

Elementary Particle Theory

Various Refs & notes

Refs

- 1) Gross + Wilczek
- 2) Politzer
- 3) t'Hooft (unpubl.)
- 4) Callan + Low. — running group singl.
- 5) Bogoz + Shirkov — extend Rgroup to vertex.
- 6) Shirkov
- 7) Bogoz Shirk + Ginz
- 8) BS book, review's the above work.
- 9) Weinberg — Weinberg's Theorem — do justify HE same as mass $\rightarrow 0$.
- 10) Johnson Baker Willey — used Rgroup to do "finite QED" (5 papers).
- 11) Bjorken — currents at short distances
- 12) Bjorken — scaling.
- 13) Wilson — theo. framework for scaling.
- 14) short-distance $\langle JJ \rangle$ theory.
- 15) Feynman — perturbation approach to scaling.
- 16) Callan + Gross — quarks seem to be spin $-\frac{1}{2}$.
- 17) Callan
- 18) Symanzik — diff equation with mass present. $\rightarrow$ RGE as $m \rightarrow 0$.
- 19) Ossola + Lüscher — unnoticed RGE paper.

20) Wilson

21) Sym

22) Callan

23) Christ, Hassl, Mueller

Realized that RGE is key to
using Wilson deal to
get deep inelastic !

24) Ferrara, Gellio, Parisi, Gatto

25) Parisi

26) Callan Gross

— proof of this fact

27) Zee

— showed scalar QED is not AF

28) Coleman Gross

— Theorem: no NAness $\rightarrow$ not AF

29) X

30) Gross + Wilczek

— applications studied

31) Feynman & PP

— ghosts.

32) — def. of "except. momenta"

33) t-Hod - Velt

— regularization, gauge invariant!

34) Slavnov.

35) Lee, ZinnJustin — another way to regularize. Add derivative terms to Lag. somehow. Gauge invariant also

36) See ↑ for review of renorm of QAGT.

37) Coleman "Dilatation" notes, (unpubl.). 1971 Ethne.

38)

39)

BEHAVIOUR OF LEADING INFRA-RED DIVERGENCES OF QCD IN THE AXIAL GAUGE

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We identify in the axial gauge a simple set of diagrams which yield the exponential of leading infra-red corrections to massive quark scattering in QCD. The relevance of the Ward identities is pointed out.

Recently it was shown [1] that the leading infra-red divergent corrections to massive quark scattering by an external colourless potential in QCD are very simple to fourth order, both for exclusive and inclusive reactions. This analysis was carried out in the Feynman gauge and use has been made of dimensional regularization with space-time dimension $4 + \epsilon$. In the regime when the momentum transfer $|q|$ is of the same order of magnitude as the quark mass m it was found that, to this order, the scattering amplitude obeys the equation:

$$\frac{\partial}{\partial t} S(q^2, t) = C_F N(q^2) \bar{g}^2(t) S(q^2, t). \quad (1)$$

C_F is the quadratic Casimir operator in the quark representation and

$$N(q^2) = \frac{1}{(2\pi)^2} \left(\frac{1+r^2}{2r} \ln \frac{1+r}{1-r} - 1 \right), \quad r = (1 + 4m^2/q^2)^{-1/2}. \quad (2)$$

For the exclusive reaction, with which we will be concerned here, $t = -\epsilon^{-1}$. Finally $\bar{g}(t)$ is the effective coupling constant of the Pure Yang-Mills theory as defined by the renormalization group. In the above reference, as well as in ref. [2], several reasons were given which suggest that eq. (1) might be true to all orders. In this case it implies the exponentiation of leading infrared divergences.

Since in the axial gauge [3], $\bar{g}(t)$ is closely connected with the self-energy correction to the gluon propagator, it is natural to try to understand eq. (1) in this gauge^{†1}.

In this note we would like to exhibit precisely the set of diagrams which yield eq. (1) in this gauge. To this end, we shall first illustrate to the 4th order, the relevance of the Ward identities. Consider for definiteness, the self-energy insertion on the quark leg of momentum p shown in fig. 1. Graphs containing crossed gluon lines have been omitted since they are non-leading in our regime [4]. Also only the graphs contributing to the one particle irreducible self-energy Σ^* have been explicitly shown. The reason is that the complete contribution is given by the inverse of the function C which is determined by the behaviour of Σ^* near mass shell:

$$\Sigma^*(p, n) = iC(p, n)(i\not{p} + m), \quad (3)$$

where n denotes a general fixed four vector specifying the gauge. The function C is obtained by differentiating Σ^* with respect to the momentum and taking the on-shell matrix elements of the resulting expression. In this way, $C(p, n)$ is given to 4th order by the set of diagrams shown in fig. 2, where only the diagrams containing leading divergences have been explicitly shown. By a completely similar procedure we also obtain $C(p', n)$. Furthermore, considering gluon self-energy as well as fermion vertex and self-energy corrections to the skeleton diagram, one

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^{†1} The convenience of this gauge has also been pointed out in this connection by Cornwall in ref. [2].

Asymptotically Free Gauge Theories. I*

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Asymptotically free gauge theories of the strong interactions are constructed and analyzed. The reasons for doing this are recounted, including a review of renormalization-group techniques and their application to scaling phenomena. The renormalization-group equations are derived for Yang-Mills theories. The parameters that enter into the equations are calculated to lowest order and it is shown that these theories are asymptotically free. More specifically the effective coupling constant, which determines the ultraviolet behavior of the theory, vanishes for large spacelike momenta. Fermions are incorporated and the construction of realistic models is discussed. We propose that the strong interactions be mediated by a "color" gauge group which commutes with $SU(3) \times SU(3)$. The problem of symmetry breaking is discussed. It appears likely that this would have a dynamical origin. It is suggested that the gauge symmetry might not be broken and that the severe infrared singularities prevent the occurrence of noncolor singlet physical states. The deep-inelastic structure functions, as well as the electron-positron total annihilation cross section are analyzed. Scaling obtains up to calculable logarithmic corrections, and the naive light-cone or parton-model results follow. The problems of incorporating scalar mesons and breaking the symmetry by the Higgs mechanism are explained in detail.

I. INTRODUCTION

In recent years the renormalization group has played an increasingly important role in the study of the asymptotic behavior of renormalizable field theories. This approach has acquired new importance due to the recent discovery that non-Abelian gauge theories are asymptotically free.¹⁻³ In this paper we shall amplify and extend the results reported in Ref. 1.

The renormalization group dates from the fundamental work of Gell-Mann and Low,⁴ who studied the asymptotic behavior of the photon propagator in quantum electrodynamics. The remarkable discovery of Gell-Mann and Low was that the asymptotic form of the photon propagator was determined by the zeros of a certain, calculable, function of the coupling constant and not by the actual value of the charge. The renormalization-group equations were extended by Bogolubov and Shirkov to the vertex function⁵ and employed to analyze the ultraviolet and infrared behavior of quantum electrodynamics and other field theories.^{6,7} (For a review of this work see Ref. 8.)

The basic idea underlying the renormalization-group equations is very simple. A renormalizable field theory contains two types of parameters—masses or coupling constants with positive dimensions of mass (i.e., due to $M\bar{\psi}\psi$ or $\lambda\phi^3$ terms in the Lagrangian) and dimensionless coupling constants (i.e., due to $\lambda\phi^4$ or $\bar{\psi}\lambda^{\mu\nu}\psi A_{\mu\nu}$ terms in the

Lagrangian). Coupling constants with negative dimensions of mass give nonrenormalizable theories. If one considers a Green's function for large and spacelike momenta (so as to exclude any Landau singularities), then one would expect that the generalized mass terms in the Lagrangian ($M\bar{\psi}\psi$ or $\lambda\phi^3$) could be neglected. In other words the leading asymptotic behavior of the Green's functions should be the same as would be calculated in a massless theory. This can be proved, to any finite order in perturbation theory, by using Weinberg's theorem.⁹ The massless theory contains no dimensional parameters to set the scale of momenta, therefore one might expect that the asymptotic behavior of the amplitudes would be determined by pure dimensional analysis. This is called naive or canonical scaling. It does not occur in practice, since the massless theory does contain a hidden dimensional parameter. This parameter, μ , must be introduced in order to perform the subtractions necessary to renormalize the theory and render it finite. Due to infrared singularities these subtractions, for the massless theory, must be performed off shell, say at some spacelike momenta $p^2 = -\mu^2$. The subtractions then define the physical coupling constants and the scale of the fields (which are determined by the wave-function renormalization constants). The subtraction point, μ , is arbitrary. If we change the subtraction point the net effect is to change the value of the coupling constants and the scale of the

Derivation of the Renorm group equations.

This is the best discussion I have seen anywhere. We are talking pure Yang Mills in the Lorentz gauge with ghosts and a gauge parameter α . The basic numbers Z_1 , Z_3 , $\tilde{Z}_1$, $\tilde{Z}_3$ are clearly defined in just the way I would have done it. The renorm point is $-\mu^2$ in the Euclidean region to keep away from normal thresholds etc.

If you change u^2 from one value to another I would expect all the Z's to change. By the way, the Z's here are functions of g_u , α_u , Λ , μ^2 . I.e, you compute your unrenormed vertex, say, to some order in g_u with a cutoff. Abarbanel does all this stuff in the dim reg.

So the Z's are defined thus. Why are you allowed, eg, to insist on (2.8)? This represents the value of the "constant" term in the vertex. I.e, if you were to Taylor expand about the three $q_1^2 = -\mu^2$, the zeroth order term in your expansion is just the RHS of (2.8).

But how do you know, you might ask, that it is proportional to the bare vertex? Here is the argument: imagine expanding in the triple Taylor series just mentioned. I know that if I perform a single subtraction at the renorm point, the result will be finite. Thus, I know that the subtraction process removes precisely this first term. On the other hand, I know that subtraction means adding a counterterm to the lagrangian of the same form as the coupling already there. The coupling that is already there is "the bare vertex", thus a subtraction can only remove a multiple of the bare vertex. Since YMTH is renormalizeable, you therefore know that that first constant term in the Taylor series is a multiple of the bare vertex! We were surprised when we found this by direct calculation, but of course we should not have been surprised.

Similarly, we expect the propagator at the renorm point to be proportional to the bare propagator, and the same for the ghost functions. If this were not the case, you could not renormalize by doing the subtractions implied by counterterms.

So, in conclusion, we know that the Z's all exist and can be computed to any order, as long as you have a cutoff or a dim reg procedure. And changing the renorm point should change the Z's. You cannot just choose the Z's to be what you want, not at this stage.

What happens next? The usual multiplicative renorm program is done. You define renormed fields, coupler, and gauge parameter, and 1PI Greens functions. By realizing that the unrenormed Green Functions "never heard of μ " you get the famous R-group differential equation (2.17), see detail attached.

Compare their results of Sec III to ours in axial gauge

① They define Z_3 as: $D_{\mu\nu}^{TR} = \frac{i}{(-q^2)} Z_3 \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)$ as $q^2 = -\mu^2$.

We define Z as: $\Delta_{\mu\nu} = \frac{Z}{(-q^2)} \left(g_{\mu\nu} + \frac{k_\mu k_\nu}{(n \cdot k)^2} - \frac{(n_\mu k_\nu + n_\nu k_\mu)}{(n \cdot k)} \right)$

② What did we calculate?? Lowest order $\sim$ means $Z_1 = 1$ (a.i), so I guess we got:

$$\bar{Z}_3 = 1 + \underbrace{\text{diagram}} + \text{diagram}$$

From $\bar{Z}_3$ we concluded that $\bar{a} = a - 1 = (\bar{Z}_3 - 1) = \frac{11}{6} \pi \ln \left(\frac{\Lambda^2}{q^2} \right)$.
ignoring the (Λ^2/q^2) quadratic term which is subtracted away somehow. To this we can add $g_0^2 \times \text{Casimir}$.

The leaf $\leftarrow$ is indep of q + cannot make $\ln(\Lambda^2/q^2)$ so ignore it.

② Meanwhile, we also calculated:

$$g_0 Z_1 = 1 + \left[\text{diagram} + \text{diagram} + \dots \right] \quad \left(\frac{22}{3} \right) \frac{\pi}{4} \ln \left(\frac{\Lambda^2}{q^2} \right) \cdot g_0^3 \cdot \text{Casimir}$$

So we got:

$$\begin{aligned} (\bar{Z}_3)^{-1} &= 1 - \frac{11}{6} \pi \ln \left(\frac{\Lambda^2}{q^2} \right) \cdot g_0^2 \cdot \text{Cas} \cdot \frac{1}{2} \\ Z_1 &= 1 + \frac{11}{6} \pi \ln \left(\frac{\Lambda^2}{q^2} \right) \cdot g_0^2 \cdot \text{Cas} \cdot 2 \end{aligned} \quad \left. \begin{array}{l} \text{for judging} \\ \text{signs + i's.} \end{array} \right\}$$

Thus we found $Z_1 = Z_2 = 1 + \frac{11}{6} \pi g_0^2 \text{Cas} \cdot \ln \left(\frac{\Lambda^2}{q^2} \right)$

④ Correction: from answers, add 2π for $\int d\phi$, and $\frac{1}{16\pi^4}$ for $(2\pi)^4$.

So

$$Z_1 = Z_2 = 1 + \frac{1}{16\pi^2} \left(\frac{11}{3} \right) g_0^2 \cdot \text{Cas} \cdot \ln \left(\frac{\Lambda^2}{q^2} \right) \quad \leftarrow \text{I think this is our result.}$$

Once again, we got

$$Z_1 = Z_2 = \frac{1}{16\pi^2} \left(\frac{22}{3} \right) \cdot g_0^2 \cdot \text{Cas} \cdot \ln \frac{\Lambda}{q}$$

Recall that, in the end, the gauge terms did nothing in our calc, ie, no η^μ dependence of Z . Thus, our Z_3 should be same as GW, except they must include ghost loop. I bet that loop makes $(-3) = \left(-\frac{1}{3} \right)$ to give their $+\frac{13}{3}$.

1. Introduction. We have here a historical review of related work. I have taken notes in refs section, the first 30 refs are involved. The big recent push of course is the discovery by authors that NAGT possess the quality of asymptotic freedom.

Old work was done by Gell_Mann Low and Bogo Shirkov group in the renorm group equations, 2 references 4 thru 8. These guys wrote down RG equations and realized that your subtraction point changes your charges etc, but the work sat idle for many years. Reason: you couldn't calculate anything. Eg, GL could not compute the asymptotic behavior of QED, though they knew about invariant charges etc. Another reason was that RGE connected you to deep Euclidean region of greens, not of physical interest as in S-Matrix elements, so no one really cared.

As an aside, during this dormant period Johnson Baker Wiley developed their finite QED ideas using R-group but not in same way as used here.

Then came the experimental work in deep inelastic and various theories to try to explain scaling.: Partons, light cone, current commutators, and so on. Eg, Bjorken scaling was coined in 1969. Feynman's book reviews this era.

Along with this experimental stuff came Callan Symanzik scaling theory and Wilson operator product theory, all related to R-group. Everything seemed to fit together if only you had a UV-stable fixed point at ZERO! A theory with this property was called asymptotically free, . In 1973, Zee showed that scalar QED was not AF. Authors showed that you had to have non-abelian gauge particles to stand a chance.

Then in 1973 ~~it~~ was found a positive proof: that NAGT were AF. Of course gauge theories were useful from other viewpoints: field theories, renormalizability had been proved in 1971, Weinbergs theory looked good for weaks, SSB, etc etc. Now it looked like the deep inelastic partonic results could be connected in!!!

Intro ends with an outline of paper.

$$\int d\vec{r} \bar{\rho}(\vec{s}, \vec{r}; \tau) \bar{\rho}(\vec{r}, \vec{s}; \beta - \tau) = \bar{\rho}(\vec{s}, \vec{s}; \beta),$$

we obtain

$$C = \beta \int d\vec{s} \bar{\rho}(\vec{s}, \vec{s}; \beta) W(\vec{s}). \quad (4.4)$$

From the relation⁸

$$\bar{\rho}(\vec{r}, \vec{r}; \beta) \sim (2\pi\beta\hbar^2/m)^{-1/2} |\bar{\psi}(0, \vec{r})|^2, \quad \beta \rightarrow \infty \quad (4.5)$$

with $\bar{\psi}(0, \vec{r})$ denoting the zero-energy limit of the outgoing scattering solution $\bar{\psi}(\vec{k}, \vec{r})$ of the Schrödinger equation $(E_{\vec{k}} - \hat{H})\bar{\psi} = 0$, it then follows that

$$C \sim (2\pi\beta\hbar^2/m)^{-1/2} \alpha, \quad \beta \rightarrow \infty \quad (4.6)$$

$$\alpha = \frac{m}{2\pi\hbar^2} \int d\vec{r} \bar{\psi}^*(0, \vec{r}) W(\vec{r}) \bar{\psi}(0, \vec{r}). \quad (4.7)$$

From (4.6) and (2.4), it follows that $g(C) = -\alpha$. Since it has already been shown that $g(B) = \bar{A}_0$, and since it is clear from Eq. (2.4) that $g(B+C) = g(B) + g(C)$, we see from Eq. (3.4) that

$$g(Z_B^{(0)}) = \bar{A}_0 - \alpha.$$

In view of Eq. (4.3) and the relation

$$\bar{H}\bar{\psi}(0, \vec{r}) = 0,$$

the VUB on A_0 can also be written as

$$g(Z_B^{(0)}) = \bar{A}_0 + \frac{m}{2\pi\hbar^2} \int d\vec{r} \bar{\psi}^*(0, \vec{r}) H \bar{\psi}(0, \vec{r}), \quad (4.8)$$

which is the desired result.

¹D. Gelman and L. Spruch, *J. Math. Phys.* **10**, 2240 (1969).

²R. P. Feynman, *Rev. Mod. Phys.* **20**, 367 (1948).

³L. Spruch and L. Rosenberg, *Phys. Rev.* **116**, 1034 (1959).

⁴All of the terms and notations which we are using were defined in Ref. 1; they will not be repeated here.

⁵A brief discussion of the potential utility of the bounds

derived in Ref. 1, which is applicable also to the bound obtained herein, is given in the Introduction of Ref. 1.

⁶S. Servadio, *Phys. Rev. A* **4**, 1256 (1971).

⁷This relation follows from the equation $J(s) = J'(s)$, where J and J' are defined by Eqs. (D1) and (D2) of Ref. 1, by the method exactly analogous to that used in deriving Eq. (D14) from Eq. (D13).

⁸This follows from Eqs. (2.8) and (2.4) of Ref. 1.

Renormalization of gauge theories—unbroken and broken

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A comprehensive discussion is given of the renormalization of gauge theories, with or without spontaneous breakdown of gauge symmetry. The present discussion makes use of the Ward-Takahashi identities for proper vertices (as opposed to the identities for Green's functions) recently derived. The following features of the present discussion are significant: (1) The present discussion applies to a very wide class of gauge conditions. (2) The present discussion applies to any gauge group and any representation of the scalar fields. (3) The renormalized S matrix is shown to be gauge-independent. (4) Dependence of counterterms on the gauge chosen is discussed.

I. INTRODUCTION

In an earlier publication,¹ we have given a derivation of the Ward-Takahashi (WT) identity for the generating functional of proper vertices in non-Abelian gauge theories. Previous discussions on the renormalizability of gauge theories²⁻⁵ were based on the Ward-Takahashi identities for Green's functions.^{6,7} The renormalization procedure is usually stated in terms of proper vertices, so that the WT identities for proper vertices would facilitate enormously the discussion of renormalizability.

In this paper we shall reexamine the renormalizability of gauge theories in terms of the WT identities for proper vertices. In addition to rederiving many results of Refs. 2, 3, and 5 (which we shall refer to as LZI, LZII, and LZIV, respectively), we shall add the following elements to our discussions: (1) We shall discuss the renormalizability of gauge theories in a wide class of gauge conditions. The gauge conditions we shall consider are linear in field variables and of dimension 2 or less. Most gauge conditions considered in the literature^{4,5,8} are of this kind. (2) We shall consider

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/method works only for one loop/, says L20I

RENORMALIZATION OF MASSLESS YANG-MILLS FIELDS

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Abstract: The problem of renormalization of gauge fields is studied. It is observed that the use of non-gauge invariant regulator fields is not excluded provided that in the limit of high regulator mass gauge invariance can be restored by means of a finite number of counter-terms in the Lagrangian. Massless Yang-Mills fields can be treated in this manner, and appear to be renormalizable in the usual sense.

Consistency of the method is proved for diagrams with non-overlapping divergencies by means of gauge invariant regulators, which however, cannot be interpreted in terms of regulator fields. Assuming consistency the S -matrix is shown to be unitary in any order of the coupling constant. A restriction must be made: no local, parity-changing transformations must be contained in the underlying gauge group. The interactions must conserve parity.

1. INTRODUCTION

In recent years the Feynman rules for massless Yang-Mills fields have been established [1-5]. Naive power counting suggests a renormalizable theory; however, in order to carry through a renormalization procedure one must first define a cut-off procedure. And if the cut-off procedure breaks the gauge-invariance of the theory then it is no more clear what the Feynman rules are. The reason is that gauge-invariance, through Ward identities, is essential for the S -matrix to be unitary.

Thus the problem poses itself as follows: how to find a gauge invariant cut-off procedure. This problem is of course quite the same in quantum electrodynamics. There the problem was solved by Pauli, Villars [6] and Gupta [7] who succeeded in finding a set of regulator fields that could be coupled in a gauge invariant way. Now in the case of massless Yang-Mills fields a gauge invariant regularizing procedure also seems to exist. Unfortunately, however, this procedure cannot be interpreted in terms of fields with indefinite metric and/or wrong statistics, like in the case of electrodynamics. Hence, unitarity and causality are no longer evident.

However, it must be realized that the whole renormalization procedure involves

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RENORMALIZABLE LAGRANGIANS FOR MASSIVE YANG-MILLS FIELDS

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Abstract: Renormalizable models are constructed in which local gauge invariance is broken spontaneously. Feynman rules and Ward identities can be found by means of a path integral method, and they can be checked by algebra. In one of these models, which is studied in more detail, local $SU(2)$ is broken in such a way that local $U(1)$ remains as a symmetry. A renormalizable and unitary theory results, with photons, charged massive vector particles, and additional neutral scalar particles. It has three independent parameters.

Another model has local $SU(2) \otimes U(1)$ as a symmetry and may serve as a renormalizable theory for ρ -mesons and photons.

In such models electromagnetic mass-differences are finite and can be calculated in perturbation theory.

1. INTRODUCTION

In a preceding article [1], henceforth referred to as I, it has been shown that, owing to their large symmetry, mass-less Yang-Mills fields may be renormalized, provided that a certain set of Ward identities is not violated by renormalization effects. With this we mean that anomalies like those of the axial current Ward identities in nucleon-nucleon interactions [2-4], which are due to an unallowed shift of integration variables in the "formal" proof, must not occur. In I it is proved that such anomalies are absent in diagrams with one closed loop, if there are no parity-changing transformations in the local gauge group. We do know an extension of this proof for diagrams with an arbitrary number of closed loops, but it is rather involved and we shall not present it here.

Thus, our prescription for the renormalization procedure is consistent, so the ultraviolet problem for mass-less Yang-Mills fields has been solved. A much more complicated problem is formed by the infrared divergencies of the system. Weinberg [5] has pointed out that, contrary to the quantum electrodynamical case, this problem cannot merely be solved by some closer contemplation of the measuring process. The disaster is such that the perturbation expansion breaks down in the infrared region, so we have no rigorous field theory to describe what happens.

EQUIVALENT ALTERNATIVES TO FADDEEV AND POPOV'S *

GAUGE THEORY QUANTIZATION PROCEDURE

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ABSTRACT

An alternate path integral gauge quantization procedure, developed in a preceding publication, is generalized. Two alternate quantization schemes can be formulated each **equivalent** to that of Faddeev and Popov. The path integral representation for either of these is of the form $\int DA \delta(GA) \Delta_G[A] \exp(\int \mathcal{L}(A) + \int JF^2)$, where $\Delta_G[A] = (\tilde{S}[A])^{-1} \det(D^H(A)G^H)$ for the 'oriented' procedure; or $(\tilde{M}[A])^{-1} |\det(D^H(A)G^H)|$ for the 'nonoriented' quantization scheme. $\tilde{M}[A]$ is 'essentially' equal to the number of noncontinuous Gribov copies for A. $\tilde{S}[A]$ is also an integer. The above is the result of an analysis independent of group measure considerations, contrary to F-P's formalism. The existence of Gribov copies in the $\overline{GW}$ gauge generally implies that a gauge transformation from all of $\overline{GW}$ space into GA space, $A[W]$, will not be invertible. Thus a functional change of variables-gauge transformation, from one gauge to the other, which takes into account the noninvertibility of the mapping must be used. There are two ways of realizing this, the 'oriented' and 'nonoriented' prescriptions. Each leads to Δ s of the above form. Simple examples are given to contrast the different methods of gauge quantization.

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THE RADIATION GAUGE STATIC POTENTIAL FOR NON-ABELIAN GAUGE FIELDS

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The static potential between a fermion and an anti-fermion in a group singlet state is calculated, through two loops, in the radiation gauge first order formalism. The results of this calculation imply that the Coulomb propagator is not sufficient to determine the static potential: a new function of the coupling constant $\alpha_s(-t)$ is also required.

The non-relativistic limit of the strong interactions has attracted considerable theoretical interest recently because of its relevance for the J/ψ and now the Υ systems [1-7]. The specific object of interest has been the static potential energy between a very massive fermion and a very massive anti-fermion. In this letter we report the results of an explicit calculation of the static potential between a heavy fermion and anti-fermion in the singlet channel to sixth order in the coupling constant for non-abelian gauge fields minimally coupled to the fermions. Although perturbation theory cannot be expected to reproduce a confining potential such as that found in charmonium, it still provides some important information about the form of the potential. For example, only in the singlet sector does a static potential exist. Here we describe a detailed examination of the perturbative structure of the potential which reveals that the potential is not directly proportional to $\alpha_s(-t)$, the strong coupling constant. Additional dynamical effects, not

seen in perturbation theory below the two loop level, imply that it is proportional to $\alpha_s(-t)$ multiplied by a new dimensionless function of $\alpha_s(-t)$.

It is important to note that perturbative results must be interpreted with caution. General features of the potential, such as those described above, may be extracted from the perturbative probe of the potential, but those relationships which depend intrinsically on the smallness of the coupling constant cannot be assumed to be valid, as may be illustrated by comparison with QED. Naive perturbation theory gives in both the abelian and non-abelian theories $\alpha \sim v/c$, where v is the characteristic speed of the constituents of the bound state. In QED we expect this relationship to be valid because $\alpha \ll 1$ and because the exact non-relativistic potential is given by the single Coulomb exchange. For the non-abelian model, however, the non-relativistic potential itself is an infinite power series in the coupling constant, and only the lowest order term is given by the single Coulomb exchange. Therefore, there may exist bound states which are threshold (with small kinetic energy of the constituents) but without any simple connection between v/c and the strong coupling constant. Also, the potential need not become stronger as the coupling constant increases, unlike non-relativistic QED.

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Gibson Ambiguity

QCD IN THE AXIAL GAUGE: BOUNDARY TERMS AND POINCARÉ INVARIANCE *

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Quantum chromodynamics is investigated in the axial gauge with particular attention to boundary terms. The Poincaré algebra has anomalies and closes only in the gauge invariant physical sector. A simple method is proposed for dealing with the boundary condition $E_3^a \rightarrow 0$ as $x_3 \rightarrow \infty$ in a Hamiltonian formalism. It is found that the gauge potential has physically meaningful non-trivial asymptotic behavior related to instanton effects. This presents an obstacle to perturbation theory.

1. Introduction

Previous investigations [1] of quantum chromodynamics (QCD) in the axial gauge $A_3^a = 0$ have remained obscure with respect to boundary conditions. The purpose of the present paper is to clarify certain problems associated with asymptotic behavior. We are motivated by recent work [2] in two-dimensional QCD which has resolved certain complications with Poincaré and gauge invariance in similar circumstances. Here we investigate the nature of analogous problems as well as some new ones that arise in 4 dimensions. Furthermore it is now apparent that the traditional Coulomb gauge condition $\nabla \cdot A^a = 0$ does not uniquely determine the potentials in non-Abelian gauge theories [3]. The fact that the axial gauge does not present this problem provides us with a further motivation.

The theory is defined by the usual Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi}(\frac{1}{2}\overleftrightarrow{D} - m)\psi;$$

$$D_\mu = \partial_\mu - ig\frac{1}{2}\lambda^a A_\mu^a,$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c, \quad (1.1)$$

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Dynamical symmetry breaking of non-Abelian gauge symmetries*†

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We use the Schwinger mechanism to generate a dynamical breakdown of non-Abelian gauge symmetries. Such a breakdown is implemented by using bound-state Goldstone bosons which violate the global invariance associated with the gauge group. The usefulness of this realization of the Schwinger mechanism is that it eliminates the necessity of introducing elementary scalar particles, and, furthermore, it is a viable possibility in a pure Yang-Mills theory. Fermion and vector-meson mass relations in the pole approximation are discussed and compared with the zeroth-order mass relations in the familiar Higgs models. In addition, we obtain consistent solutions to the Bethe-Salpeter equation for the bound state in the weak-coupling limit, which yield finite vector-boson masses in the pole approximation. However, there are group-theoretical constraints which limit the possible groups and representations.

1. INTRODUCTION

Since the success of 't Hooft and Veltman¹ and Lee and Zinn-Justin² in showing the renormalizability of massless Yang-Mills Lagrangians, there has been considerable interest³ in field theories possessing local non-Abelian gauge symmetries of vector mesons. Hope has been raised that weak and electromagnetic interactions may be united by using this class of field theories.³ More recently, models have been introduced which try to incorporate strong-interaction dynamics into this scheme.⁴ The key element in this approach is the introduction of spontaneous symmetry breakdown, so that *S*-matrix elements do not exhibit all the formal symmetry properties of the Lagrangian. Excluding anomalies, we know that such circumstances demand the appearance of massless Goldstone excitations. However, since the Lagrangians are invariant under local gauge transformations, the Goldstone excitations combine with the originally massless vector mesons associated with the gauge symmetry to produce massive vector mesons.

Some time ago, Nambu and Jona-Lasinio⁵ in their pioneering work showed that a theory may possess symmetry-violating solutions by using a four-Fermi interaction to produce a chiral-symmetry-breaking bound-state Goldstone boson. The next year Schwinger⁶ made the crucial observation that it is possible to have a pole in the polarization tensor of vector mesons at $q^2 = 0$. This pole leads to spontaneous symmetry breakdown by giving a mass⁷ to otherwise massless vector mesons, a process known as the Schwinger mechanism. In Schwinger's example,⁸ two-dimensional quantum electrodynamics, the model is soluble

and the pole in the polarization tensor can be seen explicitly, but the Goldstone theorem is evaded due to anomalies which violate axial-vector conservation.⁹ Therefore symmetry violation leading to massive particles is due to anomalies, rather than bound-state Goldstone bosons.

Most of the current resurgence of interest in spontaneous symmetry breakdown has been centered around a different realization of the Schwinger mechanism, the popular Higgs mechanism.¹⁰ In this procedure scalar particles in the Lagrangian are assigned symmetry-violating vacuum expectation values, which supply the *raison d'être* for the pole in polarization tensor. However, it is desirable to produce models without fundamental scalar particles in the Lagrangian because the scalars which are needed for the Higgs case have not been observed, and furthermore, it is difficult to arrange asymptotic freedom when scalar particles are present.

Two techniques, mentioned above, can be used to avoid the use of the conventional Higgs-Kibble mechanism¹¹ in Yang-Mills theories. One, which may be peculiar to two dimensions,¹² is to employ anomalies, which destroy current conservation, to provide the pole in the polarization tensor of the vector particles. The other is to assume that the integral equations of the theory admit symmetry violation such that there are bound-state Goldstone bosons which generate the desired pole. Using the latter method, Jackiw and Johnson¹³ and Cornwall and Norton,¹⁴ extending the work of Nambu and Jona-Lasinio,⁵ showed that it is consistent for a gauge theory of fermions interacting with massless Abelian vector mesons to have symmetry-breaking solutions such that there is a finite physical mass for the fermion and for the

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THE GLUON PROPAGATOR

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ABSTRACT

The vector gluon propagator of quantum chromodynamics is investigated by means of the gauge technique. The technique provides a self consistency equation for the propagator from which one may recover the asymptotic freedom behaviour $p^2 \Delta \rightarrow [1 + \gamma \ln(-p^2/\mu^2)]^{-1}$ in the ultraviolet regime, $p^2 \gg \mu^2$. The corresponding behaviour in the infrared limit $p^2 \ll \mu^2$ is found to be $p^2 \Delta \sim [\ln(-p^2/\mu^2)]^{-2}$.

1. INTRODUCTION

It is becoming increasingly evident (Field, 1978) that perturbative computations in quantum chromodynamics (QCD) using effective coupling constants give reasonable agreement with the experimental data for large momenta. The ability to perform and believe such calculations is predicated on the phenomenon of asymptotic freedom (Gross and Wilczek, 1973; Politzer, 1973) for the ultra-violet regime whereby the effective coupling constant $g(p^2)$, for $p^2 \rightarrow \infty$, behaves as

$$g^{-2}(p^2) \sim g^{-2}(-\mu^2) + c_{\infty} \ln(-p^2/\mu^2) \quad (1)$$

with $c_{\infty} > 0$ and μ representing a renormalisation mass scale. An equivalent restatement is that the gluon propagator has the high energy behaviour.

$$\Delta(p^2) \sim p^{-2} [1 + c_{\infty} g^2(-\mu^2) \ln(-p^2/\mu^2)]^{-1} \quad (2)$$

What happens in the infrared to $g(p^2)$ or $\Delta(p^2)$ is much less clear and has become the subject of some debate. A widespread conjecture is that as $p^2 \rightarrow 0$, $\Delta(p^2) \sim p^{-4}$ in order for colour confinement to take place by a linear potential or colour flux tube. Largely because of the quark confinement question, a substantial amount of theoretical research has gone into investigating the infrared characteristics of nonabelian theories (Cornwall and Tiktopoulos, 1976; Frenkel et al., 1977), both in a perturbative framework (Appelquist et al., 1976; Konetschny, 1978) - where, according to the order of summation, new effects may or may not show up - and also on the basis of non-perturbative techniques (Pagels, 1977). As yet no definitive, universally accepted result has emerged from these various attempts.