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SECRET SYMMETRY:

AN INTRODUCTION TO SPONTANEOUS SYMMETRY BREAKDOWN AND GAUGE FIELDS*

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SECRET SYMMETRY:
AN INTRODUCTION TO SPONTANEOUS SYMMETRY BREAKDOWN AND GAUGE FIELDS*

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1. INTRODUCTION

Here are some long-standing problems in particle theory:

1) How can we understand the hierarchal structure of the fundamental interactions? Are the strong, medium strong (i.e., $SU(3)$ -breaking), electromagnetic, and weak interactions truly independent, or is there some principle that establishes connections between them?

2) How can we construct a renormalizable theory of the weak interactions, one which reproduces the low-energy successes of the Fermi theory but predicts finite higher-order corrections?

3) How can we construct a theory of electromagnetic interactions in which electromagnetic mass differences within isotopic multiplets are finite?

4) How can we reconcile Bjorken scaling in deep inelastic electroproduction with quantum field theory? The SLAC-MIT experiments seem to be telling us that [the light-cone singularities in the product of two currents are canonical in structure;] ordinary perturbation theory, on the other hand, tells us that the canonical structure is spoiled by logarithmic factors, which get worse and worse as we go to higher and higher orders in the perturbation expansion. Are there any theories of the strong interactions for which we can tame the logarithms, sum them up and show they are harmless?

Enormous progress has been made on all of these problems in the last few years. There now exists a large family of models of the weak and electromagnetic interactions that solve the second and third problem, and we have discovered a somewhat smaller family of models of the strong interactions that solve the fourth problem. As we shall see, the structure of these models is such that we are beginning to get ideas about the solution to the (very deep) first problem; connections are beginning to appear in unexpected places, and a optimist might say that we are on the road to the first truly unified theory of the fundamental interactions. All of these marvelous developments are based upon the ideas of spontaneous symmetry breakdown and gauge fields, the subjects of these lectures.

Honesty compels me to moderate the sales-pitch of the last paragraph

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Index for Notes filed under Coleman paper

- 1) summaries of all 6 sections, all together.
- 2) proof of feynman rules $\mathbb{L}$, (4.35) and (4.48) etc.
- 3) proof of why $W(J)$ is sum of connected graphs
- 4) notes on functional derivatives
- 5) derivations of equations in Section 4 (path integrals, $Q(x)$, etc)
- 6) about derivative couplings, $(\det K)^{\frac{1}{2}}$, (4.56) and (4.61)
- 7) derivation of (5.4) and (5.26); the Fadeev Popov business in QED
- 8) derivation of the renormalization group equation.

"Secret Symmetry: an introduction to SSB and gauge fields."

1. Introduction.

1. Out of all this stuff comes several renormalizable theories of weak interactions. The problem is now to do second order experiments to decide which theory is correct.

2. Certain topics will not be mentioned: fermions will be ignored; renormalization will be mostly ignored; (subject of Veltman and 't Hooft). The generalized Ward identities called Slavnov Identities will be glossed over.

2. Classical Field Theory: SSB meets Gauge Theory.

2.1 The idea that if you live in a world in which the vacuum is superconducting, you might have lots of trouble realizing that the Hamiltonian in fact has an exact symmetry. Little man in the ferromagnet. The symmetry of the Ham. is just hidden (secret) because the vacuum happens not to have the symmetry. In general, ground states often do not carry the full symmetry of the lagrangian, eg, particle of spin not equal zero at rest.

First the usual parity symmetry is considered with Goldstones humped curve. By putting the vacuum at the minimum of the potential so that $\phi = a$ is the constant field which gives you the lowest value of H (ground state), you have to shift the fields in order to get perturbation sense. New lagrangian has lost the parity symmetry, because the vacuum broke it spontaneously. This is example of a discrete symmetry breaking.

Next he does the usual $O(2)$ example. Two scalar fields A and B are converted to fields ρ and θ in obvious way. After shifting the fields, massless θ particle appears, the Goldstone Boson. Obviously this is because there is no curvature along the trough.

2.2 For general Lie group case, you start with N generators. Suppose vacuum has $n-m$ dimensional flat surface through your chosen value. Then there will be $N-m$ Goldstones because that many directions to go flat. Vacuum still has m unbroken symmetries. Theorem was proven: a field theory which is covariant and has positive-def metric must generate goldstones when you SSB. Gauge theories, however, do not satisfy these postulates. You either lose covariance or you are stuck with ghosts. Either condition violates the assumption of a normal field theory. QED is not a normal field theory, it is a gauge theory.

2.3 Higgs idea is to "elevate" your global symmetry to a local symmetry by adding some vector mesons. Then by choosing gauge functions $f(x)$ in certain simple way, you can gauge away the massless Goldstones; Higgs mechanism! Their life goes into the vectors and makes them massive.

So Higgs solves two problems at once. In gauge theory, you were always stuck with massless vector mesons to get a locally gauge invariant lagrangian. In SSB theory you were always getting stuck with massless Goldstones. When you combine the two theories, the two things combine and remove all massless vectors except the photon, if you do it right.

OK, the next step is to see what all this has to do with quantum field theory. So far has been classical field theory.

3. Secret Renormalizability.

3.1 Outline of what we are going to do here. Peculiar ordering of arguments.

3.2 Renormalization reviewed. Consider a simple ϕ^4 lagrangian. In computing the 4-point function to second order you have to add up graphs like those shown in figure. In order to block divergences, you put on an ultraviolet cutoff. Then all graphs are finite and you get (3.2). Now go back and insert cutoff-dependent counterterms in lagrangian to exactly cancel the log term. You have successfully renormalized your ϕ^4 theory.

In general, a renormalizable field theory is one in which you add a finite number of cutoff dependent counterterms and you are able to balance the theory in each order in this way. Sidney would like to go further and require that the counterterms have the same form as terms already in the lagrangian, so by adding counterterms you are really only changing parameters in the theory.

A theory ϕ^n with $n = 5$ and up is definitely not renormalizable, ie, you will keep needing more and more counterterms of higher and higher order. "disgusting".

Vector mesons present a special problem in renormalization theory. Reason is that the propagator has that extra $k_\mu k_\nu$ term in it. If meson is massless, you are OK. If meson is coupled to a conserved current like the photon field (which is also massless), again you are OK. But a massive vector meson tends to screw you up. However, later we will see that if it got its mass by SSB, then does not count as a massive meson from renorm point of view.

3.3 Functional Methods and Effective Potential. (refs given) Writes down the connected generating function which they call $W(J)$, Abers and Lee call $Z(J)$. Show the functional relation between this generating function and the theories greens functions, ie, VEV's. The first derivative of the generating function is a VEV of just one field. This is the classical field, and this is the great connection between the quantum and classical field theories. See (3.12).

There exists a similar generating function Γ called the Effective Action. When you take functional derivatives of this thing with respect to the classical field ϕ_c , you get the "proper greens functions" of the theory. (one -article irreducible etc.)

When the effective action is expanded in powers of external momenta and converted to coordinate space, the first term is the "effective potential". (This classical function of the classical field is interpreted later on).

So, out of the depths of the quantum field theory Sidney has extracted a classical field, and a classical potential. Now that you have your hands on these guys, you can plot V against ϕ and look for minima and see if you are in the SSB situation. You can then proceed to do everything you did before in the classical analysis.

*Pushover
info here.*

3.4 The loop expansion. Now the problem is how do you compute the effective potential if you are given a lagrangian? Somehow there is a way to compute it. The first contribution is U itself, ie, the potential which appears in the lagrangian. But then there are contributions from loops. If coupling is weak, you could presumably add things up to get an exact result for this effective potential V . Then you could take its derivatives to get the proper vertices, and so on.

3.5 Sample calculation. He writes down a formula for the first correction term. The correction term diverges UV , so you put in a cutoff. But these are just the kinds of terms which you can get rid of by adding counterterms to the lagrangian, specifically, adding counter terms into the potential. In a non-renormalizeable field theory, you are just plain stuck at this point. You cannot come up with a finite value for V , thus you can never plot V versus ϕ_c to look for a superconducting vacuum.

3.6 Most important part of lecture. The point is simply this: SSB does not affect the renormalizeability of a theory. After SSB theories may "look" non-renormalizeable because, eg, there are massive vectors coupling to non-conserved currents. However this is an illusion. The real Lagrangian is the thing you have before you do SSB which has massless vectors coupling to conserved currents. In Sidney's words: "secret symmetry" buys you secret renormalizeability.

3.7 Meaning of the effective potential. In argument I don't quite follow, he shows that $V(\phi_c)$ is precisely what it is in classical field theory, ie, the energy density. Imaginary part is related to chance of decay of the state in question.

3.8 How big is loop correction? In certain theories, especially gauge theories, the loop contribution to V may dominate over the tree contribution, because the loop allows new couplings to be used. I don't get the point of the $SO(5)$ business here.

3.9 Alternative approach. Sidney takes view of doing a field theory corrections first getting the renorm done, then doing the fields shift for SSB. Most people really do things in the other order, but then the logic is not as clear.

4. Functional Integration.

4.1 Integration over infinite dimensional spaces. Starts with a one-dim gaussian integral. Extends the idea to finite dimensional space where expo is now diagonal matrix element of a finite-dim operator. Actually, expo is a quadratic form in the integration variables. But you write it as an inner product. Then make the leap to infinite dimensions. The integration and determinant that you get are defined by the limit only. In the limit, A will be an operator, still must be real symmetric I think.

4.2 Functional Integrals and Generating Functionals. Defines $e^{iW(J)}$ as path integral (fields only) of the expo'd action. Just a definition. You want to actually do this integral using methods learned in last section. But, as a function of the fields ϕ , the integrand is not decaying so integral is not defined. The trick is to rotate the k^0 and x^0 components to get the Euclidean action shown in (4.30), which is now positive definite so you can do the integral. Identifying the operator A as in (4.31), you can explicitly do the integral to get (4.32). You now have the generating function not as a path integral of an expo, but as a simple expo of a double spacetime integral. Of course you knew the answer all the time was (4.28), so you have accidentally found the determinant. The point is that this is a method for actually doing the path integrals. It does not tell you how to figure the determinant, however.

The above example was a free-field scalar example. Next, allow an integration as shown in (4.35). The goal here is again to evaluate the path integral. First, you write it in the form (4.38) [note" just interchange order and apply little theorem to see that (4.38) is the same as LHS]. But the RHS of (4.38) is also obtained directly from the Dyson equation as in (4.36). Thus, in effect you have done the integral without ever doing it. You are making use of the free-field case which you already proved.

4.3 Feynman Rules. The Feynman rules are already explicit, it seems to me, in the Dyson formula (4.35) for the generating function. However, there you do have to take contractions and worry about Wick's theorem and so on. By using the little identity (4.44) which I have proved, you can come up with another version of the Feynman rules implicitly stated in (4.48), where your ϕ derivatives act as pincers and reach into the mess and tie together fields to make propagators. Wick is automatic here, although normal ordering is not automatic.

For general field theory (including derivatives) if you could only write the generating function in the path integral form, then you could use the above method to read off the Feynman rules of your theory in a naive and correct way. You do not have any problems with pushing time derivatives through time ordered products and all that mess.

However, up to this point our formalism has always assumed that $L = L(\phi)$ only, no derivatives, eg, as in (4.38) or (4.36). In gauge theory you must know how to deal

with derivative interactions, so let's now go do this in next section I hope.

4.4 Derivative Interactions. If the full lagrangian is at most quadratic in time derivatives and has no spatial derivatives well, I am not sure of the status of these spatial derivatives. A regular scalar free lagrangian has spatial derivatives in the kinetic energy part. Maybe he means this: 'integrate the lagrangian density..? No. He is simply not saying anything about the spatial derivatives here, so I don't know what the conclusion is. In a certain case, I don't know precisely what case it is, you can handle some derivatives in your lagrangian. The only effect on the functional integral representation is to add a certain $\det K$ factor in there as shown.

Amusingly, the form of this generating function is coordinate independent.

Also, it is possible to write the thing as an integral over both coordinates and momenta, as in Abers and Lee. Then you can see that the $\det K$ thing comes from integrating first over the dp stuff. Coleman does not like the full $dqdp$ form of the path integral.

4.5 Fermion fields. Very heuristic section here. Assuming fermion fields enter quadratically, you would like to be able to do a path integral of the form (4.65). The correct answer turns out to be (4.67), where $\det A$ is inverted from what it was in the boson case. This inversion has something to do with fermi statistics, but I don't follow the heuristic argument. I think e^{iW} is sum of bubbles, not W as he claims.

But I will accept (4.67) as the presumed answer for the path integral. If you really want to know this, have to study anticommuting c-number fields.

4.6 Ghost fields. Right away we are now going to make use of the last section. When we had derivative interactions, you are stuck with a big $\det K$ sitting in the integrand. You cannot set up feynman rules unless you get that $\det K$ into the exponent somehow. A trick for doing that is to write $\det K$ as the path integral of some fake fermi fields with K as the operator in the exponent. I.e., you make up dummy variable fermi fields and write (4.58). These are called ghost fields. Your new lagrangian gets modified in the obvious way shown in (4.69) and (4.70). You have to include the ghost fields in your feynman rules, I guess, as if they were true fields.

As an example of ghost fields, suppose you take a trivial lagrangian of ϕ and define a new variable A as shown in (4.72). Obviously this cannot change the theory. But, in doing this redefinition, you have introduced derivative couplings, and therefore you will have a K and $\det K$ floating around. Thus, in order to get correct Feynman rules in terms of the new field A , you have to introduce some ghost fields η to get the thing into standard form where you can then read off the feynman rules. So even a simple change of variable will require you to use ghost fields. I wonder what BD had to say about dealing with derivative couplings?

Section 5: the Feynman Rules for Gauge Theories. (Memory of Fadeev + Popov)

1. Troubles with Gauge Invariance. We open with a simple example: take QED and try to read off the photon propagator in the usual functional way. What this means is: look at the action S and identify the operator which is quadratic in the fields, in the sense of (4.51). The greens function of this operator is the propagator of interest, as in (4.52). The problem with QED is immediate: the operator in k -space is a projection operator and is therefore non-invertible. Thus, the propagator is singular, as schematically indicated by (5.5).

The reason for this problem is as follows: (1) because you have a gauge group, if you integrate over all values of the field, you have overcounted; (2) you should canonically speaking, fix a gauge from the start and proceed therefrom. I.e. find out which fields are not true variables and eliminate them. In fact, both ideas are the same as is later shown.

5.2 The Fadeev-Popov Ansatz. Just don't integrate over the non-independent fields! This idea is first expressed in (5.7). Since your action is gauge-invariant, you can add the extra constrained variables with a delta function as shown in (5.11). Finally, you can write the surface constraint in the general way of (5.13), where now you must add that determinant since you have changed variables in your delta functions. I.e., by choosing a gauge, we restrict integration to some multi-dimensional surface $F=0$. Very simple. The variables y_i (upon which the action does not depend) are the gauge functions. Shift your fields with a gauge transformation, and your action will not change at all, hence (5.6). So the result is summarized in (5.15). Fine.

Colman has shown that the integral is same regardless of what surface you restrict to. Each surface represents a different choice of gauge. Thus, the generating function is the same in all gauges. So you only have to prove it correct in one gauge.

5.3 Application of FP Ansatz. Let's go back and redo QED. Fix the gauge by requiring that $F=0$ as in (5.17). This happens to be a covariant gauge choice. Then do a little trick to get rid of the delta function in favor of an exponential as in (5.22). This causes a shift in the action, i.e., adds a correction term to the lagrangian as in (5.24). Then, when you include this term, you can go back and recompute your photon propagator. You find that the thing which was not-invertible now is invertible. You get (5.26) and QED is made well again. Of course this same propagator comes out when you properly quantize in this same gauge, although BD don't quite use this gauge.

So, now that we have done QED, let's apply the same idea to a real, non-abelian gauge theory to get out the Feynman rules. This time, when you use the same kind of covariant gauge, the little Faddeev-Popov determinant is not independent of the fields because of the extra terms you always get in the non-abelian theory. So now you have a non-trivial determinant in your generating functional integral. Thus, you must use the trick of ghost fields to exponentiate the thing, as in (5.29). This makes its own correction to the lagrangian, in addition to the other correction arising from getting rid of the delta function. So the fully corrected "effective" lagrangian is given in (5.32). The lesson here is simple: if you want the correct propagator to use in your Feynman rules, you cannot simply ignore gauge invariance. One way or another you must account for it. In the functional scheme, the accounting method is called Faddeev-Popov.

5.4 Justification of the Ansatz. Take same non-abelian gauge theory and go to the so-called axial gauge where you just set all A_3^a components = 0. This is of course very non-covariant, but big deal. It turns out very quickly that in this gauge, the determinant is not field-dependent so you don't have to have Faddeev-Popov ghosts. The FPA predicts (5.35). Now, what does the old canonical quantization method tell us? Go to the same gauge and write out lagrangian. Identify the independent quantum fields. Then write path integral for them, as in (5.41). Then soup this up to get form (5.42), which you then find agrees with the FPA.

5.5 Conclusions: The FPA has now been proved from canonical methods. It provides an extremely easy way to move from one gauge to another! That is its strength.

Section 6 Asymptotic Freedom.

6.1 Operator Products. In deep inelastic, the photon-hadron process is described by structure functions of the two invariants called q^2 , the photon mass, and the Feynman x variable. At large $-q^2$, these functions become independent of variable q^2 and seem to depend only on x . This is Bjorken scaling, how do you explain it? Integrate over x to define moments $F(q^2)$, then want to explain why these moments are constant in q^2 .

(2) For large q^2 in field theory there are some theorems for expanding products of operators, invented by Wilson. Apply this to deep inelastic where the produced operators are the J^μ electromagnetic currents. This gives 6.4, which is not very enlightening. Now want to know why the $F(q^2)$ things are constants. In field theory you can calculate these functions. In fact they do depend on q^2 as shown in (6.5), and are constants only in free field theory. I.e., Bjorken scaling suggests that the field theory is getting free at large q^2 .

But is it possible that if you sum the series shown in (6.5) that you might still get something independent of q^2 ? How can this series possibly make sense for large q^2 ??? And what about large g ?

6.2 Massless Field Theory and Renormalization Group. In high q^2 limit, mass does not matter, seems very reasonable. So why not first work with massless field theories and see what you can do. You need a scale parameter called M , say, which sets the scale of the inverse propagator. I.e., you set the thing at $p^2 = M^2$ or something.

Obviously nothing physical can depend on the value of M^2 . This statement leads to the equation (6.8) which says in effect that $\delta f = 0$ under a change in M^2 . Notice what the various parameters are:

$$\epsilon = \frac{\delta m}{m} \quad \beta = \frac{dg}{g} = m \frac{dg}{dm} \quad \gamma = \frac{\delta A}{A} = \frac{m}{A} \frac{dA}{dm}$$

6.3 Exact and Rough Solution to the RGE. The equation is first order, two-variable partial differential equation, solution is not obvious. General solution is (6.13) where F is any function of a single variable, and where g' is solution of 6.12.

This function $g'(g, t)$ has two arguments. The first is the value of g' when $t=0$. The second argument t is $\ln(Q/M)$. Thus, g is the special value that g' takes when $Q = M$. I think g' is called the running coupling constant, or the invariant charge or something like that. It seems that g' is the coupling constant at some particular Q .

Now assume you know BETA for small g as in (6.14). Then for small g' you can solve for g' as in (6.15). Solution is shown in (6.17). If b is negative, as it was discovered to be in gauge theories, then g' is small as Q goes large! I.e., field theory approaches a free field theory.

6.4 Asymptotic Freedom. This simply means that g' goes to zero at large Q . In principle you can compute from lagrangian what field theory does in this limit by computing BETA.

Argument is extended to claim that result holds even if g is not small.

So, what do you predict for Bjorken scaling in an AF theory? Look back at solution given in (6.13). You get mainly (6.25). Things are not constants but are slow logs, which is consistent with data.

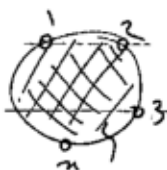
Facts: all gauge theories with only gauge mesons (and no abelian factor groups) are asymptotically free. Fermions added to the theory tend to make BETA more positive, but you can accommodate some fermions. Scalar particles like Higgs bosons are hard to evaluate as to their effect on AF. Also: if you have no gauge fields, theory cannot be AF. Conclusion: if a field theory explains strong interactions and if AF explains Bj scaling, then strong interactions must be a non-abelian gauge theory.

Jim Dec 4, 1977

Interacting Case, (4.35) etc.

- ② (4.35) is obvious because it is a definition of e^{iW} .
Or, if e^{iW} is defined such that derivs = greens, then you prove (4.35) as follows:

$$\left. \frac{\delta^n e^{iW}}{\delta J(x_1) \dots \delta J(x_n)} \right|_{J=0} = N' \underbrace{T \langle 0 | \phi(x_1) \dots \phi(x_n) | 0 \rangle}_{i \int dx' L'} = \text{Dyson-Feynman-Wick formula for sum of all Feynman graphs}$$

$$= N' \times \text{diagram} = N' \text{ Greens}(x_1, x_2, \dots, x_n)$$


- ③ How does (4.36) reproduce (4.35)? Use a theorem proved earlier:

$$F\left(\frac{\int}{i \delta J(x)}\right) e^{i \int dx' J(x') \phi(x')} = F(\phi(x)) e^{\text{same.}}$$

Or, in regular form:

$$F\left(\frac{d}{dx}\right) e^{ax} = F(a) e^{ax}$$

And this you can prove a la Fourier

$$F(x) = \sum F_k e^{ikx} \Rightarrow F(\partial_x) = \sum F_k e^{ik \partial_x}$$

$$\text{So LHS} = \sum F_k e^{a(x+ik)} = e^{ax} \left(\sum F_k e^{iak} \right) = \text{RHS} \checkmark$$

Then, we have $F\left(\frac{1}{i\sigma}\right) = e^{i \int dy L'(-i/2\sigma)}$

So (4.36) does become

$$F(a) = e^{i \int dy L'(q)} e^{i \int dy \phi}$$

⑤ Comment: notice that e^{iW} contains only the L_{int} , so also does (4.30). In contrast, the path integral formulas contain all of L .

Explain why Coleman (4.48) yields Feynman rules.

- ① Feynman Rules give amplitude for a Feynman graph. But Green's Function is a sum of all Feynman graphs. We want first to see how to pick out one graph from (4.48). First, decide how many external legs you want. Let's say 4 as an example. Then here is what you know:

$$\frac{\delta^4 e^{iW[\phi]}}{\delta J(x_1) \delta J(x_2) \delta J(x_3) \delta J(x_4)} \bigg|_{J=0} = N' T \langle 0 | \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) e^{i \int d^4x \mathcal{L}_I} | 0 \rangle$$

$$= \text{[Diagram: A square with diagonal lines]} = \text{[Diagram: Two parallel horizontal lines]} + \text{[Diagram: Two vertical lines]} + \text{[Diagram: A cross]} + \dots (\lambda^0)$$

+ nothing of order λ^1 (I am assuming $\mathcal{L}_I = \lambda \phi^3$) because $\eta = \text{odd}$

$$\text{[Diagram: A cross with a dot in the center]} + \text{[Diagram: A cross with a dot in the center and four external legs labeled 1, 2, 3, 4]} + \text{[Diagram: A cross with a dot in the center]} + \text{[Diagram: Two vertical lines with a dot in the middle]} + \text{[Diagram: A cross with a dot in the center and a loop]} + \dots$$

I.e., I am using Wick's Theorem here. In general, each external leg has one line going to it, each internal dot has n lines in $\lambda \phi^n$ theory. ~~So~~ The disconnected graphs can be ignored, self-energy tadpoles out by normal ordering $\mathcal{L}_I$.

- ② So the above was just a review. Each connected graph at level λ^2 with $N_{\text{ext}} = 4$ consists of 5 Δ_F propagators. Each set of 5 pairs is a particular "contraction".

③ Now how does (4.48) do all this work? First, you take your $\delta\delta$ derivatives to get

$$Z(x_1, x_2, x_3, x_4) = N \exp \left[\frac{1}{2} \int dx dy \Delta_F(x-y) \frac{\delta}{\delta \phi(x)} \frac{\delta}{\delta \phi(y)} \right] \cdot \phi(x_1) \phi(x_2) \phi(x_3) \phi(x_4) \cdot e^{i \int dz \mathcal{L}'(\phi)} \Big|_{\phi=0}.$$

So we first select the # external lines, already done. Next, we select the order of interest. Eg, for λ^2 we keep $[]^2$ term in second exponential, thus

$$Z(1,2,3,4) \stackrel{\lambda^2}{=} N \cdot \exp \left[\frac{1}{2} \int dx dy \Delta_F \delta\delta \right] \underbrace{\phi_1 \phi_2 \phi_3 \phi_4 \int dz_1 dz_2 \lambda \phi^3}_{\text{second exponential}}$$

④ Now you see that there are 10 ϕ -fields here. Each power of $\left[\frac{1}{2} \int dx dy \Delta_F \delta\delta \right]$ connects two fields into a Δ_F . I.e.,

$$\left[\frac{1}{2} \int dx dy \Delta_F^{(x-y)} \frac{\delta}{\delta \phi(x)} \frac{\delta}{\delta \phi(y)} \right] [\phi(x_1) \phi(x_2)] \cdot \text{rest} = \Delta_F(x_1 - x_2) \cdot \text{rest}.$$

Of course the first attack of $\left[\int dx dy \dots \right]$ could grab any pair of ϕ 's. Then the next attack grabs two more, until all the fields are gone. This, the $[]^5$ term of the first exponential is what contributes. Lower powers like $[]^2$ give 0 due to $| \phi = 0$.

Higher powers like $[]^6$ give nothing because all derivatives beyond $[]^5$ vanish because no ϕ 's left!

⑤ Now try to summarize. The operator $[]$ is a machine which grabs two fields and sews them together with a Δ_F . For each Δ_F in your desired diagram, $[]$ must act. But first you must do your δ/δ 's to pull down your desired $\phi(x_i)$ fields.

So the action of the Δ_F sewing machine replaces Wick's theorem, a simpler formalism in fact.

⑥ Now that all this is said, the "Feynman rules" for a $\lambda\phi^n$ theory are: Δ_F for each prop, $i\lambda$ for each interaction along with $\int d^4x$. You must remove tadpoles etc by hand:

Why is $W(\mathcal{T}) = \text{sum of all connected graphs?}$

$$e^{iW[\mathcal{T}]} = T \langle 0 | e^{i \int d^4x (\mathcal{L}(\phi) + \mathcal{J}\phi)} | 0 \rangle$$

So perhaps $W[\mathcal{T}] = T \langle 0 | i \int d^4x \mathcal{L}(\phi) + \mathcal{J}\phi | 0 \rangle = 0$ in ϕ^3 , so not true.

$$e^{iW} = T \langle 0 | 1 + i \int d^4x \mathcal{L}(\phi) + \mathcal{J}\phi + i^2 \int \int \dots + \dots | 0 \rangle$$

We must mean that $W[\mathcal{T}] \sim$ well (3.11) states W .

⊗ Look this up in BD: Discussion is on page 189. First of all, I see this idea:

$$e^{iW(0)} = T \langle 0 | e^{i \int d^4x \mathcal{L}(\phi)} | 0 \rangle$$

$$= T \langle 0 | 0 \rangle + \int \langle 0 | i : \lambda \phi^4 : | 0 \rangle + \int \int \langle 0 | \lambda' : \phi^4(x) : \phi^4(y) | 0 \rangle + \dots$$

$$= 1 + \int + \text{[diagram: two vertices connected by a line]} + \text{[diagram: two vertices connected by two lines]}$$

↑ no, N.O. kills

$$+ \text{[diagram: two separate loops]} + \text{[diagram: two vertices connected by two lines, with a loop on one vertex]} \quad \text{first disconnected vacuum graphs.}$$

So, in $\lambda \phi^4$, $\exp[iW(0)] = \text{sum of all vacuum graphs, connected + disconnected}$

⊗ Let's try to reconstruct the BD argument. Then get it again in the fctd notation.

Consider the Dyson numerators: but only to order λ . Or maybe we want all graphs whose connected part is order λ . Then

$$\begin{aligned} \text{this} &= T \langle 0 | \phi_1 \dots \phi_n \cdot \mathcal{O} \rangle \Big|_{\text{order } \lambda} \\ &= \sum_{p=0}^{\infty} \frac{i^p}{p!} T \langle 0 | \phi_1 \dots \phi_n \int dx_1 \dots dx_p \lambda \mathcal{L} \dots \mathcal{L} \rangle \\ &\quad \int dx_1 \dots dx_p \lambda \mathcal{L} \mathcal{L} \dots \mathcal{L} \rangle \end{aligned}$$

Why are you allowed to insert $|0\rangle\langle 0|$? The fields ϕ_i do not couple at all into the second set of $\mathcal{L}$'s. The second set of $\mathcal{L}$'s all self couple. Now, do things go down λ ? I.e., can we use Wick?

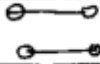
$$\langle 0 | T (\phi_1 \dots \phi_n \mathcal{L} \dots \mathcal{L} \mathcal{L} \dots \mathcal{L}) | 0 \rangle$$

$$= \langle 0 | T (\phi_1 \mathcal{L}) | 0 \rangle \dots \dots \dots \langle 0 | \dots$$

Probably, it's a step along way to Wick, so

$$\begin{aligned} [\text{order } \lambda]_{\text{next from step}} &= \sum_p \frac{i^p}{p!} \int T \langle 0 | \phi_1 \dots \phi_n \lambda \mathcal{L} \dots \mathcal{L} | 0 \rangle \\ &\quad \times \binom{p}{s} \langle 0 | \mathcal{L} \mathcal{L} \dots \mathcal{L} | 0 \rangle \lambda^{p-s} \end{aligned}$$

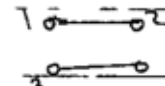
$$= (\text{order } \lambda) (\text{all vac. bubbles})$$

On:  is connected!

What becomes of a graph which is disconnected in this sense?

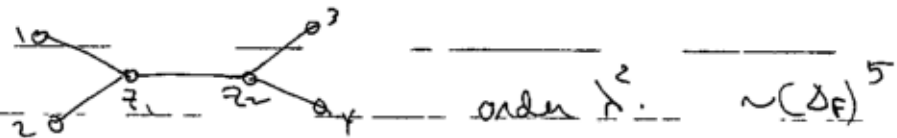
$$T(0 | (d_1, d_2, d_3, d_4) | 0) = \tau(1, 2, 3, 4) = \text{complete greens}$$

$$\equiv T(0 | d_{in}(1) d_{in}(2) d(3) d(4) \in \frac{y}{x_{dE}} | 0)$$

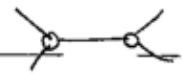
Well, such a graph as  is order λ^0 . The S-matrix contribution is?

$$T(0 | d_1, d_2, d_3, d_4) \equiv \Delta_F(1, 2) \Delta_F(3, 4)$$

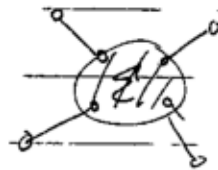
OK, this is a forward scattering. In general, term looks like



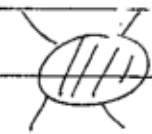
Then you remove all ext pops to get



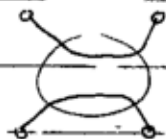
No. (16.81) says:



amplitude to get



Contains all graphs. Every part has an x_i . Well, when you take



and remove 2 pops, you get

$\vec{K} \vec{K}$. s . s .

Probably nothing left.

① So at any rate, "disconnected part" means a ~~graph~~ vacuum bubble. "Connected graph" means no vacuum bubbles

② So, Dyson-Wick says

$$G_n(x_1 \dots x_n) = \frac{\langle 0 | T \{ \phi_1 \dots \phi_n e^{iL} \} | 0 \rangle}{\langle 0 | T (e^{iL}) | 0 \rangle}$$

The top may be written: $\sum_s (\text{connected})_s \cdot \frac{[\text{all vac bubbles}]_s}{[\text{all vacuum bubbles}]}$

So good. Now

$$e^{iW(0)}$$

$$= \langle 0 | T$$

$$\left. \frac{\delta^n W}{\delta J \dots} \right|_{J=0} = \text{connected greens}$$

$$\text{So } \left. \frac{\delta^n W[J]}{\delta J(x_1) \dots \delta J(x_n)} \right|_{J=0} = G_n^c(1 \dots n) = \frac{\langle 0 | T (\phi_1 \dots \phi_n e^{iL}) | 0 \rangle}{\langle 0 | e^{iL} | 0 \rangle}$$

This is (3.11) Coleman + (11.38) BD.

It would then seem that $W[J] \Big|_{J=0} = 1$.

⊙ Look back at Abus+Lee: They say:

$$i^n G(x_1 \dots x_n) = \frac{\delta^n W[J]}{\delta J(x_1) \dots} \Big|_{J=0}$$

So AL say: but G is already only connected, say $\mathcal{G}$.

Well, let us suppose that what AL mean is this

$$\frac{\delta^n W[J]}{\delta J(x_1) \dots \delta J(x_n)} \Big|_{J=0} \equiv i^n \times T \langle 0 | \phi_1 \dots \phi_n | 0 \rangle$$

so that the denom is not there. Then

$$W[0] = T \langle 0 | e^{iS} | 0 \rangle$$

So you could say

$$\frac{1}{W} \frac{\delta^n W}{\delta J \dots} \Big|_{J=0} = i^n \cdot G_n(x_1 \dots x_n).$$

$$\text{Let } W = e^{iZ} \quad \text{Then } \frac{1}{W} \frac{\delta^n W}{\delta J \dots} = e^{iZ} \frac{\delta^n e^{iZ}}{\delta J \dots}$$

So now things are clear. Turn page:

Abram+Lee must be saying:

$$1. \quad \frac{\delta^n W[J]}{\delta J(x_1) \dots \delta J(x_n)} \Big|_{J=0} = i^n \langle 0 | T(\phi(x_1) \dots \phi(x_n) e^{+i \int \mathcal{L}_I}) | 0 \rangle$$

Therefore, this includes disconnected. The parallel statement is

$$2. \quad W[J] = T \langle 0 | \exp [i \int d^4x (\mathcal{L}_I(\phi) + \phi(x) J(x))] | 0 \rangle$$

$$3. \quad \text{and } W(0) = T \langle 0 | e^{+i \int \mathcal{L}_I} | 0 \rangle = \text{sum of all vac bubbles}$$

$$\text{Next, let } W = e^{iZ} \quad \text{then} \quad dW = W \cdot i dZ.$$

$$d^n W = W i$$

$$1. \quad \frac{\delta^n W}{W \delta J \dots} = \frac{\delta^n Z}{\delta J \dots} \quad \text{So:}$$

$$4. \quad \frac{\delta^n Z[J]}{\delta J(x_1) \dots \delta J(x_n)} \Big|_{J=0} = \frac{i^n \langle 0 | T(\phi_1 \dots e^{+i \int \mathcal{L}_I}) | 0 \rangle}{\langle 0 | T e^{+i \int \mathcal{L}_I} | 0 \rangle} = i^n G_n(x_1 \dots x_n)$$

$$\text{then } Z[J] = ?$$

$$5. \quad Z[0] = 1$$

$$\text{Well: } \frac{1}{W(0)} \frac{\partial^2 W(x,y)}{\partial x \partial y} =$$

$$\frac{\partial W}{\partial x} = W i \frac{\partial Z}{\partial x}$$

$$\frac{\partial^2 W}{\partial y \partial x} = W i \frac{\partial^2 Z}{\partial y \partial x} + i \frac{\partial Z}{\partial y} \frac{\partial Z}{\partial x}$$

Relation not yet clear.

$$\frac{1}{W} \frac{\partial^2 W(x,y)}{\partial x \partial y} \Big|_{x=y=0} = 0$$

Let $W(x,y) = e^{iZ(x,y)}$. Then $\frac{\partial W}{\partial y} = W \cdot i \frac{\partial Z}{\partial y}$

$$\frac{\partial^2 W}{\partial x \partial y} = \frac{\partial}{\partial x} \left[i W \frac{\partial Z}{\partial y} \right] = i W \frac{\partial^2 Z}{\partial x \partial y} + i \frac{\partial Z}{\partial y} \cdot W \cdot i \frac{\partial Z}{\partial x}$$

Does not seem to work right: Seems like

$$\frac{1}{W} \frac{\partial^2 W(x,y)}{\partial x \partial y} \Big|_{x=y=0} = i \frac{\partial^2 Z}{\partial x \partial y} + (i^2) \left(\frac{\partial Z}{\partial y} \frac{\partial Z}{\partial x} \right)$$

Well maybe Schrodinger idea is better:

$$\frac{1}{W} \frac{\partial^2 W[\gamma]}{\partial \gamma(x_1) \partial \gamma(x_2)} \quad ?$$

$$W[\gamma] = e^{iZ[\gamma]} \Rightarrow \frac{\partial W[\gamma]}{\partial \gamma(x_2)} = W[\gamma] \cdot \frac{\partial Z[\gamma]}{\partial \gamma(x_2)}$$

$$\Rightarrow \frac{\partial^2 W[\gamma]}{\partial \gamma(x_1) \partial \gamma(x_2)} = \frac{\partial}{\partial \gamma(x_1)} \left[W[\gamma] \frac{\partial Z[\gamma]}{\partial \gamma(x_2)} \right]$$

= two terms + no? go to scratch + return.

Try again, Abers + Lee use:

$$W[J] \equiv \langle 0 | T (e^{i \int d^4x [\mathcal{L}_I + \phi J]}) | 0 \rangle$$

$$\Rightarrow \left. \frac{\delta^n W}{\delta J \dots} \right|_{J=0} = i^n \langle 0 | T (\phi_1 \dots \phi_n e^{i \int d^4x \mathcal{L}_I}) | 0 \rangle$$

= sum of all graphs including disconnected.

$$\Rightarrow W[0] = \langle 0 | T (e^{i \int d^4x \mathcal{L}_I}) | 0 \rangle = \text{sum of all vac. bubbles}$$

$$\Rightarrow \left. \frac{1}{W[J]} \frac{\delta^n W[J]}{\delta J(x_1) \dots} \right|_{J=0} = \frac{(i)^n \langle 0 | T (\phi_1 \dots \phi_n) | 0 \rangle}{\langle 0 | T (e^{i \int d^4x \mathcal{L}_I}) | 0 \rangle}$$

= sum of all connected graphs only.

Notes on the Manipulation of Functional Derivatives

- ① Let $x =$ a set of variables, $f(x) =$ a function of these variables.
Let $F[f]$ be a functional of f . Then you can have:

$$I = \int dx F[f(x)]$$

as a well-defined object. Eg, $F[f] = f^2$ is possible.

- ② Now define

$$\frac{\delta I}{\delta f} = \frac{\delta}{\delta f} \left[\int dx F[f(x)] \right] = \frac{dF[f(x)]}{df} = F'[f(x)].$$

The basic idea is this

$$\boxed{\frac{\delta f(x)}{\delta f(x')} = \delta(x-x')}$$

Thus,
$$\frac{\delta I}{\delta f(x')} = \int dx \cdot \frac{dF}{df} \cdot \frac{\delta f(x)}{\delta f(x')} = \int dx F'[f] \delta(x-x') = F'[f(x')].$$

- ③ To summarize:

$$\boxed{\frac{\delta}{\delta f(x)} \left[\int dx' F[f(x')] \right] = \frac{dF}{df(x)}} \\ \text{functional derivative of integral of a functional} \\ = \text{derivative of fctnl wrt the function.}$$

↑ How to do a first fctnl deriv on an integral.

- ④ Now consider:

$$\int dx \left[\frac{\delta}{\delta f(x)} \right] \cdot \int dx' F[f(x')] = \int dx \cdot \frac{dF}{df} = \int dx F'(x)$$

⑤ Now some specific examples:

$$\begin{aligned} \frac{\delta}{\delta J(x)} e^{\int dx' J(x') \phi(x')} &= \frac{\delta}{\delta J(x)} \sum_n \frac{1}{n!} \left[\int dx' J(x') \phi(x') \right]^n \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} n \left[\int dx' J(x') \phi(x') \right]^{n-1} \cdot \phi(x) \\ &= \sum_{n=0}^{\infty} \frac{1}{(n-1)!} \left[\int dx' J(x') \phi(x') \right]^{n-1} \cdot \phi(x) = e^{\int dx' J(x') \phi(x')} \phi(x) \end{aligned}$$

$$\text{So } \frac{\delta}{\delta J(x)} e^{\int dx' J(x') \phi(x')} = \phi(x) e^{\int dx' J(x') \phi(x')}$$

$$\left[\frac{\delta}{\delta J(x)} \right]^n e^{\int dx' J(x') \phi(x')} = [\phi(x)]^n e^{\int dx' J(x') \phi(x')}$$

Therefore:

$$\boxed{F\left(\frac{\delta}{\delta J(x)}\right) e^{\int dx' J(x') \phi(x')} = F(\phi(x)) e^{\int dx' J(x') \phi(x')}} \quad \text{}$$

With 'Orie theorem, (12.16) is easily explained:

$$\exp \left[i \int dx L_2 \left(\frac{\delta}{\delta \phi} \right) \right] \int [d\phi] \exp \{ i \int dx [L_0 + J\phi] \}$$

$$= \int [d\phi] \cdot \exp [i \int dx L_2(\phi)] \exp \{ \text{rest} \} = \int [d\phi] \exp \{ i \int dx [L_0 + L_2 + J\phi] \}$$

Calculation of the path integral (4.17) using $S = \dots$ (4.30)

~~Interpret (4.30) in the determinant sense.~~

$$iS = - \int d^4x \left(\frac{1}{2} [\partial_\mu \phi]^2 + (\nabla \phi)^2 + \mu^2 \phi^2 \right) - \int \phi$$

$= -\frac{1}{2} (\phi, A \phi)$ somehow. Well actually it is not a perfect square. Go back to page 35. Let

$$(x, Ax) = \sum_{i=1}^n x_i A_{ij} x_j$$

Then calculate: $\int dx_1 dx_2 \dots dx_n e^{-\frac{1}{2} x_i A_{ij} x_j}$

Example: $\int dx_1 dx_2 e^{-\frac{1}{2} [x_1 A_{11} x_1 + x_1 A_{12} x_2 + x_2 A_{21} x_1 + x_2 A_{22} x_2]}$

$= \int dx_1 dx_2$. Try to diagonalize A so that

$$y_i = V_{ij} x_j \quad \text{or} \quad y = Vx.$$

Want $(x, Ax) = (\bar{V} y, A \bar{V} y) = (y, V A \bar{V} y) = (y, A' y)$

Recall that A must be real symmetric and then V will be real orthogonal so $V^\dagger = V^T = V^{-1}$. The diagonal elements of A' are the eigenvalues. So

$$(y, A' y) = \sum_{i=1}^n \lambda_i y_i^2 = y_i A'_{ij} y_j$$

Now, $y = Vx$, how is $[dx]$ related to $[dy]$?

$$\frac{\partial y_i}{\partial x_j} = V_{ij} \quad \text{so} \quad dx \dots = \frac{\partial(x \dots)}{\partial(y \dots)} dy \dots$$
$$= |\det V|$$

So I seem to get $\det \bar{V}$ and not $\det A$. But consider

$$\bar{V} = V^+ = V^T \text{ so } \det \bar{V} = \det V^T = \det V. \text{ Next,}$$

$$A' = \bar{V} A V^+ \Rightarrow \det A' = \det \bar{V} \det A \det V$$

$$\Rightarrow \det V = 1. \text{ So:}$$

$$\int dx_1 \dots dx_n e^{-\frac{1}{2}(x, Ax)} = \int dy_1 \dots dy_n e^{-\frac{1}{2}(y, A'y)}$$

$$= \int dy_1 \dots dy_n e^{-\frac{1}{2} \sum_{i=1}^n \lambda_i y_i^2} = \prod_{i=1}^n \left(\int dy_i e^{-\frac{1}{2} \lambda_i y_i^2} \right)$$

$$= \prod_{i=1}^n \left[\frac{\pi}{\lambda_i} \right]^{\frac{1}{2}} = \frac{(2\pi)^{n/2}}{[\lambda_1 \lambda_2 \dots \lambda_n]^{\frac{1}{2}}} = \frac{(2\pi)^{n/2}}{[\det A]^{\frac{1}{2}}}$$

Thus we have derived this fact:

$$\left[\int dx_1 \dots dx_n e^{-\frac{1}{2}(x, Ax)} = (2\pi)^{\frac{n}{2}} / \sqrt{\det A} \right] \quad (4.2)$$

Must assume that $\lambda_i > 0$ [ie, $A = \text{real, sym, pos def}$] to make integrals converge. Now rewrite

$$\int \frac{dx_1 dx_2 \dots dx_n}{\sqrt{2\pi} \sqrt{2\pi} \dots \sqrt{2\pi}} e^{-\frac{1}{2}(x, Ax)} = (\det A)^{-\frac{1}{2}}$$

$$\text{or: } \prod_{i=1}^n \left(\frac{dx_i}{\sqrt{2\pi}} \right) e^{-\frac{1}{2}(x, Ax)} = (\det A)^{-\frac{1}{2}}$$

Now take $n \rightarrow \infty$ in both sides:

$$\left[\int [dx] e^{\frac{1}{2}(x, Ax)} = (\det A)^{-\frac{1}{2}} \right] \quad (4.4)$$

How relate this to a function space?

$$(f, g) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \Delta x f(x_i) g(x_i) = \int f(x) g(x) dx.$$

Make easy. What becomes of dx ? $[df]$

$$[df] ? \quad x_i \rightarrow f(x_i) \quad \text{so}$$

$$[df] = \lim_{n \rightarrow \infty} \int \frac{df(x_1) df(x_2) \dots df(x_n)}{\sqrt{2\pi}}$$

Finally, what is $(\det A)$? What is A_{ij} ?

A_{ij} was $(e_i, A e_j)$. Here A_{ij} ... how does a matrix operator go into an integral operator?

$$A f(x) = \int dy a(x, y) f(y) = \sum_j a_{ij} f(y_j)$$

So seems like $A_{ij} \rightarrow a(x, y)$. Certainly

$$h(A_{ij}) = \sum_i A_{ii} \rightarrow hA = \int dx a(x, x).$$

But what about $\det a$? Just like a Fredholm det. Hard to calculate!

100% 90% 80% 70% 60% 50% 40% 30% 20% 10% 0%

$$-\frac{b}{2}$$

.....



$$= -\frac{1}{2}ay'$$

↑

.....

doi:10.1371/journal.pone.0141405.g002



$$= \frac{1}{2} \sum_n A_{in} x_n + \frac{1}{2} \sum_m x_m A_{mi} + b_i$$

$$= \sum_n A_{in} x_n + b_i = (Ax + b)_i$$

Set $\Rightarrow Ax + b = 0 \Rightarrow \bar{x} = -A^{-1}b$. This $\bar{x}$ minimizes $Q(x)$. So $[\bar{x} = \text{a vector}]$. In

$$Q(x) = \frac{1}{2} (x - \bar{x}, A(x - \bar{x})) + Q(\bar{x})$$

$$= \frac{1}{2} (x, Ax) - \frac{1}{2} (\bar{x}, Ax) - \frac{1}{2} (x, A\bar{x}) + \frac{1}{2} (\bar{x}, A\bar{x}) \\ + \frac{1}{2} (\bar{x}, A\bar{x}) + (b, \bar{x}) + c.$$

$$= (-A\bar{x}, \bar{x}) = -(\bar{x}, A\bar{x})$$

Now $(b, x) = (-A\bar{x}, x) = -(\bar{x}, Ax) = -(\bar{x}, A\bar{x})$
 $= (x, A\bar{x})$. cancel other two

$$\Rightarrow Q(x) = \frac{1}{2} (x, Ax) + (b, x) + c \quad \checkmark$$

So $\frac{1}{2} (x - \bar{x}, A(x - \bar{x}))$ is trivial, just shift! Result:

$$\int [dx] e^{-Q(x)} = e^{-Q(\bar{x})} (\det A)^{-\frac{1}{2}} \quad (4.9)$$

where $Q(x) = (x, Ax) + (b, x) + c$

and where $\bar{x} = -A^{-1}b$

⑧ Now back to (4.30)

$$I' = \int [d\phi] e^{iS(\phi, J)} = ?$$

$$x \rightarrow \phi \quad Q(x) \rightarrow -iS(\phi)$$

paths

$$So \quad Q(\phi) = \int d^4x \left(\frac{1}{2} \phi [-\Box_E + \mu^2] \phi - J\phi \right)$$

$$Define \quad (a, b) = \int d^4x \quad a(x) b(x). \quad So$$

$$Q(\phi) = \frac{1}{2} (\phi, (-\Box_E + \mu^2) \phi) - (J, \phi)$$

$\Rightarrow A = -\Box_E + \mu^2 \quad b = -J \quad c = 0$. So we now do the integral to get? Well, what is $\bar{x} = \bar{\phi}$?

$$\bar{\phi} = -\bar{A}^{-1} b = +\bar{A}^{-1} J \Rightarrow A \bar{\phi} = J$$

$$\Rightarrow (-\Box_E + \mu^2) \bar{\phi} = J \quad \bar{\phi} = \text{solution}$$

But $(-\Box_E + \mu^2) \Delta_E(x) = -\delta^4(x)$ has a unique solution.

⑨ Study this greens equation for a while:

$$[(\Box_E - \mu^2) \Delta_E(x) = \delta^4(x)] \quad (4.26)$$

How do you solve this? Try fourier analysis

$$\Delta_E(x) = \int d^4k \quad \tilde{\Delta}_E(k) e^{ik \cdot x}$$

Then $\square_E^2 e^{-ik_E x_E} = \sum_{i=1}^4 \partial_i^2 e^{-ik_E x_E} = (-ik_i)^2 e^{-ik_E x_E}$

$= \left(\sum_{i=1}^4 \partial_i^2 \right) e^{-ik_E x_E} = \square^2 e^{-ik_E x_E}$

$= \partial_i^2 e^{-ik_E x_E} = \partial_i \partial_i e^{-ik_E x_E} = \partial_i [-ik_i e^{-ik_E x_E}] = (-ik_i)^2 e^{-ik_E x_E}$

So $\sum_{i=1}^4 (-ik_i)^2 = -k \cdot k = -k_E^2$ So

$\square_E^2 e^{-ik_E x_E} = -k_E^2 e^{-ik_E x_E}$

So we get: $\int d^4 k_E (-k_E^2 - \mu^2) \tilde{\Delta}_F(k) e^{-ik_E x_E} = \delta^4(x)$

$= \int d^4 k_E e^{-ik_E x_E} (2\pi)^4$

So seems that $-(k^2 + \mu^2) \tilde{\Delta}_F(k) = (2\pi)^4$

$\Rightarrow \tilde{\Delta}_F(k) = \frac{1}{(2\pi)^4 (k^2 + \mu^2)}$

Then $\left[\Delta_E(x) = \int \frac{d^4 k_E}{(2\pi)^4} \frac{e^{-ik_E x_E}}{(k_E^2 + \mu^2)} \right] \quad (4.25)$

But $k_E^2 > 0$ so no ambiguity.

⊗ Now back to $A\bar{\phi} = b = J$

so $(-\square_E^2 + \mu^2) \hat{\phi}(x) = J(x) \Rightarrow \hat{\phi}(x) = \int d^4 y_E J(y_E) \Delta_E(x-y)$

So I would say

$$\left[Z' = \int [d\phi] e^{-Q(\phi)} = e^{-Q(\bar{\phi})} (\det A)^{-\frac{1}{2}} \right]$$

$$\text{Now } Q(\bar{\phi}) = Q(\int d^4y J(y) \Delta_F(y-x))$$

$$= \frac{1}{2} (\bar{\phi}, \dots) = -(\bar{J}, \bar{\phi})$$

$$= - \int d^4x J(x) \cdot \int d^4y J(y) \Delta_F(y-x)$$

$$\text{So } e^{-Q(\bar{\phi})} = e^{-\int d^4x d^4y J(x) \Delta_F(x-y) J(y)}$$

what about the $\frac{1}{2}$?

Well be careful:

$$Q(\phi) = \frac{1}{2} (\phi, A\phi) - (\bar{J}, \phi)$$

$$Q(\bar{\phi}) = \frac{1}{2} (\bar{\phi}, A\bar{\phi}) - (\bar{J}, \bar{\phi}) = \frac{1}{2} (\bar{\phi}, \bar{J}) - (\bar{J}, \bar{\phi})$$

$$= -\frac{1}{2} (\bar{J}, \bar{\phi}) = -\frac{1}{2} \int d^4x d^4y J(x) \Delta_F(x-y) J(y)$$

here is the $\frac{1}{2}$.

$$\text{So: } \left[e^{-Q(\bar{\phi})} = e^{-\frac{1}{2} \int d^4x d^4y J(x) \Delta_F(x-y) J(y)} \right]$$

Summary: The formula (4.9) allows you to calculate path integrals when $Q(x)$ has a certain form. The integral (4.17) for $\exp[iW[\phi]]$ in terms of the action has this form at least when the Lagrangian is the free Lagrangian shown in (4.30). You never need compute $\det A$.

Exercise: do the p integrations in (4.61) to derive (4.56).

① Lagrangian is $L(q, \dot{q}) = \frac{1}{2} \langle \dot{q}, K(q) \dot{q} \rangle + L(q) \dot{q} - U(q)$, contains at most quadratic time derivatives. Convert this to $L'(p, q)$, i.e., replace $\dot{q}$ with p . Note that, from (4.62),

$$p = \frac{\partial L(q, \dot{q})}{\partial \dot{q}} = ?$$

Assume only one field, so don't need a, b, c stuff. K could be a differential operator! Maybe I should revert to discrete notation. Then

$$\frac{1}{2} \langle \dot{q}, K(q) \dot{q} \rangle = \frac{1}{2} \sum_{i,j=1}^n \Delta t \dot{q}(t_i) \left(\sum_{j=1}^n K_{ij}(q) \dot{q}(t_j) \right)$$

$$\begin{aligned} \text{Then } \frac{\partial \langle \cdot \rangle}{\partial \dot{q}(t_i)} &= \frac{1}{2} \Delta t \dot{q}(t_i) (K_{ij} \dot{q}(t_j)) + \frac{1}{2} \Delta t \sum_{j=1}^n \dot{q}(t_i) K_{ij} \dot{q}(t_j) \\ &= \Delta t \sum_j K_{ij} \dot{q}_j = K(q) \dot{q} \end{aligned}$$

So you use the symmetry of the operator to show desired result.

② ~~Wait~~ If K is a fn of the q 's, then not an operator.

K is just a function, so (4.54) then follows at once. Also, does it make sense! Let $K(q) = q$. Then

Let $K(q) = ?$ needs two args?.....

$$\text{So } p_a = K_{ab} \dot{q}^b + L_a \Rightarrow K_{ba} (p_a - L_a) = \dot{q}^b$$

$$\text{Then } L = \frac{1}{2} K^{-1}(p-L) K \cdot K^{-1}(p-L) = \frac{1}{2} K^{-1}(p-L)^2$$

So you will get

$$\int (dp) \mathcal{C} \int d^4 p \, K^1_p \sim \int (dp) \mathcal{C} \sim (\det K^1)^{-\frac{1}{2}}$$

Here, K must be an operator in p -space!

$$K(q) = K(\cdot) \text{ , is independent of } q!$$

But I see $\sim [\det K]$ will arise from the p -integration in the obvious way

I cannot carry this thru at present

A series of exercises to study Fadeev-Popov idea for QED.

① Start by verifying (5.4). We have at the start

$$\begin{aligned}
 S &= \int d^4x \left(-\frac{1}{2}\right) [\partial_\mu A_\nu] [\partial^\mu A^\nu - \partial^\nu A^\mu] \\
 &= \left(-\frac{1}{2}\right) \int \frac{d^3k}{2E} [k_\mu \tilde{A}_\nu] [k^\mu \tilde{A}^\nu - k^\nu \tilde{A}^\mu] = -\frac{1}{2} \int \frac{d^3k}{2E} [k_{\mu\nu}] [k^\mu a^\nu - k^\nu a^\mu] \\
 &= \left\{-\frac{1}{2} \int \frac{d^3k}{2E}\right\} a_\nu [k^2 a^\nu - k_\mu k^\mu a^\nu] = \int [dk] \cdot a_\nu k^2 \left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right] a_\mu \\
 &= \int [dk] a_\nu k^2 P_T^{\mu\nu} a_\mu = \int a_\nu k^2 P_T^{\nu\nu'} P_T^{\mu\mu'} a_\mu = \int P_T^{\nu\nu'} a_{\nu'} k^2 P_T^{\mu\mu'} a_\mu \\
 &= \int a_\nu^\top k^2 a_\nu^\top = -\frac{1}{2} \int d^4x A_\nu^\top \partial_\mu \partial^\mu A_\nu^\top = +\frac{1}{2} \int d^4x (\partial_\mu A_\nu^\top)^2.
 \end{aligned}$$

So we learn that you can replace: $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \rightarrow +\frac{1}{2} (\partial_\mu A_\nu^\top)(\partial^\mu A^{\top\nu})$ and you get the same action!

② Now backstep to here $\rightarrow$ to get $S = \langle a_\nu, k^2 P_T^{\mu\nu} a_\mu \rangle$ so that the operator "A" which determines the propagator is:

$$(k^2 P_T^{\mu\nu})_{\text{DF}}(k) = -i\delta^{\nu\mu}.$$

But, the matrix $(k^2 P_T^{\mu\nu})$ cannot be inverted, so the solution DF must be singular. I.e., the transverse propagator is $\propto$, because $a_\nu^\top \square a_\nu^\top$ gives easy answer. Obviously, if you cannot invert your operator, you are dead!

③ Show that $\det P^T = 0$.

$$\begin{aligned}\det P^T &= \det \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) = \det \left(g^{\nu\nu'} \left(g^{\mu\mu'} - \frac{k^\mu k^{\mu'}}{k^2} \right) \right) \\ &= \underbrace{\det(g)}_{=-1} \cdot \underbrace{\det(1 - K)}_{=1 - \text{trace } K} = 1 - \sum_{\mu=0}^3 \frac{k^\mu k_\mu}{k^2} = 1 - 1 = 0 \checkmark\end{aligned}$$

Thus, $\det P^T = 0$ and P^T is non-invertible.

④ How is theory changed by (5.24)? Well:

$$\mathcal{L}_{\text{new}} = \frac{1}{2} (\partial_\mu A^\mu)^2 - \frac{1}{2\alpha} (\partial_\mu A^\mu)^2$$

Go to k -space $\mathcal{L}_{\text{new}} = \frac{1}{2} (k_\mu A^\mu k^\mu A^\mu - \frac{1}{2} k_\mu A^\mu \cdot k^\mu A^\mu)$

$$= \frac{1}{2} [A_\nu k^2 A_\nu^T - \frac{1}{2} A_\nu k^2 A_\nu^L] \quad \text{since} \quad P^{T^2} = P^T, P^{L^2} = P^L$$

$$= \frac{1}{2} A_\nu k^2 \left[P^T - \frac{1}{2} P^L \right] A_\nu$$

Propagator is now: $(P^T - \frac{1}{2} P^L) A_F = \frac{-i}{k^2}$

But now the operator is invertible:

$$A = P^T - \frac{1}{2} P^L$$

$$A^{-1} = P^T - \alpha P^L$$

$$\Rightarrow D_F = (P^T - \alpha P^L) \left(\frac{-i}{k^2} \right) = (5.26)$$

Derivation of a General Renorm Group Equation

① Let f = some operator, a function of the field operators. Let f = observable. Field theory is massless, or we are in deep Euclidean limit of a massive field theory. M = an artificial renormalization point, eg, when p.p. = M^2 , $\Delta f = 1$ a something. Obviously f cannot change if you change M .

② What happens when M changes? All fields get rescaled a certain amount because, eg, your propagators are renormalized. So if A is some field:

$$M \rightarrow M(1+\epsilon) \\ \Rightarrow A \rightarrow A(1+\gamma_A \epsilon)$$

γ_A is called the dimension of field A . By playing with def. of f , you could find its dimension.

$$f \rightarrow f(1+\gamma_f \epsilon) \quad \text{as} \quad M \rightarrow M(1+\epsilon)$$

③ Coupling g , since its defined "at M " somehow, also changes:

$$g \rightarrow g + \beta \epsilon$$

④ Imagine this change $M \rightarrow M(1+\epsilon)$ as a scale change, some $\lambda \rightarrow \lambda + \epsilon$. Each field is a function of scale parameter λ .

So consider f as

$$f = f(g, M, \lambda)$$

Then under scale change, f should be invariant,
 $\delta f = 0$, since no change in theory

$$\delta f = \frac{\partial f}{\partial g} \frac{\partial g}{\partial \epsilon} \delta \epsilon + \frac{\partial f}{\partial M} \frac{\partial M}{\partial \epsilon} \delta \epsilon + \frac{\partial f}{\partial \lambda} \frac{\partial \lambda}{\partial \epsilon} \delta \epsilon$$

$$\Rightarrow ? \quad \frac{\partial g}{\partial \epsilon} = \beta \quad \frac{\partial M}{\partial \epsilon} = M \quad \frac{\partial \lambda}{\partial \epsilon} = 1$$

So

$$\left[\beta \frac{\partial f}{\partial g} + M \frac{\partial f}{\partial M} + \frac{\partial f}{\partial \lambda} \right] = 0$$

but $f \rightarrow f(\lambda \epsilon) \Rightarrow \delta f = \lambda \frac{\partial f}{\partial \lambda} \delta \epsilon = f \delta \epsilon$

I.e. $\frac{\partial f}{\partial \epsilon} = \delta f$. So

$$\boxed{\left[\beta \frac{\partial}{\partial g} + M \frac{\partial}{\partial M} + \delta_f \right] f(g, M) = 0}$$

Really this is scale invariance, (6.8) is just a special case

④ Maybe $f = f[g(M)]$ is the way to write it. Given M , everything in this field theory depends only on g . Thus, $\beta = \beta(g)$ $\delta = \delta(g)$

Steven Weinberg

1974

Recent progress in gauge theories of the weak, electromagnetic and strong interactions*†

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A review is presented of progress since mid-1972 in the development of gauge theories of the weak, electromagnetic, and strong interactions. A brief introduction to the history and the fundamentals of the subject is also provided.

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I. INTRODUCTION

Renormalizable gauge theories of the weak and electromagnetic interactions have been with us for six years or so, and have been popular at least since mid 1971. There are by now a number of excellent surveys of the subject, including Ben Lee's comprehensive report (1972a) at NAL and the more recent articles by Bernstein (1974), Sehgal (1973b), Abers and Lee (1973), and Llewellyn Smith (1973a). I intend in the body of this review to concentrate on the exciting new developments which have occurred since the preparation of Lee's report, some of which are only a few months old. However, before getting into this new material, which is the real subject of this article, it may be well to take a brief look in this Introduction at the history and the fundamentals of gauge theories.

The history of attempts to unify weak and electromagnetic interactions goes back to Fermi's (1934) proposal of a four-vector lepton-hadron beta decay interaction, analogous to the vector interaction of electrons with the electromagnetic field. Shortly thereafter Yukawa (1935) suggested that the beta decay interaction was carried by a spin-one boson, analogous to the photon, but with a large mass. (Yukawa intended this vector boson to explain nuclear forces as well as beta decay, so he gave it what in modern terms would be called a strong interaction with hadrons and a superweak interaction with leptons, and a mass of order 100 MeV.) The subsequent

discovery of a change in nuclear spin in beta decays of such nuclei as He^6 , F^{18} , and Al^{26} made it clear that the beta decay interaction was *not* pure vector, and for many years the favored combination was S, T, P, which of course would quite destroy the analogy with electromagnetism.

The idea of a unified theory was revived in the late 1950's, after the form of the beta decay interaction finally settled down to a combination of vector and axial vector. Specific models were proposed by several authors, notably Schwinger (1957), Bludman (1958), Glashow (1961), and Salam and Ward (1964). However, two great obstacles stood in the way of a synthesis. One was the obvious discrepancy in mass between the photon and the intermediate vector boson W ; if the W couplings are comparable with the electronic charge e , then its mass is determined by the condition that e^2/m_W^2 should be of the order of the Fermi coupling constant G_F , so that

$$m_W \approx (137 \times 10^{-5} m_p^{-2})^{-1/2} \approx 30 \text{ GeV}. \quad (1.1)$$

A less obvious though no less important difficulty had to do with the problem of high-energy behavior. Although the introduction of an intermediate vector boson ameliorated the bad asymptotic behavior of processes such as lepton-lepton scattering, there were known to be others, such as the process $\nu + \bar{\nu} \rightarrow W^+ + W^-$ considered by Gell-Mann, Goldberger, Kroll, and Low (1969), where the Born approximation amplitude grows so rapidly with energy that unitarity forces a failure of perturbation theory at energies above 300 GeV. Even worse, the occurrence of such rapidly growing matrix elements as pieces of higher-order diagrams (as in the example of $\nu - \bar{\nu}$ scattering considered by Low, 1970) invalidates perturbation theory at all energies.

In 1967 a way out of these difficulties was suggested (Weinberg, 1967b; see also Salam, 1968). It was proposed that the photon and the intermediate vector bosons should arise as quanta of the Yang-Mills vector fields¹ associated with some exact local gauge invariance of nature. In order to avoid the problem of ultraviolet divergences, it was proposed that the theory should be constructed to be renormalizable. Finally, in contrast to earlier theories of intermediate vector bosons, it was proposed that their mass should arise from a spontaneous breakdown of the gauge group.

It was already known since 1964 through the work of

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† Note: This review is based in part on a talk given at the *11^e Conférence Internationale d'Aix-en-Provence sur les Particules Elementaires*, September 6, 1973. A preliminary version of this review is included in the conference proceedings, published by the Journal de Physique, Comptes-Rendus des Colloques no. 1 (1973). Permission to reprint parts of the preliminary version is gratefully acknowledged.

¹ The first example of a theory based on a non-Abelian gauge group was that of Yang and Mills (1954). This theory was generalized to arbitrary semisimple Lie groups by Utiyama (1956) and Gell-Mann and Glashow (1961).

1. Introduction. Several other reviews are listed, one by L-Smith I have not seen. Then a little review of past attempts to unify EM and Weak. The problem is that photon and W have such different mass, and that you have high energy UV problems. Hard to understand the severe mass difference if theory is to be unified in some way.

In 1964 Higgs mechanism appeared, but people were happy with the pion as a Goldstone and did not want to get rid of goldstones so bad. In 1967 came the Weinberg and Salam models in which the Higgs thing was used to get mass onto the vector bosons by using SSB. By thus avoiding an explicit massive vector bosons, there was at least a hope of renormalizability. In 1967 it simply was not known whether or not such SSB theories were renormalizable. The real problem was the SSB part of it.

In 1971 't Hooft did it, showed that SSB theories are renormalizable. Then other people carried on his start and showed proofs for weak and EM theories (Ben Lee Zinn Justin, Veltman 't Hooft).

Finally, the Xi Gauge thing is mentioned as a way to debug calculations.

At present there are no definitely accepted gauge theories. But the idea that the various conserved weak currents are gauge currents is attractive. And then there is the neutral currents recently found (see below).

2. Models and Experiments. How do you test these theories? First, look for the W bosons, but we cannot do that in 1973 yet. Next, look for the various kinds of neutral currents.

A CERN exper in 1973 showed one event of the type $\bar{\nu}_\mu e$ elastic scattering which seems to be an honest event. The absence of other neutral events puts certain limits on the Weinberg angle.

Things are easier when you go to semileptonic and look for neutral current events. People are doing neutrino-induced deep inelastic events to get high statistics. From CERN it seems that there definitely are these semileptonic $dS=0$ currents, because they have $R = .21 \pm .03$, and $R=0$ means no neutral currents with ν induction. The $\bar{\nu}$ induced $\bar{R} = .45 \pm .09$, so again you have definite neutral currents.

You can interpret these hadronic neutral currents by putting some quarks into the Weinberg model in naive way. Then model predicts R and $\bar{R}$ as functions of θ . If $\sin^2\theta$ is in the range of .3 to .4, then you get agreement. Looks good.

What are the faults of the Weinberg model? First, it has $dS=1$ neutral currents at first order, and these predict things that should not happen when you add quarks in the naive way, eg, K_0 decay into muons. Now Weinberg discussed several ways to "fix things up" one of which is to add more quarks. This review is pre-PSI; now of course this is clearly the way to fix things up. The other possibilities listed sound a little obscure.

Another problem with the theory is that it just isn't basic because it has two independent couplings, ie, it has that free parameter the Weinberg angle. It would be a lot nicer if you could use a simple gauge group which then has one parameter.

The most famous model that does this is the Georgi Glashow which uses only $SU(2)$. In this model, however, the neutral gauge field is the photon and there is nothing else, this neutral currents can eliminate this nice model.

Another idea is to consider the $SU(2) \times U(1)$ group as being a subgroup of some larger group. Weinberg gives an example which then predicts that $\sin^2\theta = .25$, which matches experiment!

Other problems are: Weinberg theory can have the anomalies; it does not give the CP violation in a nice way.

Generally, the model is not "natural". This is the main problem. It does not predict masses say of the muon, or anything basic.

3. Can you derive gauge invariance from HE constraints? It can be roughly shown that the only possible theory of vector particles which are "perturbatively unitary" are the gauge theories. Recall that in principle any field theory is unitary formally if you sum to all orders, but if it is non-renormalizeable you are cooked.

I am confused as to the relation between renormalizeability and perturbative unitarity? Can a theory be renormalizeable, and yet violate unitarity at some given order, but be unitary to all orders? References given for pursuing this interesting subject.

4. Regge and gauge theories. Can a particle be elementary in the way it appears in a Lagrangian, but at the same time lie on a Regge trajectory? Answer is yes. In fact, even the electron lies on a regge trajectory. Refs given for pursuing this also interesting path.

5. Dynamical Symmetry Breakdown? Usually you do your SSB by adding some Higgs scalars and shifting some of the fields. One problem is that you are always left over with some scalars that don't seem to match any physically observed particles. Is there some way to get SSB without introducing these explicit scalars into the lagrangian? Apparently there is a way called "dynamical SSB" and some references are given. Somehow you are getting Goldstones out without putting any bosons in. I don't follow this section, but the subject matter is clear enough.

6. Neutral Currents and Astrophysics. If there are in fact neutral currents, this changes the decoupling times for γ versus ν in the early cosmology. However, this does not seem to make much difference in early universe history because the basic facts don't depend on whether things are coupled in thermal equilibrium, or decoupled.

Next, what about "neutrino cooling" of stars by obvious neutrino loss. Clearly any such calculation should be affected by neutral currents. Calculations show that neutral currents can change the neutrino cooling rate by a factor of 10.

Finally, supernova theory is affected. One theory is this: you have a star with an iron core and an envelope. The core is large enough to begin shrinking down to the neutron star limit; as it shrinks, neutrinos are emitted which are then captured in the envelope, causing the envelope to fly off, leaving a neutron star.

Such envelope-blow-off theories are obviously affected by neutral currents.

7. Natural Symmetries and Sym Breaking. skip

8. Strong Interactions and Hadron Symmetries.