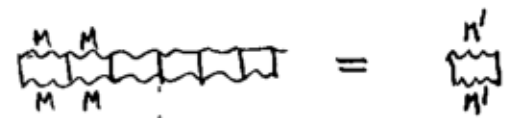
$$\begin{aligned} \text{Diagram} &= \{ \text{Diagram 1} \dots \} + \{ \text{Diagram 2} \dots \} \\ &\equiv \{ \text{Diagram 3} \dots \} \end{aligned}$$

The second implication of reggeons as links, and the driving motivation for using them as links in the first place, is the possibility of "bootstrapping" these reggeons. The "input" reggeons are the links, and the "output" reggeons are the multiperipheral ladders. The self-consistency integral equation is something like,



$$\alpha_{in} = \alpha_{out}.$$

Chew and Pignotti attempt such a bootstrap calculation.⁹ They allow the pomeron and one ~~symbolic~~ representative meson trajectory in their model. Long segments of pure meson ladder are found to telescope down to one renormalized rung,



Since the pure meson chain does contribute to the result, and since the result must respect the Froissart bound, they find an upper bound on their vertex coupling constant, $g_m^2 \leq 2(1-\alpha_m)$, where α_m is the "bare" input trajectory.

Since all meson chain segments can be replaced with an M' rung, the entire problem of the pomeron contributions ^{is reduced} to a consideration of chains with ^{an} alternating PMPMPM... structure. Although Chew and Pignotti consider these contributions, they fit the pp inelastic data without them.

The Froissart result $g_m^2 \leq 2(1-\alpha_m)$ is pinned to the equality by the requirement of constant or near-constant total cross section (ie, $\alpha'_m = 1$). A fit to $\sigma_{tot, inelastic}^{pp}$ in the range 10-30 GeV gives the results $\alpha'_m = .43$ and $g_m^2 = 1.14$.*

6. Application to Oscillations in σ_{tot}^{pp} .

As discussed in Horn⁴ page 212, the multiperipheral equation

* Other bootstrap efforts have placed the pomeron at $\alpha_p(0) = .97 \pm .01$ and $\alpha'_p(0) = .47 \pm .09$. (Discussed in references 11)

if written in the strong-ordering limit with no off-shell mass dependence on any of the lines, takes the simple form, *

$$\left\{ \begin{array}{l} \star \rightarrow \text{blob } A = \text{blob } A_2 \text{ --- blob } A + \text{blob } A_2 \\ A(S,t) = \beta_A \beta_B A_2(S,t) + \frac{1}{\pi} \int_{-1}^S \frac{ds'}{s'} A_2\left(\frac{s}{s'}, m^2, t\right) A(s', t) \end{array} \right.$$

The driving term A_2 is a Born term, like $\pi g^2 \delta(s-m^2)$. This equation has the delightful property of becoming a trivial algebraic equation under the Mellin transform. Thus, in the notation of reference 12, we have,

$$F(s) = F_0(s) + \int_{-1}^s \frac{ds'}{s'} K\left(\frac{s}{s'}\right) F(s')$$

and

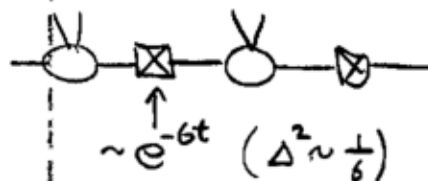
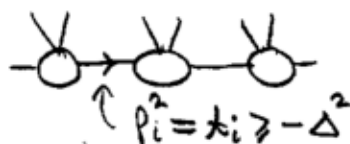
$$f(j) = f_0(j) \times \left[\frac{1}{1 - k(j)} \right],$$

where f and k are the Mellin transforms of F and K .

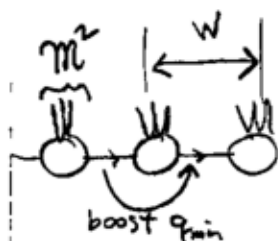
This simple multiperipheral equation is one ingredient necessary for the oscillation prediction. The other is the presence of $N\bar{N}$ thresholds within the central blobs in the chain.

In a multiperipheral amplitude, the links are usually endowed with cutoffs in their momentum transfers. The ~~the~~ cutoffs must actually reside in the adjacent blobs, although sometimes a special cutoff blob is inserted directly into the link itself,

* I have the feeling that there is a hidden assumption here about $A_2^j(s,t)$ being a factorizeable kernel in the diagonalized (t -channel partial waves) integral equation, but I do not know where it fits in.



This cutoff directly implies a minimum rapidity gap separating two blobs on the chain. It also implies a minimum value for the vertex boost q around a blob. This minimum is given by,



$$\cosh W \equiv \cosh(y_2 - y_1)_{\min} \approx 1 + \frac{m^2}{2\Delta^2} = \min[\cosh q]$$

Thus, a multiperipheral blob is "born" into the chain at this minimum size characterized by the boost q . Such a birth is rather naturally interpreted as a production threshold of some sort.

Chew and Snider propose that it is the $N\bar{N}$ threshold that is making its appearance.¹² With the assumption that the first baryon-antibaryon pair occurs in the center of the chain (not on the end), and taking a rather simple kernel,

$$K(s) = \begin{cases} \delta(\text{a real constant}) & \ln s > w \quad (Y > q_{\min}) \\ 0 & \ln s < w \end{cases}$$

they make the following simple calculation,

$$\begin{aligned} k(j) &= \int_1^\infty dx \, \bar{x}^{j-1} [\delta \Theta(\ln x - w)] \\ &= \delta \int_w^\infty dx \, \bar{x}^{j-1} = \delta j^{-1} e^{-wj} \end{aligned}$$

Thus, the partial wave amplitude is,

$$f(j) = \frac{f_0(j) \cdot j}{j - \gamma e^{-w j}}$$

where

$$f_0(j) = \gamma_0 s_0^{-j} \quad \text{since} \quad F_0(s) = \gamma_0 s_0 \delta(s - s_0)$$

The poles of this amplitude are at values of α which solve:

$$\alpha = \gamma e^{-w \alpha} \Rightarrow \alpha^* = \gamma e^{-w \alpha^*}$$

Since γ and w are real, it is clear that all complex solutions occur as conjugate pairs.

In order that the leading pole be at $j = 1$, we have

$$1 = \gamma e^{-w} \Rightarrow \gamma = e^w$$

so the spectrum of regge poles is defined by:

$$\alpha = e^{w(1-\alpha)}$$

Chew and Snider take the value $w = 1.5$ and find the following poles:

$$\begin{aligned} \alpha_0 &= 1 \\ \alpha_1, \alpha_1^* &\approx .23 \pm 3i \\ \alpha_2, \alpha_2^* &\approx -.2 \pm 6i \end{aligned}$$



The effect of a pair of complex poles on the total cross section is,

$$\begin{aligned} \sigma_T &\approx A s^{\alpha_0-1} + \underbrace{\sum_{i=1}^{\infty} (\alpha_i s^{\alpha_i} + \alpha_i^* s^{\alpha_i^*})}_{= 2 \sum |\alpha_i| s^{\text{Re}[\alpha_i]-1} \cos[\text{Im}(\alpha_i) \ln(s) + \phi_i]} \\ &= 2 \sum |\alpha_i| s^{\text{Re}[\alpha_i]-1} \cos[\text{Im}(\alpha_i) \ln(s) + \phi_i] \end{aligned}$$

Thus the poles α_1 and α_1^* will have the leading influence in the second term since they have the largest real parts. This term will oscillate

in the variable $\ln s$ with a period

$$\Delta(\ln s) = \frac{2\pi}{\text{Im}(\alpha_1)}$$

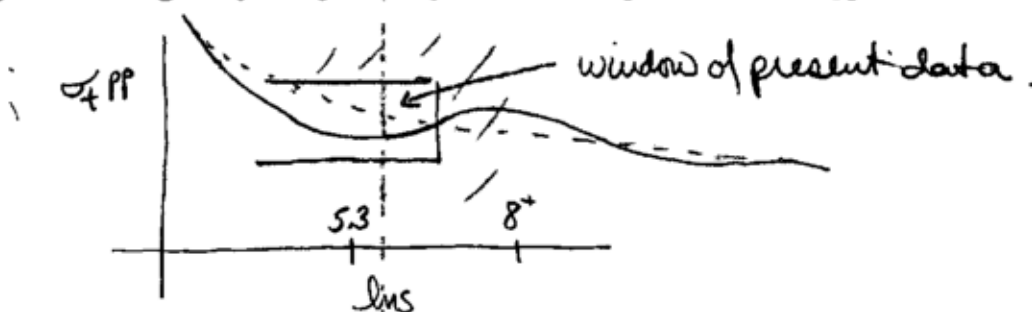
Using the approximation $\text{Im} \alpha_1 = \frac{3\pi}{2w}$ we find,

$$\sigma_{\pi}(s) \approx A \bar{s}^{\epsilon} + B \bar{s}^{[\text{Re}(\alpha_1)-1]} \times \cos\left(\left[\frac{3\pi}{2w}\right] \ln s + \phi\right)$$

Then with the α_1 obtained from $w = 1.5$ we get

$$\Delta(\ln s) \approx 2$$

The whole point of this analysis is that the rising total cross sections may be rising only temporarily. The full picture is supposed to be this:

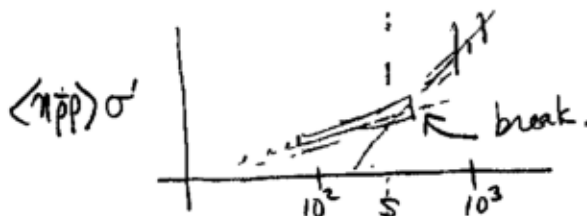


Chew and Sniders' paper was written before the p-p rise was discovered. They were thinking more in terms of the rising K^+p and π^+p total cross sections. (Has the NAL data come in on this? I notice these experiments among those listed in the approved NAL proposal No. 104, Kycia et al..)

There are a few papers which do mention oscillations and $n\bar{n}$ thresholds ~~for~~ the p-p data. Joel Koplik¹³ repeats the Chew-Snider argument, downplays the triple pomeron contribution to the rise, but

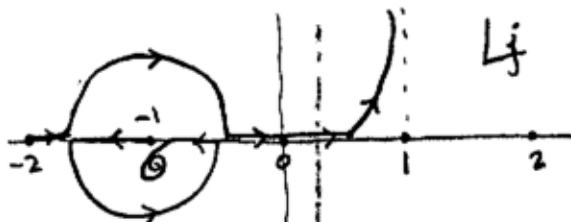
postpones a fit to a later publication now in the works.

Gaisser and Tan look at the ISR inclusive $\bar{p}$ data¹⁴



and, being careful to include non-multiperipheral contributions to σ_{pp}^{tot} , try two separate fits. Their question is whether to interpret the break in the data as the first or second $n\bar{n}$ threshold. They seem to lean toward the former conclusion. Their rather broad fits are shown on the last page of this report.

As a final remark, we note that complex pole pairs are fairly common in Regge theory. Bali et al show that complex pole pairs in the left half j -plane do very exciting things.¹⁵ In a series of truly exotic illustrations suggestive of a Nyquist daydream they analyze the Regge trajectories of a dual* Yukawa potential in various cases. When the coupling is weak, the poles zoom around in the left half plane, ~~always~~ ^{often} in complex pairs, but they never penetrate $\text{Re}(j) = -\frac{1}{2}$. When the coupling is sufficiently strong, one (or more) of the poles breaks into the right half plane and becomes a conventional Regge Pole exhibiting a bound state at $l=0$ (all spinless) and a threshold breakaway into the first quadrant. Here is an example taken from their paper:



* not Veneziano dual but dual in that there are two Yukawa potentials added together, one attractive and one repulsive.

Gaussian + Tan like
to σ_{pp}

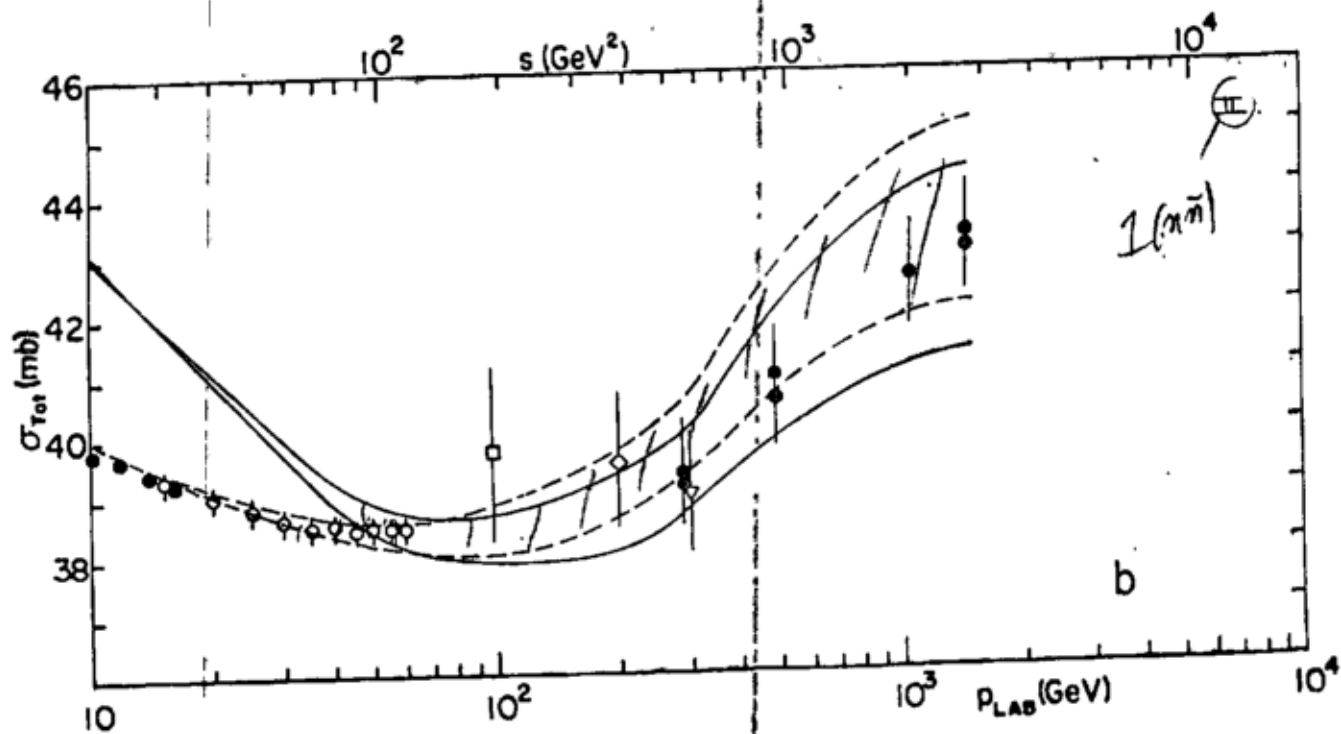
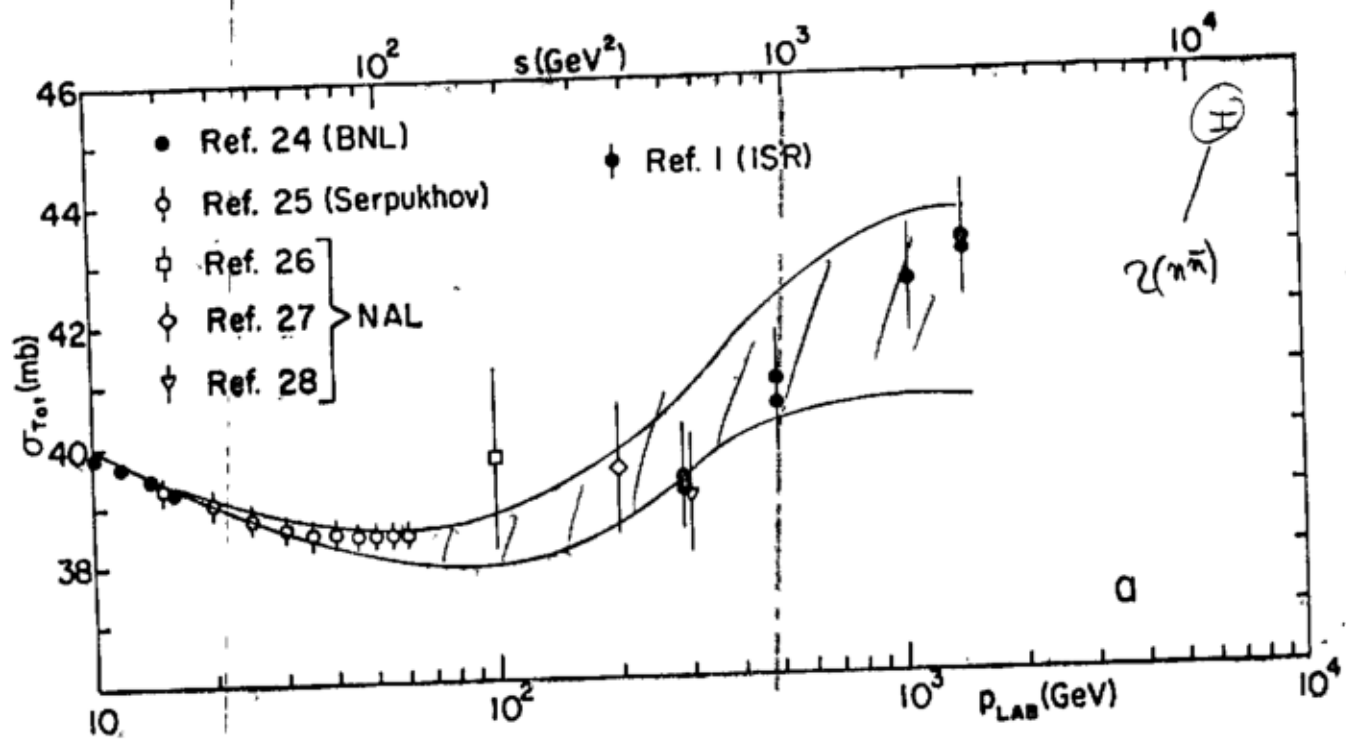


FIG. 2

FIG. 1

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- Chew Low 1959 — relate $\pi N \rightarrow \pi N$ to $\sigma(\pi N)$ via OPE
- Trieman Yang 1962 — azimuthal uniformity test of OPE
- Ferrari Selleri 1961, 1963 — add form factors to get cutoff
- Amaldi Selleri 1964 — explicit form factor
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- Hearn Drell 1967 — phases
- Clegg 1964 — angular momentum barrier, kinematics provide form factor
- Durr Pilkuhn 1965 — add some spin structure
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